MATH 106: Calculus

Final - Fall 2010
Duration : 150 minutes

NAME ____________________

STUDENT ID ____________________

SIGNATURE ____________________

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- Put your name, student ID and signature in the space provided above.
- No calculators or any other electronic devices are allowed.
- This is a closed-book and closed-notes exam.
- Show all of your work; full credit will not be given for unsupported answers.
- Write your solutions clearly; no credit will be given for unreadable solutions.
- Mark your section below.

  SECTION 1 (AZADEH NEMAN, MW 12:30-13:45) ____________________
  SECTION 2 (EMRE MENGI, MW 15:30-16:45) ____________________
  SECTION 3 (SELDA KÜÇÜKÇIFÇI, MW 11:00-12:15) ____________________
  SECTION 4 (SELDA KÜÇÜKÇIFÇI, MW 14:00-15:15) ____________________
  SECTION 5 (AZADEH NEMAN, TUTH 12:30-13:45) ____________________
  SECTION 6 (AZADEH NEMAN, TUTH 15:30-16:45) ____________________

Question 1. Let $f(x) = \frac{1}{\sqrt{x}}$.

(a) Use a linear approximation to estimate $\frac{1}{\sqrt{1.1}}$.

(b) Find the Taylor series for $f(x)$ centered at 1.

(c) Determine the interval and radius of convergence for the Taylor series that you determined in part **(b)**.

Question 2. Recall the power series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$$

provided $|x| < 1$.

(a) Find the power series for $f(x) = \ln(1+x)$ about 0 and its radius of convergence.

(b) Using your solution to part **(a)** find a power series whose sum is $\ln(1.5)$.

Question 3.

(a) Find the x-coordinates of the critical points of the function

$$F(x) = \int_1^{x^2} \frac{\sin t}{t}\, dt$$

defined for $x > 1$.

(b) Evaluate the limit

$$\lim_{h \to 0} \frac{\displaystyle\int_{-h}^{0} \sin t\, dt}{h^2}.$$

Question 4.

(a) Evaluate the indefinite integral given below.

$$\int x \cos(7x)\, dx$$

(b) Is the integral given below convergent or divergent?

$$\int_{-\infty}^{\infty} f(x)\, dx, \quad \text{where} \quad f(x) = \begin{cases} e^{2x} & \text{if } x < 0 \\ e^{-x} & \text{if } x \geq 0 \end{cases}$$

Evaluate the integral if it is convergent. State your reasoning and show the details of your work.

(c) Evaluate the definite integral given below.

$$\int_2^3 \frac{x}{x^3 - 1}\, dx$$

Question 5. Consider the region $\mathcal{R}$ bounded by the curves $y = e^{2x}$ and $y = e^{-x}$, and the vertical lines $x = -1$ and $x = 1$.

(a) Find the area of the region $\mathcal{R}$.

(b) Find the volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

Question 6. In each part indicate whether the series is convergent or divergent. Explain your answer. In particular state explicitly which test you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3 \cdot \sqrt{n+1}}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{5}{k \cdot (\ln k)^6}$

(c) $\displaystyle\sum_{n=0}^{\infty} \sin n$

Question 7. A spherical balloon is inflated so that its volume increases at a constant rate of 1 cm^3/min. Find the rate at which its radius increases when its radius is 20 cm.

Question 8. Indicate whether each of the following sentences about a sequence $\{a_n\}$ or a series $\sum_{n=1}^{\infty} a_n$ is true in general or not necessarily true by circling out the appropriate choice. You do not need to justify your answer.

(a) If the sequence $\{|a_n|\}$ is convergent, then the sequence $\{a_n\}$ is also convergent.
True in general **Not necessarily true**

(b) If the sequence $\{|a_n|\}$ is convergent, then the sequence $\{1/a_n\}$ is divergent.
True in general **Not necessarily true**

(c) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then the series $\sum_{n=1}^{\infty} 2a_n$ is also convergent.
True in general **Not necessarily true**

(d) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then the series $\sum_{n=1}^{\infty} a_n^2$ is also convergent.
True in general **Not necessarily true**