
KOÇ UNIVERSITY
MATH 102 - CALCULUS
Midterm I March 18, 2009

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: _____

Signature: _____

Section (Check One):

- Section 1: Aybike Özer M-W (15:30) _____
- Section 2: Burak Özbağcı M-W (14:00) _____
- Section 3: E. Şule Yazıcı Tu-Th(11:00) _____
- Section 4: E. Şule Yazıcı Tu-Th(14:00) _____
- Section 5: Sinan Ünver M-W(11:00) _____

PROBLEM	POINTS	SCORE
1	20	
2	25	
3	20	
4	15	
5	20	
TOTAL	100	

Problem 1. Calculate the following limits (specify infinite limits):

(a) (5 pts) $\lim_{x \rightarrow \pi^-} \frac{x^2 - 8x + 1}{\sin x}$

(b) (5 pts) $\lim_{x \rightarrow \infty} \sqrt{x^2 + 1} - x$

(c) (5 pts) $\lim_{x \rightarrow 0} \ln(\tan^2 x)$

(d) (5 pts) $\lim_{x \rightarrow 1} \frac{x^3 - x}{x^2 - 1}$

Problem 2. a. (10 pts) Let

$$f(x) = \begin{cases} x^2 + 3x + 6 & \text{for } x \leq 1 \\ 2x + c^2 & \text{for } x > 1 \end{cases}$$

where c is a fixed real number.

Find all the values of c such that f is continuous for all real numbers.

b. (15 pts) Let $f(x) = x^3 - x^2 + x$. Show that there exists a real number c such that $f(c) = 10$.

Problem 3. Find the derivative of the following function f .

(a) (5 pts) $f(x) = \sin(x^2)$

(b) (5 pts) $f(x) = \frac{1}{\sqrt{1 + \tan x}}$

(c) (5 pts) $f(x) = \frac{\cos x}{2 + \sin x}$

(d) (5 pts) $f(x) = 1 + x^2 e^{-x}$

Problem 4. (15 pts) Let f and g be two differentiable functions. Assume $r = fog$ and;
 $g'(1) = 3$, $g(1) = 5$ and $f'(5) = 11$. Compute

$$\lim_{h \rightarrow 0} \frac{r(h+1) - r(1)}{h}$$

Problem 5. (20 pts) Use implicit differentiation to find the equation of the tangent line at the point $(3, 1)$ to the curve defined by the equation

$$2(x^2 + y^2)^2 = 25(x^2 - y^2)$$