KOÇ UNIVERSITY

MATH 106 - CALCULUS I

Midterm I October 22, 2014

Duration of Exam: 75 minutes

INSTRUCTIONS: CALCULATORS ARE NOT ALLOWED FOR THIS EXAM.

No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

| Surname, Name: | |
|---|--|
| Signature: | |
| Section (Check One): | |
| Section 1: E. S. Yazıcı (Mon-Wed 16:00) | |
| Section 2: E. Ş. Yazıcı (Mon-Wed 13:00) | |
| Section 3: Doğan Bilge (Mon-Wed 11:30) | |
| Section 4: Doğan Bilge (Mon-Wed 14:30) | |
| Section 5: Altan Erdoğan (Tu-Th 16:00) | |

| PROBLEM | POINTS | SCORE |
|---------|--------|-------|
| 1 | 40 | |
| 2 | 16 | |
| 3 | 15 | |
| 4 | 15 | |
| 5 | 15 | |
| TOTAL | 101 | |

1. Compute the following limits if they exist. You are not allowed to use the L'Hospital Rule. Specify any infinite limits.

a) (8 points)
$$\lim_{x\to 0} \frac{\sqrt{\cos x + x^2} - \sqrt{\cos x - x^2}}{\sin^2 x} =$$

b) (8 points)
$$\lim_{x\to 0} \frac{1}{|x-1|-|x+1|} =$$

c) (8 points)
$$\lim_{x \to \infty} \frac{x - x^3}{x^2 + 5} =$$

d) (8 points)
$$\lim_{x\to 1} \frac{x+1}{1-x^4} =$$

e) (8 points)
$$\lim_{x \to 1} \frac{\ln x}{x - 1} =$$

2. Differentiate the following functions. (Note: $\sin^{-1} x = \arcsin x$)

a) (8 points)
$$f(x) = \ln \frac{\sin(2x)}{\sin^{-1} x}$$

b) (8 points) $f(x) = \cos(2^{x^2})$

3. (15 points) Let $f(x) = e^x$ and $g(x) = \sin(\ln x)$. Find equations for two distinct parallel lines l_1 and l_2 where; l_1 is tangent to f at x = 0 and l_2 is tangent to g.

4. (15 points) Show that there exist a sphere with radius $r \in (0,1)$ and a cube with side $r + \frac{1}{2}$ with the same volume. (Volume of the sphere with radius r is $\frac{4}{3}\pi r^3$, and the volume of a cube with side a is a^3).

5. (15 points) Let x > 0. Find y' using implicit differentiation if $x^y + y - 1 = 0$.