SOLUTIONS for Midterm 1
Math 106-Fal

1\. a. $\lim_{x\to\infty} x\cdot\tan\frac{1}{x} = \lim_{x\to\infty}\frac{\tan\frac{1}{x}}{\frac{1}{x}} \underset{\text{L'Hopital}}{=} \lim_{x\to\infty}\frac{\frac{1}{\cos^2\frac{1}{x}}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = 1$

b. $\lim_{x\to-\infty}\frac{x^{1/3}-x^{1/5}}{x^{1/3}+x^{1/5}} = \lim_{x\to-\infty}\frac{x^{1/3}(1+x^{-2/15})}{x^{1/3}(1+x^{-2/15})} = 1$

c. $\lim_{x\to1^+}\frac{\sqrt{2x}\,(x-1)}{|x-1|} = \lim_{x\to1^+}\frac{\sqrt{2x}\,\cancel{(x-1)}}{\cancel{x-1}} = \sqrt{2}$

d. $\lim_{x\to\infty}\frac{x^2+x+1}{x\sin x-1} =$ No limit $\quad x=\left(2n\pi+\frac{\pi}{2}\right) \Rightarrow \sin x = 1 \Rightarrow \lim\ +\infty$

$x=\left((2n+1)\pi+\frac{\pi}{2}\right) \Rightarrow \sin x=-1 \Rightarrow \lim\ -\infty$

e. $\lim_{x\to\frac{\pi}{2}}\frac{\sin^4x-1}{\cos^3x} \underset{\text{L'Hopital}}{=} \lim_{x\to\frac{\pi}{2}}\frac{4\sin^{\cancel{3}2}x\cdot\cancel{\cos x}}{3\cos^{\cancel{2}}x\cdot\cancel{\sin x}} \underset{\text{L'Hopital}}{=} \lim_{x\to\frac{\pi}{2}}\frac{4\sin^2x}{3\cos x} =$ No limit

$\lim_{x\to\frac{\pi}{2}^+}\frac{4\sin^2x}{3\cos x} = -\infty$

$\lim_{x\to\frac{\pi}{2}^-}\frac{4\sin^2x}{3\cos x} = +\infty$

2\. a. $x^3+y^3=16 \Rightarrow 3x^2+3y^2y'=0 \Rightarrow y'=\frac{-3x^2}{3y^2}=-\frac{x^2}{y^2}$ at $(2,2) \Rightarrow y'=-1$

b. $6x+6y(y')^2+3y^2y''=0 \Rightarrow y''=\frac{-(6x+6y(y')^2)}{3y^2} = -\frac{(12+12)}{3\cdot4} = -2$

3\. (a+b) $4x^2+y^2=9 \Rightarrow y^2=9-4x^2$

$d=\sqrt{(x-1)^2+y^2}$

$d(x)=\sqrt{(x-1)^2+(9-4x^2)}$

maximize $f(x)=(x-1)^2+(9-4x^2)$ on $\left[-\frac{3}{2},\frac{3}{2}\right]$

$=x^2-2x+1+9-4x^2$

$=-3x^2-2x+10$

$f'(x)=-6x-2 \Rightarrow x=-\frac{1}{3}$

critical pts: $-\frac{3}{2}, -\frac{1}{3}, \frac{3}{2}$

$d\left(-\frac{3}{2}\right)=\sqrt{\frac{25}{4}}=\frac{5}{2}$

$d\left(-\frac{1}{3}\right)=\sqrt{\frac{16}{9}+\frac{77}{9}}=\sqrt{\frac{93}{9}}=\boxed{\sqrt{\frac{31}{3}}}$ max

$d\left(\frac{3}{2}\right)=\boxed{\frac{1}{2}}$ min

4. ~~By~~ $f(x) = x^3 - 3x + 1$

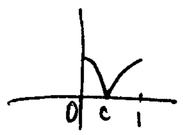
$f(0) = 1$
$f(1) = -1$
$\Rightarrow$ by IVT, $\exists$ there is a c in $(0,1)$ $f(c) = 0$

Near c

$|x^3 - 3x + 1| = \begin{cases} +(x^3-3x+1) & x \le c \\ -(x^3-3x+1) & x > c \end{cases}$

$f'(x)$ $x \to c^+$
$-3x^2 - 3$

$f'(x)$ $x \to c^-$
$3x^2 - 3$



$(-3c^2 - 3) \neq (3c^2 - 3)$ $(c \neq \pm 1)$

$\Rightarrow$ f' does not exist at c.

5. $f(x) = x^4 + 4x^3$

a. $f'(x) = 4x^3 + 12x^2 = 4x^2(x+3)$ $\Rightarrow$ critical pts $0, -3$

b. f' — + + ; -3, 0

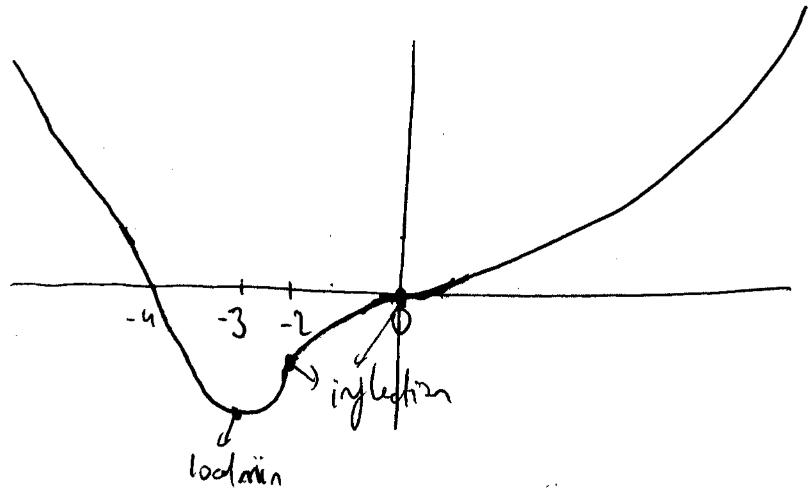
$(-\infty, -3)$ ↘
$(-3, \infty)$ ↗

c. $f'' = 12x^2 + 24x = 12x(x+2)$ f'' + − + ; -2, 0

concave up $(-\infty, -2) \cup (0, \infty)$
concave down $(-2, 0)$

d. f'' change sign at $-2, 0$ $\rightarrow$ inflection pts.

e. No asymptote



6. $x + 2x^{3/2} = t^2 + t$

$y\sqrt{t+1} + 2t\sqrt{y} = 4$

$x' + 2\cdot\frac{3}{2}x^{1/2}\cdot x' = 2t+1 \Rightarrow x' = \frac{2t+1}{1+3\sqrt{x}}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$y'\sqrt{t+1} + y\frac{1}{2\sqrt{t+1}} + 2\sqrt{y} + 2t\cdot\frac{y'}{2\sqrt{y}} = 0 \Rightarrow y' = \frac{-\left(\frac{y}{2\sqrt{t+1}} + 2\sqrt{y}\right)}{\left(\sqrt{t+1} + \frac{t}{\sqrt{y}}\right)}$

$t = 0 \Rightarrow x + 2x\sqrt{x} = 0$

$x(1+2\sqrt{x}) = 0$

$\boxed{x = 0}$

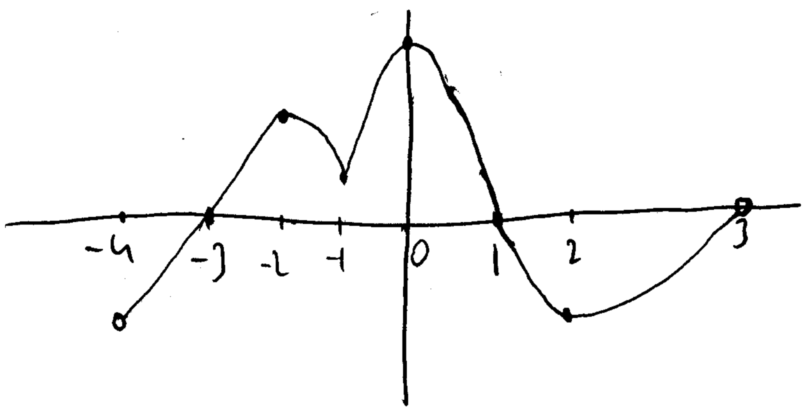
$y\sqrt{t+1} + 2t\sqrt{y} = 4$

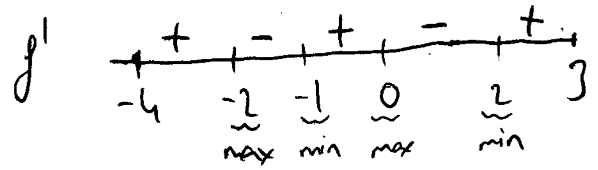
$\boxed{y = 4}$

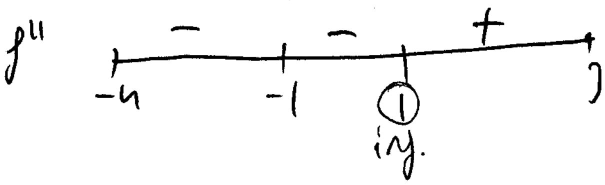
$$\frac{dy}{dx} = \frac{-\left(\frac{y}{2\sqrt{t+1}} + 2\sqrt{y}\right)}{\left(\sqrt{t+1} + \frac{t}{\sqrt{y}}\right)} = \frac{-(2+4)}{1+0} = -6$$

$\frac{y-4}{x-0} = -6 \Rightarrow y = -6x + 4$

7. a.







b. -2, -1, 0, 2

c. 1

d. 0

e. No abs. min.

0, cts at 0 $\Rightarrow$ $3 = -0^2 + 3.0 + a$ $\Rightarrow$ $\boxed{a = 3}$

diff at 1 $\Rightarrow$ $-2x + 3 = m$ at 1

$\Rightarrow$ $\boxed{m = 1}$

cts at 1 $\Rightarrow$ $-x^2 + 3x = x + b$ at 1

$-1 + 3 = 1 + b$ $\Rightarrow$ $\boxed{b = 4}$