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KOÇ UNIVERSITY  
MATH 106 - CALCULUS  
Midterm II                      April 9, 2007  
Duration of Exam: 75 minutes

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**INSTRUCTIONS:** Calculators may not be used on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print and sign your name, and indicate your section below.**

Surname, Name: \_\_\_\_\_

Signature: \_\_\_\_\_

Section (Check One):

Section 1 - 11:30                      \_\_\_\_\_

Section 2 - 14:30                      \_\_\_\_\_

PROBLEM	POINTS	SCORE
1	27	
2	30	
3	22	
4	26	
<b>TOTAL</b>	<b>105</b>	

1. (a) (10 points) Find  $\frac{dy}{dx}$ , where  $x^2y^2 - 2x = 4 - 4y$ .

(b) (7 points) Evaluate the limit  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2}$ .

(c) (10 points) Find the slope of the tangent line to the curve  $F(x)$  at  $x = 1$ , where

$$F(x) = \int_1^{x^2} \sqrt{t^2 + 1} \, dt.$$

- 2.** (30 points) Sketch the graph the function  $f(x) = \frac{x^2 + 1}{x^2 - 1}$ . Draw all asymptotes, write their equations, and include the coordinates of any local extrema and inflection points.

3. (a) (12 points) Find the point on the parabola  $y = 9 - x^2$  closest to the point  $(3, 9)$ .

(b) (10 points) Show that if  $f'' < 0$  throughout an interval  $[a, b]$ , then  $f'$  has at most one zero in  $[a, b]$ .

4. (a) (10 points) Let  $f(x) = x^3$ . Find  $\int_0^2 f(x)dx$  using upper Riemann sums with subintervals of equal length.  $\left( \text{Hint} : \sum_{k=1}^n k^3 = \left( \frac{n(n+1)}{2} \right)^2 \right)$

(b) (6 points) Evaluate the indefinite integral  $\int (3 \sin x + 4)^5 \cos x \, dx$ .

(c) (10 points) Find the area of the region bounded by the graphs  $y = x^2$ ,  $y = 2 - x$ , and  $y = 0$ .