# KOÇ UNIVERSITY

# MATH 106 - CALCULUS I

# Midterm II December 14, 2011

## Duration of Exam: 90 minutes

**INSTRUCTIONS: CALCULATORS ARE NOT ALLOWED FOR THIS EXAM**. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

**Surname, Name:** _____

**Signature:** _____

**Section (Check One):**

Section 1: S. Küçükçifçi (Mon-Wed-Fri 12:30) ___
Section 2: E. Ş. Yazıcı(Mon-Wed-Fri 14:30) ___
Section 3: S. Küçükçifçi (Mon-Wed-Fri 10:30) ___
Section 4: E. Ş. Yazıcı(Mon-Wed-Fri 11:30) ___
Section 5: T. Etgü (Tue-Thu 12:30) ___

| PROBLEM | POINTS | SCORE |
|---|---|---|
| 1 | 23 | |
| 2 | 15 | |
| 3 | 18 | |
| 4 | 15 | |
| 5 | 12 | |
| 6 | 9 | |
| 7 | 9 | |
| **TOTAL** | **101** | |

**1.** Compute the limits in a and b. Specify any infinite limits.

a) (9 points) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\sin x}{x^5}\right)=$

b) (9 points) $\lim_{x\to\infty}(x+e^x)^{1/x}=$

c) (5 points) If the linearization of $f(x)$ at x=0 is $L(x)=ax+b$, find $\lim_{x\to 0}\frac{2f(x)}{ax+b}$ .

**2.** (15 points) While a race car is moving at a speed of 10 m/s on a straight track, the driver's friend is watching it 120 m away from the track. How fast is the distance between the friend and the car changing when they are 150 m away from each other?

**3.** (18 points) Sketch the graph of a function $f$ satisfying the following properties. Identify local maximum, local minimum and inflection points on the graph, if they exist.

- domain($f$)= $\mathbb{R} \setminus \{3\}$, $\quad$ $f$ is twice differentiable on its domain.
- $\lim_{x \to 3^-} f(x) = -\infty$, $\quad$ $\lim_{x \to 3^+} f(x) = +\infty$
- $\lim_{x \to -\infty} f(x) = 1$, $\quad$ $\lim_{x \to +\infty} f(x) = -2$
- $f'(x) > 0$ on $(-\infty, 1)$ and $(5, \infty)$

  $f'(x) < 0$ on $(1, 3)$ and $(3, 5)$
- $f''(x) > 0$ on $(-\infty, 0)$ and $(3, 7)$

  $f''(x) < 0$ on $(0, 3)$ and $(7, \infty)$

**4.** (15 points) Find the critical points, the intervals where function is increasing and decreasing, local minimum, maximum and inflection points, if they exist and the intervals where the function is concave up and down for the function $f(x) = x^{2/3}\left(\frac{5}{2} - x\right)$.

**5.** (12 points) Find the absolute maximum and minimum values of $f(x) = \dfrac{x^2+4}{2x}$ on $[1, \infty)$, if they exist.

**6.** (9 points) If $f$ is differentiable and $f(0) < f(1)$ then there is a number $c$ with $0 < c < 1$ such that $f'(c) > 0$.

**7.** (9 points) Find the antiderivative $F$ of $f(x) = \sin x + \dfrac{1}{1+x^2}$ satisfying $F(0) = 2$.