
KOÇ UNIVERSITY

MATH 106 - CALCULUS

Final Exam June 1, 2007

Duration of Exam: 135 minutes

INSTRUCTIONS: Calculators may not be used on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print and sign your name, and indicate your section below.**

Surname, Name: _____

Signature: _____

Section (Check One):

Section 1 - 11:30 _____

Section 2 - 14:30 _____

PROBLEM	POINTS	SCORE
1	20	
2	18	
3	24	
4	13	
5	20	
6	13	
TOTAL	108	

1. Evaluate the limits in (a)–(c). Specify infinite limits and if the limit does not exist give the reason.

(a) (5 points) $\lim_{x \rightarrow 0} \frac{2^{\sin x} - 1}{e^x - 1}$

(b) (5 points) $\lim_{x \rightarrow 0} (1 + 2x)^{1/x}$

(c) (5 points) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{\sqrt{x+1} - 1}$

(d) (5 points) Give an example of a function $f(x)$, where $f(x)$ is continuous at $x = 1$, but $f(x)$ is not differentiable at $x = 1$.

2. (a) Find $\frac{dy}{dx}$ in (i)-(ii), where:

(i) (5 points) $y = \frac{(x^2 + 1)}{\cos 2x}$

(ii) (5 points) $y = (\sin x)^x$

(b) (8 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose and what are its dimensions?

3. Calculate the following integrals.

(a) (6 points) $\int \frac{\tan(\ln x)}{x} dx$

(b) (6 points) $\int_0^{\pi/2} \cos^2(3x) dx$

(c) (6 points) $\int \frac{2x+1}{x^2-7x+12} dx$

(d) (6 points) $\int_0^1 \frac{e^x}{e^x-1} dx$

4. Consider the function $f(x) = x - \ln x$.

(a) (5 points) Find the intervals on which the function f is increasing and decreasing.

(b) (5 points) Determine where the graph of f is concave up and where it is concave down.

(c) (3 points) Determine the local extreme values of the function f .

5. Which of the following series converge and which diverge in (a)-(c)? Give reasons for your answers.

(a) (5 points) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

(b) (5 points) $\sum_{n=1}^{\infty} \frac{n^n}{2^{(n^2)}}$

(c) (5 points) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3 + 1}$

(d) (5 points) Find the values of x for which the series $\sum_{n=0}^{\infty} 3 \left(\frac{x-1}{2} \right)^n$ converges.

6. (a) (8 points) Find the Maclaurin series of $f(x) = \sin 3x$.

(b) (5 points) Give an example of a series $\sum_{n=1}^{\infty} a_n$ which converges, but $\sum_{n=1}^{\infty} |a_n|$ does not converge.