MATH 106: Calculus

Final - Fall 2009

Duration : 180 minutes

NAME ____________________

STUDENT ID ____________________

SIGNATURE ____________________

#1	10	
#2	10	
#3	15	
#4	10	
#5	10	
#6	10	
#7	15	
#8	10	
#9	10	
Σ	100	

- Put your name, student ID and signature in the boxes above.
- No calculators or any other electronic devices are allowed.
- This is a closed-book and closed-notes exam.
- Show all of your work; full credit will not be given for unsupported answers.
- Write your solutions clearly; no credit will be given for unreadable solutions.
- Mark your section below.

SECTION 1 (SULTAN ERDOĞAN DEMIR, MW 11:30-13:20) ____________________

SECTION 2 (SULTAN ERDOĞAN DEMIR, MW 14:30-16:20) ____________________

SECTION 3 (EMRE MENGI, MW 9:30-11:20) ____________________

SECTION 4 (EMRE MENGI, MW 14:30-16:20) ____________________

SECTION 5 (KAZIM BÜYÜKBODUK, TuTh 11:30-13:20) ____________________

SECTION 6 (KAZIM BÜYÜKBODUK, TuTh 14:30-16:20) ____________________

Question 1. Determine whether each of the following series is convergent or divergent. Explain your answer fully.

(a) $\sum_{n=2}^{\infty}(-1)^n \frac{\sqrt[3]{n}}{\ln n}$ $\qquad a_n = (-1)^n \frac{\sqrt[3]{n}}{\ln n} = (-1)^n \frac{n^{1/3}}{\ln n}$

$$\lim_{n\to\infty} \frac{n^{1/3}}{\ln n} \underset{\uparrow \text{ L'HR}}{\overset{\frac{\infty}{\infty}}{=}} \lim_{n\to\infty} \frac{\frac{1}{3}n^{-2/3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{3} n^{1/3} = \infty$$

$\Rightarrow a_{2n} \to +\infty$ and $a_{2n+1} \to -\infty$ as $n\to\infty$

Thus, $\lim_{n\to\infty} a_n$ does not exist.

Then, by Test for Divergence, the series is divergent.

(b) $\sum_{n=1}^{\infty} \frac{\cos\sqrt{n}}{n^3}$ $\qquad a_n = \frac{\cos\sqrt{n}}{n^3} \Rightarrow |a_n| = \frac{|\cos\sqrt{n}|}{n^3}$

Let $b_n = \frac{1}{n^3}$, then since $0 \leqslant |\cos x| \leq 1$ for all x, we have:

$$0 \leqslant \frac{|\cos\sqrt{n}|}{n^3} \leqslant \frac{1}{n^3}, \quad \text{that is,} \quad 0 \leq |a_n| \leq b_n$$

We know that $\sum b_n$ is a p-series with $p=3>1$, so it is convergent. Then, by Comparison Test, $\sum |a_n|$ is convergent. So, $\sum a_n$ is absolutely convergent, which means that $\sum a_n$ is convergent.

(c) $\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{n^3}\right)$, $\qquad a_n = \sin\left(\frac{\pi}{n^3}\right)$, $\quad 0 < \frac{\pi}{n^3} \leqslant \pi$ for all $n \geq 1$.

So, $a_n = \sin\left(\frac{\pi}{n^3}\right) \geq 0$ for all $n \geq 1$. Let $b_n = \frac{1}{n^3}$, $b_n > 0$ for all $n \geq 1$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n^3}\right)}{\frac{1}{n^3}} \underset{\text{L'H.R}}{\overset{\left(\frac{0}{0}\right)}{=}} \lim_{n\to\infty} \frac{\cos\left(\frac{\pi}{n^3}\right).\pi.(-3.n^{-4})}{(-3.n^{-4})}$$

$= \lim_{n\to\infty} \pi.\cos\left(\frac{\pi}{n^3}\right) = \pi$. Since $0 < \pi < \infty$, and $\sum b_n$ is a convergent p-series ($p=3>1$), by Limit Comparison Test, $\sum a_n$ is convergent.

Question 2. In (a) and (b) below, find the indicated <u>area</u> or <u>volume</u> by first expressing it as a definite integral, and then evaluating the definite integral.

(a) The <u>area</u> of the region between $x = y^2 - 6y$ and $x = 4y - y^2$.

Let $f(y) = y^2 - 6y$ and $g(y) = 4y - y^2$. Intersection points:

$f(y) = g(y) \Rightarrow y^2 - 6y = 4y - y^2 \Rightarrow 2y^2 = 10y \Rightarrow 2y(y-5) = 0$

$\Rightarrow y = 0$ or $y = 5$.

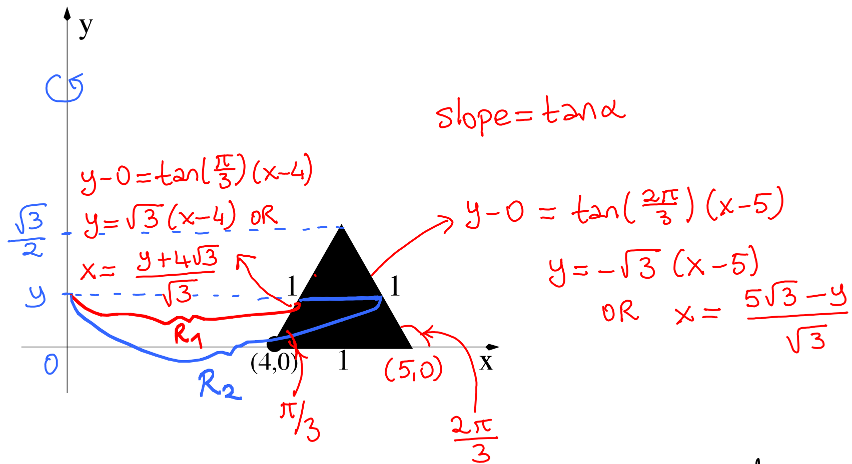
$f(y) - g(y) = 2y(y-5)$

y		0		5	
$f(y)-g(y)$	$+$	0	$-$	0	$+$

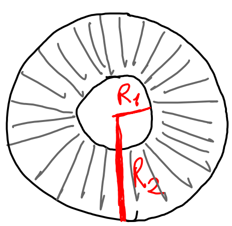
On $[0,5]$, $f(y) - g(y)$ is negative so $|f(y) - g(y)| = g(y) - f(y) = 10y - 2y^2$

$$\text{Area} = \int_0^5 |f(y) - g(y)|\,dy = \int_0^5 (10y - 2y^2)\,dy = \left(5y^2 - \frac{2y^3}{3}\right)\Big|_0^5 = \frac{125}{3}$$

(b) The <u>volume</u> obtained by rotating the equilateral triangle shown in the figure below about the y-axis.
(Remark: The equilateral triangle lies above the x-axis except its base which lies on the x-axis. Each side of the equilateral triangle is of length 1. The left-most corner of the equilateral triangle has coordinates $(4, 0)$.)



We obtain a typical cross-section by rotating the blue line segment (for arbitrary $y \in (0, \frac{\sqrt{3}}{2})$ about y-axis.



$$R_1 = \frac{y + 4\sqrt{3}}{\sqrt{3}} \qquad R_2 = \frac{5\sqrt{3} - y}{\sqrt{3}}$$

Cross-section Area: $A(y) = \pi(R_2^2 - R_1^2)$

$$= \pi\left(\frac{75 + y^2 - 10\sqrt{3}y}{3} - \frac{y^2 + 48 + 8\sqrt{3}y}{3}\right)$$

$$= \pi(9 - 6\sqrt{3}y)$$

$$\text{Volume} = \int_0^{\frac{\sqrt{3}}{2}} A(y)\,dy$$

$$= \int_0^{\sqrt{3}/2} \pi(9 - 6\sqrt{3}y)\,dy = \pi\left(9y - 3\sqrt{3}y^2\right)\Big|_0^{\sqrt{3}/2} = \frac{9\sqrt{3}\pi}{4}$$

Question 3.

(a) Evaluate the limit $\lim_{x\to 0} \dfrac{x \cdot \int_0^x \tan(t^2)\,dt}{\sin(x^2)}$.

$$\left(\int_a^a f(x)\,dx = 0 \right)$$

$$\lim_{x\to 0} \frac{x \int_0^x \tan(t^2)\,dt}{\sin(x^2)} \left(\frac{0}{0}\right) \underset{\substack{\text{L'HR} \\ + \\ \text{FTC}}}{=} \lim_{x\to 0} \frac{1.\int_0^x \tan(t^2)\,dt + x.\tan(x^2)}{2x.\cos(x^2)} \left(\frac{0}{0}\right) \underset{\text{L'HR + FTC}}{=}$$

$$= \lim_{x\to 0} \frac{\tan(x^2) + 1.\tan(x^2) + x.\sec^2(x^2).2x}{2.\cos(x^2) + 2x.(-\sin x^2).2x} \underset{\substack{\uparrow \\ \text{Direct} \\ \text{Substitution}}}{=} \frac{0+0+0}{2+0} = 0$$

(b) Find the function defined by

$$F(t) = \int_{\sqrt{t}}^{t} \frac{d}{dx}\left(e^{x^{2x}}\right)\,dx$$

for all $t \geq 0$. Your answer should <u>not</u> involve an integral nor a derivative.

$e^{x^{2x}}$ is an antiderivative for $\frac{d}{dx}(e^{x^{2x}})$. Then, by FTC Part II, we have:

$$F(t) = \int_{\sqrt{t}}^{t} \frac{d}{dx}(e^{x^{2x}})\,dx = e^{x^{2x}} \Big|_{\sqrt{t}}^{t} = e^{t^{2t}} - e^{(\sqrt{t})^{2\sqrt{t}}}$$

(c) Find the function defined by

$$G(t) = \frac{d}{dt}\left(\int_{\sqrt{t}}^{t} e^{x^{2x}}\, dx\right)$$

for all $t > 0$. Your answer should <u>not</u> involve an integral or a derivative.

$$\int_{\sqrt{t}}^{t} e^{x^{2x}} dx = \int_{\sqrt{t}}^{1} e^{x^{2x}} dx + \int_{1}^{t} e^{x^{2x}} dx = \int_{1}^{t} e^{x^{2x}} dx - \int_{1}^{\sqrt{t}} e^{x^{2x}} dx$$

$$\frac{d}{dt}\left(\int_{\sqrt{t}}^{t} e^{x^{2x}} dx\right) = \frac{d}{dt}\left(\int_{1}^{t} e^{x^{2x}} dx\right) - \frac{d}{dt}\left(\int_{1}^{\sqrt{t}} e^{x^{2x}} dx\right)$$

$$\frac{d}{dt}\left(\int_{1}^{t} e^{x^{2x}} dx\right) = e^{t^{2t}} \quad \text{by FTC Part I}$$

$$\frac{d}{dt}\left(\int_{1}^{\sqrt{t}} e^{x^{2x}} dx\right) = \frac{d}{du}\left(\int_{1}^{u} e^{x^{2x}} dx\right) \cdot \frac{du}{dt} \quad \text{where } u = \sqrt{t} \quad \left(\begin{array}{c}\text{chain}\\ \text{Rule}\end{array}\right)$$

$$= e^{u^{2u}} \cdot \frac{1}{2\sqrt{t}} = \frac{e^{(\sqrt{t})^{2\sqrt{t}}}}{2\sqrt{t}} \quad (\text{By FTC Part I})$$

$$\Rightarrow G(t) = e^{t^{2t}} - \frac{e^{(\sqrt{t})^{2\sqrt{t}}}}{2\sqrt{t}}$$

Question 4. Prove that the polynomial $P(x) = x^3 + 2x + 3$ has exactly one root in $(-\infty, \infty)$.

Since $P(x)$ is a polynomial, it is continuous and differentiable for all $x \in (-\infty, \infty)$.

$P(-2) = -8 - 4 + 3 = -9 < 0$

$P(0) = 0 + 0 + 3 = 3 > 0$

Then by Intermediate Value Theorem, there is at least one point $c \in (-2, 0)$ such that $P(c) = 0$ (Actually, $P(-1) = 0$)

Assume that there were two distinct points c_1 and c_2 with $P(c_1) = P(c_2) = 0$, then by Rolle's Theorem, there exists a point d between c_1 and c_2 such that $P'(d) = 0$.

But $P'(x) = 3x^2 + 2 \geq 2$ for all $x \in (-\infty, \infty)$, it is never zero.

This is a contradiction to our assumption. Therefore there is exactly one point $c \in (-\infty, \infty)$ with $P(c) = 0$.

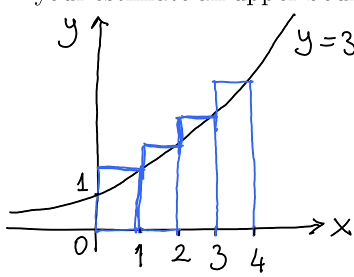
(-1 is the only root of $P(x) = x^3 + 2x + 3$)

Question 5.

(a) Estimate the integral

$$\int_0^4 3^{\sqrt{x}}\,dx$$

using a right-sum (i.e., the heights of the rectangles are given by the values of the function at the right end-points) with $n = 4$ rectangles of width $\Delta x = 1$. Is your estimate an upper bound or a lower bound for the exact integral? Explain.



Sum of the areas of rectangles:

$$\sum_{i=1}^{4} \Delta x . f(i) = \sum_{i=1}^{4} 1. f(i) = f(1) + f(2) + f(3) + f(4)$$

$$= 3^{\sqrt{1}} + 3^{\sqrt{2}} + 3^{\sqrt{3}} + 3^{\sqrt{4}}$$

$$= 12 + 3^{\sqrt{2}} + 3^{\sqrt{3}}$$

$$\int_0^4 3^{\sqrt{x}}dx \approx 12 + 3^{\sqrt{2}} + 3^{\sqrt{3}}$$

This is an upper bound for the integral since the sum of the areas of rectangles is greater than the area under the curve $y = 3^{\sqrt{x}}$ on $[0,4]$.

(b) Evaluate the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(1+\frac{i}{n}\right)\ln\left(1+\frac{i}{n}\right)}{n}$$

by interpreting it as a definite integral and then calculating the value of the integral.

Let $x_i = 1 + \frac{i}{n}$ then $\Delta x = x_{i+1} - x_i = \frac{1}{n}$, $x_0 = 1$, $x_n = 2$

$$f(x_i) = \left(1+\frac{i}{n}\right)\ln\left(1+\frac{i}{n}\right) = x_i . \ln(x_i)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(1+\frac{i}{n}\right)\ln\left(1+\frac{i}{n}\right)}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x . f(x_i) = \int_{x_0}^{x_n} f(x)dx = \int_1^2 x . \ln x\, dx$$

$$\int_1^2 x\ln x\,dx = \frac{x^2}{2}.\ln x \Big|_1^2 - \int_1^2 \frac{x^2}{2}.\frac{dx}{x} = 2\ln 2 - \frac{x^2}{4}\Big|_1^2 = 2\ln 2 - 1 + \frac{1}{4} = 2\ln 2 - \frac{3}{4}$$

Int. by Parts

let $u = \ln x$ and $x dx = dv$, then $du = \frac{dx}{x}$ and $\frac{x^2}{2} = v$

Question 6. Compute the following integrals. Show all your reasoning clearly.

(a) $\displaystyle\int_0^{\pi/2} \sin^4(x)\cos^3(x)\,dx \;=\; \int_0^{\pi/2} \sin^4 x \cdot \cos^2 x \cdot \cos x\,dx = \int_0^{\pi/2} \sin^4 x \cdot (1-\sin^2 x)\cdot \cos x\,dx$

let $u = \sin x$ then $du = \cos x\,dx$, $x = 0 \Rightarrow u = \sin 0 = 0$, $x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$

$$\int_0^{\pi/2} \sin^4 x\,(1-\sin^2 x)\cos x\,dx = \int_0^1 u^4(1-u^2)\,du = \int_0^1 (u^4 - u^6)\,du = \left(\frac{u^5}{5} - \frac{u^7}{7}\right)\Bigg|_0^1 = \frac{2}{35}$$

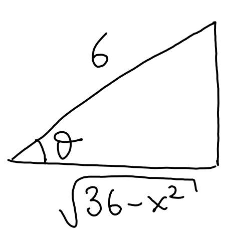
(b) $\displaystyle\int \frac{1}{x^2\sqrt{36-x^2}}\,dx$ Let $x = 6\sin\theta$, $\underbrace{-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}$

then $dx = 6\cos\theta\,d\theta$, $x^2 = 36\sin^2\theta$ and

$$\sqrt{36-x^2} = \sqrt{36 - 36\sin^2\theta} = \sqrt{36(\underbrace{1-\sin^2\theta}_{\cos^2\theta})} = 6|\cos\theta| = 6\cdot\cos\theta$$

$$\int \frac{1}{x^2\sqrt{36-x^2}}\,dx = \int \frac{6\cdot\cos\theta}{36\cdot\sin^2\theta\cdot 6\cos\theta}\,d\theta = \frac{1}{36}\int \frac{1}{\sin^2\theta}\,d\theta = \frac{1}{36}\int \csc^2\theta\,d\theta$$

$$= -\frac{1}{36}\cot\theta + C = -\frac{\sqrt{36-x^2}}{36x} + C$$



$\sin\theta = \dfrac{x}{6}$

$\Downarrow$

$\cot\theta = \dfrac{\sqrt{36-x^2}}{x}$

Question 7.

(a) Find the Taylor series $T(x)$ for $\cos x$ centered at $\pi/3$. $f(x)=\cos x$, $T(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(\pi/3)}{n!}(x-\frac{\pi}{3})^n$

$$\left.\begin{aligned} f(x)&=\cos x \Rightarrow f(\tfrac{\pi}{3})=\cos\tfrac{\pi}{3}=\tfrac{1}{2}\\ f'(x)&=-\sin x \Rightarrow f'(\tfrac{\pi}{3})=-\sin\tfrac{\pi}{3}=-\tfrac{\sqrt{3}}{2}\\ f''(x)&=-\cos x \Rightarrow f''(\tfrac{\pi}{3})=-\cos\tfrac{\pi}{3}=-\tfrac{1}{2}\\ f'''(x)&=\sin x \Rightarrow f'''(\tfrac{\pi}{3})=\sin\tfrac{\pi}{3}=\tfrac{\sqrt{3}}{2}\end{aligned}\right\}\quad \begin{aligned} f^{(2n)}(\tfrac{\pi}{3})&=(-1)^n\tfrac{1}{2}\\ f^{(2n+1)}(\tfrac{\pi}{3})&=(-1)^{n+1}\tfrac{\sqrt{3}}{2}\end{aligned}$$

$\vdots$

Repeats itself

$$T(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{2.(2n)!}(x-\frac{\pi}{3})^{2n}+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}.\sqrt{3}}{2(2n+1)!}(x-\frac{\pi}{3})^{2n+1}$$

(b) Show that the Taylor series $T(x)$ that you determined in part **(a)** satisfies $\cos x = T(x)$ for all $x \in (-\infty, \infty)$.

$f^{(n+1)}(x)$ is either $\mp\sin x$ or $\mp\cos x$, so $|f^{(n+1)}(x)|\leq 1$ for all x $(M=1)$

By Taylor's Inequality: $|R_n(x)|\leq \frac{1}{(n+1)!}(x-\frac{\pi}{3})^{n+1}$ for all x.

Since $\lim_{n\to\infty}\frac{(x-\pi/3)^{n+1}}{(n+1)!}=0$ for all x, by Squeeze Thm. $\lim_{n\to\infty}|R_n(x)|=0$.

Since $-|R_n(x)|\leq R_n(x)\leq |R_n(x)|$ for all x, by Squeeze Thm $\lim_{n\to\infty}R_n(x)=0$.

Thus $f(x)=T(x)$.

Question 8.

(a) Find the radius and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n(n+1)}$$

$a_n = \frac{(x+3)^n}{2^n(n+1)}$. Apply Ratio Test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x+3)^{n+1}}{2^{n+1}(n+2)} \cdot \frac{2^n(n+1)}{(x+3)^n}\right| = \frac{|x+3|}{2} \cdot \frac{(n+1)}{(n+2)} \to \frac{|x+3|}{2} \text{ as } n \to \infty$$

According to Ratio Test:

① If $\frac{|x+3|}{2} < 1$ then the series is absolutely convergent.

$\Rightarrow |x+3| < 2 \Rightarrow -5 < x < -1$, center $= -3$, radius $R = 2$

② If $\frac{|x+3|}{2} > 1$ then the series is divergent, that is, if $x > -1$ or $x < -5$ then the series is divergent.

③ If $\frac{|x+3|}{2} = 1$ then the test is inconclusive, So for $x = -1$ and $x = -5$ try other tests.

a) $x = -1 \Rightarrow \sum_{n=0}^{\infty} \frac{1}{n+1}$, $a_n = \frac{1}{n+1}$, $b_n = \frac{1}{n}$, $a_n, b_n > 0$, $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$

and $\sum b_n$ is divergent, so by Lim. Comp. Test, the series is divergent.

b) $x = -5 \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$, This is an alternating series with $b_n = \frac{1}{n+1}$

$b_n > 0$, $b_{n+1} < b_n$ and $\lim_{n\to\infty} b_n = 0$

So by Alt. Ser. Test, the series is convergent.

Hence the interval of convergence is $[-5, -1)$

(b) Newton discovered that

$$\frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2}\ x^n$$

for $-1 < x < 1$.

(i) Using this formula, find a power series expansion for $\arcsin x$.

$$\arcsin x = \int \frac{1}{\sqrt{1-x^2}}\,dx = \int \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2} x^n = C + \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (n+1)} x^{n+1}$$

to find C, let $x=0$ then $\arcsin 0 = C+0 \Rightarrow C=0$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (n+1)} x^{n+1} \qquad \text{for } -1<x<1.$$

(ii) Use your power series from part **(i)** with $x = 1/\sqrt{2}$ to find a power series whose sum is π.

$$\frac{\pi}{4} = \arcsin\left(\frac{1}{\sqrt{2}}\right) = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (n+1)} \left(\frac{1}{\sqrt{2}}\right)^{n+1}$$

$$\Rightarrow \pi = 4 \cdot \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (n+1)} \left(\frac{1}{\sqrt{2}}\right)^{n+1} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^{n-1} (n!)^2 (n+1)} \left(\frac{1}{\sqrt{2}}\right)^{n+1}$$

Question 9. Determine whether the following improper integrals are convergent or divergent. Evaluate them when they are convergent. Show all your reasoning.

(a) $\int_1^\infty \frac{1}{(x+2)(x+3)(x+4)}\,dx = \lim_{R\to\infty} \int_1^R \frac{1}{(x+2)(x+3)(x+4)}\,dx$

$$= \lim_{R\to\infty} \int_1^R \left[\frac{1}{2(x+2)} - \frac{1}{x+3} + \frac{1}{2(x+4)}\right]dx = \lim_{R\to\infty} \left(\frac{1}{2}\ln|x+2| - \ln|x+3| + \frac{1}{2}\ln|x+4|\right)\Bigg|_1^R$$

$$= \lim_{R\to\infty} \left(\ln\left(\frac{|x+2|^{1/2}\cdot|x+4|^{1/2}}{|x+3|}\right)\right)\Bigg|_1^R = \lim_{R\to\infty}\left(\ln \underbrace{\frac{\sqrt{|(R+2)(R+4)|}}{|R+3|}}_{\to 1 \text{ as } R\to\infty} - \ln\frac{\sqrt{15}}{4}\right) = \ln\frac{4}{\sqrt{15}}$$

CONVERGENT

$$\frac{1}{(x+2)(x+3)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+3)} + \frac{C}{(x+4)} = \frac{1}{2(x+2)} - \frac{1}{(x+3)} + \frac{1}{2(x+4)}$$

$A(x+3)(x+4) + B(x+2)(x+4) + C(x+2)(x+3) = 1$

$x^2(A+B+C) + x(7A+6B+5C) + (12A+8B+6C) = 1$

$$\left.\begin{array}{l} A+B+C=0 \\ 7A+6B+5C=0 \\ 12A+8B+6C=1 \end{array}\right\} \Rightarrow \begin{array}{l} A=\frac{1}{2} \\ B=-1 \\ C=\frac{1}{2} \end{array}$$

(b) $\int_{-1}^{1} \frac{1}{x^{4/3}}\,dx$ This integral is improper at 0. since $\lim_{x\to 0} \frac{1}{x^{4/3}} = \infty$

$$\int_{-1}^{1} \frac{1}{x^{4/3}}dx = \int_{-1}^{0} \frac{1}{x^{4/3}}dx + \int_{0}^{1} \frac{1}{x^{4/3}}dx$$

$$\int_{0}^{1} \frac{1}{x^{4/3}}dx = \lim_{R\to 0^+} \int_{R}^{1} \frac{1}{x^{4/3}}dx = \lim_{R\to 0^+}\left(\frac{x^{-1/3}}{-1/3}\Bigg|_R^1\right) = \lim_{R\to 0^+} -3\left(1 - \frac{1}{\sqrt[3]{R}}\right) = \infty$$

Since the limit does not exist $\int_0^1 \frac{1}{x^{4/3}}dx$ is divergent.

No need to check $\int_{-1}^{0} \frac{1}{x^{4/3}}dx$.

$\int_{-1}^{1} \frac{1}{x^{4/3}}dx$ is divergent.