Question 1: (15)

Evaluate the integral $\int_0^1 (-\ln(x))dx$

Question 2: (15Points +5Points)

The continuous function $f(x)$ has horizontal tangents at $x=-2$, $x=1$ and $x=3$. It has a vertical tangent at $x=0$, and no tangent at $x=2$. It is concave up for $-3<x<0$ and $5<x<\infty$. It is concave down for $0<x<2$ and $2<x<5$.

a) Identify all critical points, maxima, minima, and inflection points of $f(x)$. Justify your answers.

b) In which intervals is $f(x)$ decreasing, in which intervals is it increasing?

Question 3: (30 Points)

Evaluate the following integrals:

a) $\int \frac{\ln(x)}{\sqrt{x}}\,dx$

b) $\int \frac{2}{(x^2+4)^2}\,dx$

c) $\int \frac{\cos(2x)}{\cos(x)}\,dx$

d) $\int \frac{1}{\sqrt{x^2+3x+2}}\,dx$ (Hint: Use completing the square and trigonometric substitution)

e) $\int \frac{x+3}{(x+1)^2(x+2)}\,dx$

Question 4: (10 Points + 5 Points):

Show that $\int e^{-\sqrt{x}}\,dx = -2\sqrt{x}e^{-\sqrt{x}} - 2e^{-\sqrt{x}} + c$ by

(a) integrating the left side using a substitution and/or Integration by Parts.
(b) using any other method

Question 5: (10 Points)

Find an integral expression for the volume of the solid obtained by rotating the triangle enclosed by the lines $y = -x + 3$, $y = 2$, and the y-axis around

a) the y-axis

b) the line $y = 1$

Do NOT evaluate the integrals!

Question 6: (10Points)

Find the derivative of $f(x) = \int_{x}^{\tan x} \arctan(y)dy$.

Trigonometry Part: (10 Points + 10 Points)

1) Find all $x \in [-\pi, 2\pi]$ so that $\cos(2(x-\pi/3)) = -0.5$

2) Show that

a) $\cos(3x) = 4\cos^3(x) - 3\cos(x)$

b) $\cos^2(\tan^{-1}(x)) = \frac{d}{dx}\tan^{-1}(x)$