KOÇ UNIVERSITY

MATH 102 - CALCULUS

Midterm I April 13, 2010

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: _____

Surname: _____

Signature: _____

Section (Check One):

Section 1: Sultan Erdoğan M-W (14:00) ___
Section 2: Benjamin Smith M-W (17:00) ___
Section 3: Selda Küçükçifçi T-Th (11:00) ___
Section 4: Selda Küçükçifçi T-Th (14:00) ___
Section 5: Sultan Erdoğan T-Th(12:30) ___

PROBLEM	POINTS	SCORE
1	14	
2	24	
3	8	
4	18	
5	20	
6	16	
TOTAL	**100**	

1. Let $f(x) = \dfrac{x-1}{x^2+2x-3}$

(a) (6 points) Find the domain and the range of $f(x)$.

(b) (8 points) Let $g(x)$ be the function that corresponds to the graph obtained by first reflecting the graph of $f(x)$ about the x-axis, then shifting the reflected graph 3 units right and finally shifting 1 unit up the graph obtained after the horizontal shift. Determine $g(x)$.

2. (24 points) Evaluate the following limits, if they exist. If the limit is infinity, specify whether it is positive or negative infinity. (Do not use l'Hospital's Rule):

(a) $\lim_{x\to 0} \dfrac{\sin x}{\sqrt{x+3}-\sqrt{3-2x}}$

(b) $\lim_{x\to 0} \dfrac{x^2-2x}{x^3+x^4}$

(c) $\lim_{x\to 1} \dfrac{x^{101}-1}{x-1}$ (Hint: Use definition of derivative.)

(d) $\lim_{x\to 1} f(x)$, if $4\arctan x \le f(x) \le \pi \ln(e/x)$ for $x > 0$.

3. (8 points) Find the constants a and b that makes the following function continuous for all real numbers:

$$f(x) = \begin{cases} x^2 - x \ , & x \leq 0 \\ ax + b \ , & 0 < x \leq 1 \\ \dfrac{x^2 - 1}{x - 1} \ , & 1 < x \end{cases}$$

4. (18 points) Consider the function $f(x)$ that satisfies the given conditions:

$\lim_{x\to 0^-} f(x) = \infty$, $\lim_{x\to 0^+} f(x) = -\infty$,

$\lim_{x\to 1^-} f(x) = -\infty$, $\lim_{x\to 1^+} f(x) = \infty$,

$\lim_{x\to \pm\infty} f(x) = 0$,

$f'(x) < 0$ on $(1/2, 1) \cup (1, \infty)$,

$f'(x) > 0$ on $(-\infty, 0) \cup (0, 1/2)$,

$f''(x) < 0$ on $(0, 1)$,

$f''(x) > 0$ on $(-\infty, 0) \cup (1, \infty)$.

(a) Find the horizontal and vertical asymptotes of $f(x)$.

(b) Determine where f is increasing or decreasing.

(c) Determine where f is concave upward or concave downward.

5. (20 points) Find the derivative of the following function f.

(a) $f(x) = 1 + 2x^2 + \dfrac{5e^x}{x}$

(b) $f(x) = \sqrt{1 - \sqrt{x}}$

(c) $f(x) = \ln \dfrac{(2x+1)^3}{(x-1)^4}$

(d) $f(x) = \left(\frac{1}{x}\right)^x$

6. (16 points) (a) Use implicit differentiation to find $\dfrac{dy}{dx}$ if

$$\sin(x^2 + y) = -e^y \cos(x^2).$$

(b) Find an equation of the tangent line to the curve $y = \dfrac{x-3}{x^2-3}$ at the point $(2, -1)$.