

Midterm 1.

Math 106 (23.04.06)

1. For which value of b is the function

$$g(x) = \begin{cases} x + 2, & \text{if } x \leq 0, \\ b^{\frac{\sin(2x)}{x}}, & \text{if } x > 0 \end{cases}$$

continuous at the point $x = 0$.

2. Use the l'Hopital's rule to find the limit

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$$

3. State the Rolle's theorem and use it to show that the function $f(x) = x^4 - 4x + 1$ has exactly two zeros on the real line.

4. Sketch the graph of the function

$$f(x) = \frac{x}{x - 2}.$$

Indicate any horizontal or vertical asymptotes and write the equation of the tangent line to this graph at the point $(3, 3)$.

5. Find the extreme values of the function and where they occur

$$a) \quad f(x) = 2x^2 - 8x + 9, \quad b) \quad g(x) = x + \frac{4}{x},$$

$$c) \quad \text{find absolute minimum of } g(x) \quad \text{for } 0 < x < \infty$$

6. Find antiderivatives of the following functions

$$a) \quad \sin(2x), \quad b) \quad \frac{1}{x^4} \quad c) \quad \cos^2(3x)$$