
KOÇ UNIVERSITY
MATH 102 - CALCULUS
Midterm I March 17, 2006

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: _____

Signature: _____

Section (Check One):

- Section 1: S. Küçükçifci _____
Section 2: T. Albu (9:30) _____
Section 3: E.Ş. Yazici (15:30) _____
Section 4: T. Albu (12:30) _____
Section 5: E.Ş. Yazici (11:00) _____

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

Problem 1. Calculate the following limit or show that it does not exist:

(a) (5 pts) $\lim_{x \rightarrow 2} \frac{|4 - 2x|}{3x - 6} (1 - 2x)$

(b) (5 pts) $\lim_{x \rightarrow -\infty} \frac{-5x^4 + x^2 - 3}{3x^4 - 4x + 7}$

(c) (5 pts) $\lim_{x \rightarrow 0} \frac{\sin^2(15x)}{\tan(3x) \tan(5x)}$

(d) (5 pts) $\lim_{x \rightarrow 1} \frac{x^{10} - 2x - 3}{x^2 + 4x - 5}$

Problem 2. Let

$$f(x) = \begin{cases} 2x^3 - 3x + 5 & \text{for } x \leq 0 \\ 7 \sin^2(3x) + a^2 & \text{for } x > 0 \end{cases}$$

where a is a fixed real number.

- (a) (6 pts) Show that f is continuous on $(-\infty, 0) \cup (0, \infty)$ for any value of a .
- (b) (8 pts) Find all the values of a such that f is continuous at $x = 0$.
- (b) (2 pts) Find all the values of a such that f is discontinuous at $x = 0$.
- (c) (4 pts) Find all the values of a such that f is continuous on $(-\infty, \infty)$.

Explain all your answers.

Problem 3. Find the derivative of the following function f . Simplify your answers.

(a) (5 pts) $f(x) = \sqrt[5]{x^3} + \frac{1}{\sqrt[3]{x^5}}$

(b) (5 pts) $f(x) = \frac{3x}{\sqrt{2-x^2}}$

(c) (5 pts) $f(x) = \cos(3x) \sin^3 x$

(d) (5 pts) $f(x) = (3 + \tan^4 x)^5$

Problem 4. Consider the function

$$f(x) = \sqrt{2x + 7}$$

- (a) (3 pts) What is the domain of f ?
- (b) (3 pts) What is the range of f ?
- (c) (8 pts) Find the derivative f' of f using the definition of the derivative.
- (d) (4 pts) Find the derivative f' of f using the differentiation rules, and compare the result with the one you found in (c).
- (e) (2 pts) What is the domain of f' ?

Problem 5.

(a) (12 pts) Use implicit differentiation to find $\frac{dy}{dx}$ if

$$y^3 \sin y = 2x^3 + 3y^2 - 5$$

(b) (8 pts) Find the equation of the tangent line at the point $P(0, 1/2)$ to the curve defined by the equation

$$y = \sin(2x + (\pi/6))$$