PART 1: 40 Minutes

Question 1: (10)

a) Find x such that $90 < x < 180$ and $\cos(2x) = -0.8$ if $\cos(37^\circ) = 0.8$

b) What is sin(2x) ?

Question 2: (10)

A continuous and differentiable function is described by $f(x) = e^{2x} - 1$ for $-\infty < x \leq 0$, $f(x) = ax^3 + bx^2 + cx + d$ for $0 < x < 1$, and $f(x) = \ln(x)$ for $1 \leq x < \infty$.

Find *a, b, c, d.*

Question 3: (10)

a) Find $\lim_{x\to 0}\dfrac{\tan 3x}{\sin 2x}$

b) Find $f(x)$ and x_0 if $\lim_{z\to 0}\dfrac{\sqrt{3+z}-\sqrt{3}}{z} = f'(x_0)$

Question 4: (10)

Show that all such points on the curve $x^2y^2 + xy = 2$ where the slope of the tangent to the curve is -1, have x-coordinates that satisfy the equation $x^4 + x^2 - 2 = 0$.

Question 5: (10)
The following is a portion of the graph of a function. Here C, D, F are inflection points. At C there is a horizontal tangent, and at D there is a vertical tangent.

Indicate whether the following are positive (+), negative (–), zero (0), or do not exist (dne):
$f'(A)$ $f''(A)$ $f''(C)$ $f''(B)$ $f''(G)$ $f'(H)$ $f'(D)$ $f''(F)$

PART 2: 40 Minutes

Question 1: (20) (Do two of these)

a) Evaluate $\int \cos(\sqrt{x})dx$

b) Evaluate $\int 1 \cdot \arcsin(x)dx$

c) Evaluate $\int \frac{x-1}{x(x+1)^2}dx$

Question 2: (20)

Given the integral $\int \frac{r^3}{\sqrt{4+r^2}}dr$, evaluate it using

a) r^2 as one of the functions in Integration by Parts
b) the substitution $r = 2\tan x$

Question 3: (10)

Find the area between $y = 1 - \sqrt{x}$ and the y-axis for $0 \le x \le 4$. (The top and bottom boundaries are horizontal lines)

PART 3: 40 Minutes

Question 1:(15 Points)
Find the sum of the following series, if they exist:

a) $1-1+1-1+\cdots$

b) $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\cdots$

c) $\sum_{n=2}^{20}\frac{1}{(n+1)(n+2)}$

Question 2:(10 Points)
The approximation $\sin(x) \cong x$ for small x is quite often used.
a) Explain this using the MacLaurin Series for $\sin(x)$.
b) Is $(\sin(x) - x)$ always positive, or always negative? Why?

Question 3:(10 Points)
For what values of x will $\sum_{n=1}^{\infty} \frac{nx^n}{3^n}$ converge?

Question 4:(15 Points)

Find the first three terms of the Taylor series for $f(x)$ centered at the given value of a.

a) $f(x) = \sqrt{x}$, $a = 1$

b) $f(x) = \sin^2(x)$, $a = 0$

c) $f(x) = \dfrac{2x}{(1+x^2)^2}$, $a = 0$ (Hint: This function is the derivative of $-(1+x^2)^{-1}$)

<u>PART 4: 40 Minutes</u>

Question 1: (10)

a) Find the cosine of the angle of the vector $(-1,2,2)$ with the z-axis

b) Find the sine of the angle between the vectors $(-1,2,2)$ and $(0,4,3)$

Question 2: (5)

Find the cosine of the angle between the lines $y = -2x + 3$ and $y = x - 1$ using the dot product of two vectors.

Question 3: (20)

Given are the lines $g_1 : (a,0,1) + t(-1,b,2)$ and $g_2 : (1,1,3) + t(-1,1,2)$.

a) For what values of a and b will g_1 and g_2 be the same line?

b) For what values of a and b are g_1 and g_2 parallel, but not the same?

c) If $b = 2$, for what value of a will g_1 and g_2 intersect?

d) For $a = 1$ and $b = 2$, find the equation of the plane that includes g_1 and is parallel to g_2.

Question 4: (15)

a) Find the intersection of $x + 2y - z = 4$ with $-x - y + 3z = 5$.

b) Show that $x - y + z = 3$ and g_2 in question 2 are parallel. Then find the distance between them.