KOÇ UNIVERSITY

MATH 106

FIRST MIDTERM NOVEMBER 3, 2007

Duration of Exam: 120 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________________

Student ID no: ______________________________

Signature: ______________________________

(Check One):

(Barış Çoskunüzer – 11:30-13:20) : ___
(Barış Çoskunüzer – 14:30-16:20) : ___
(Burak Özbağcı – 14:30-16:20): ___
(Demircan Canadinç 14:30-16:20): ___
(Burak Özbağcı – 10:30-12:20): ___

PROBLEM	1	2	3	4	5	6	7	8	TOTAL
POINTS	15	10	15	10	10	15	10	15	100
SCORE									

Problem 1 (15 pts) Find the limit if it exists.

a) $\lim\limits_{x\to\infty} x\tan\frac{1}{x}$

b) $\lim\limits_{x\to-\infty} \frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}}$

c) $\lim\limits_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$

d) $\lim\limits_{x\to\infty} \frac{x^2+x+1}{x\sin x-2}$

e) $\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin^4 x-1}{\cos^3 x}$

Problem 2a (5 pts) Find $\frac{dy}{dx}$ at $(x, y) = (2, 2)$ if $x^3 + y^3 = 16$.

Problem 2b (5 pts) Find $\frac{d^2y}{dx^2}$ at $(x, y) = (2, 2)$ if $x^3 + y^3 = 16$.

Problem 3a (8 pts) Find the points on the ellipse $4x^2 + y^2 = 9$ which is farthest from $(1, 0)$. Prove your answer.

Problem 3b (7 pts) Find the points on the ellipse $4x^2 + y^2 = 9$ which is nearest to $(1, 0)$. Prove your answer.

Problem 4 (10 pts) Show that there exists a point $c \in (0, 1)$ such that

$$f(x) = |x^3 - 3x + 1|$$

is not differentiable at $x = c$.

(HINT: Do not try to determine c, instead use the Intermediate Value Theorem!)

Problem 5 (10 pts) Let $f(x) = x^4 + 4x^3$.

a) Find all the critical points of f.

b) Find the intervals on which f is increasing & decreasing.

c) Find the intervals on which f is concave up & concave down.

d) Find all the inflection points of f.

e) Sketch the graph of f.

Problem 6 (15 pts) Given $x + 2\sqrt{x^3} = t^2 + t$ and $y\sqrt{t+1} + 2t\sqrt{y} = 4$.

Find an equation of the line tangent to the graph of this parametric curve at $t = 0$.

Problem 7 (10 pts) Suppose that $f(x)$ is a function defined on the interval $(-4, 3)$, which is differentiable except at $x = -1$ such that

$\lim_{x \to -4^+} f(x) = -2 \quad \lim_{x \to 3^-} f(x) = 0; \quad f(-3) = f(1) = 0; \quad f(-2) = 2;$

$f(-1) = 1; \quad f(0) = 3; \quad f(2) = -1$; the first derivative of f is positive on the intervals $(-4, -2), (-1, 0)$ and $(2, 3)$ and negative on $(-2, -1)$ and $(0, 2)$ with $f'(-2) = f'(0) = f'(2) = 0$; the second derivative of f is positive on $(-4, -1)$ and $(-1, 1)$ and negative on $(1, 3)$.

a) Sketch the graph of f

b) List all the critical points of f

c) Find all the inflection points of f

d) Find the absolute maximum of f on $(-4, 3)$ if it exists

e) Find the absolute minimum of f on $(-4, 3)$ if it exists

Problem 8 (15 pts) For which values of a, m and b is the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \leq x \leq 2 \end{cases}$$

continuous for every x in $[0, 2]$ and differentiable for every x in $(0, 2)$?