

**Question 1: ( 20 Points)**

Evaluate the following integrals :

(a)  $\int \sin^3\left(\frac{x}{2}\right)dx$

(b)  $\int \sin(2x) \cos(2x)dx$

(c)  $\int (\tan^2(x) + \tan(x))dx$

**Question 2: ( 15 Points)**

Evaluate

$$\int \frac{-x^3 + 2x^2 - 3x + 6}{(x^2 + 1)(x - 1)^2} dx$$

**Question 3: ( 10 Points + 5 Points):**

Show that  $\int e^{-\sqrt{x}} dx = -2\sqrt{x}e^{-\sqrt{x}} - 2e^{-\sqrt{x}} + c$  by

- (a) integrating the left side using a substitution and/or Integration by Parts.
- (b) using any other method

**Question 4: ( 10 Points)**

Find the derivative of  $f(x) = (\sin x)^{\tan x}$

(Hint: Express the function  $f(x)$  as  $e^{h(x)}$ , or take the logarithm of both sides and use implicit differentiation)

**Question 5: ( 10 Points)**

Evaluate the following integral:

$$\int \frac{2dx}{\sqrt{6x - 2x^2 + 7}}$$

**Question 6: ( 10 Points)**

The graph of  $f(x) = x^3 + bx^2 + cx + d$  is increasing in the interval  $x < -1$ , decreasing in the interval  $-1 < x < 3$ , and increasing in the interval  $x > 3$ . The graph is concave down for  $x < 1$ , and concave up for  $x > 1$ . The inflection point is on the  $x$ -axis. Find the constants  $b$ ,  $c$ , and  $d$ .

**Question 7: ( 10 Points)**

Find an expression for the volume of the solid generated by revolving the region bounded by  $y = \tan(x)$ ,  $y = -1$ ,  $x = 0$ , and  $x = \pi/4$  about the line  $y = -1$ . Do NOT evaluate the expression.

**Question 8: ( 5 Points)**

Find  $\lim_{x \rightarrow \infty} \frac{x^3}{3^x}$

**Question 9: ( 5 Points)**

Find  $f'(3)$  for any function  $f$  whose domain is  $\mathbb{R}$  satisfying the inequality  $|f(x) + x^2 - 2x + 4| \leq \sin^2(x - 3)$  for all real numbers  $x$ .

**Question 10: ( 10 Points)**

Find the length of the curve  $y = f(x)$  between  $-3 \leq x \leq -2$ , if  $f'(x) = \sqrt{x^2 - 1}$ .