KOÇ UNIVERSITY
MATH 106 - CALCULUS I
FINAL EXAM   January 14, 2009
Duration of Exam: 150 minutes

INSTRUCTIONS: CALCULATORS ARE NOT ALLOWED FOR THIS EXAM.
No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. **Print (use CAPITAL LETTERS)** and sign your name, and indicate your section below.

Surname: —————————————————————

Name: ————————————————————

Signature: ———————————————————

Section (Check One):

Section 1: E. Ceyhan (Mon-Wed 11:30)   ——
Section 2: E. Ceyhan (Mon-Wed 14:30)   ——
Section 3: E. Ş. Yazıcı (Mon-Wed 14:30)   ——
Section 4: V. Kalantorov (Mon-Wed 14:30)   ——
Section 5: E. Ş. Yazıcı (Mon-Wed 10:30)   ——
Section 6: V. Kalantorov (Mon-Wed 10:30)   ——

PROBLEM	POINTS	SCORE
1	7	
2	7	
3	7	
4	5	
5	10	
6	7	
7	15	
8	5	
9	5	
10	4	
11	13	
12	5	
13	10	
Bonus	5	
TOTAL	**105**	

1. (7 points) Find the limit if it exists.

$$\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) =$$

2. (7 points) Where does the tangent line to the curve $y = x^4 - x^2 + 3x$ at $x = 1$ cross the x-axis?

3. (7 points) Let f' is continuous function on $[a, b]$ and, $f(a) = 1$ and $f(b) = 2$. Show that

$$2 \int_a^b f'(x) f(x) dx = 3.$$

4. (5 points) Assume that $f'(b) = f''(b) = 0$. Show that

$$\int_a^b (x - a)^2 f'''(x) dx = f(b) - f(a)$$

.

5. (10 points) Show that $f(x) = |\sin x|$ is continuous but not differentiable at $x = 0$.

6. (7 points) Find the maximum and minimum values of the function $f(x) = 2 + 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 2\sqrt{2}]$.

7. Compute the following integrals

a) (5 points) $\int_{-\infty}^{1} xe^{2x}dx$

b) (5 points) $\int_{1}^{2} \frac{dx}{\sqrt{x-1}}$

c) (5 points) $\int \frac{3dx}{(x-1)^2(x+1)}$

8. (5 points) Find the Maclaurin series expansion of $f(x) = \sin^2 x$.

9. (5 points) Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(n+1)x^{2n}}{3^n}$$

10. (4 points) Find the sum of the following series

$$1 - \ln 3 + \frac{(\ln 3)^2}{2!} - \frac{(\ln 3)^3}{3!} + \, ...$$

11. Determine whether the given series converge or diverge. Explain.

a) (4 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$

b) (4 points) $\displaystyle\sum_{n=1}^{\infty} (\frac{n^2 + 1}{n^2 + 4n})$

c) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{2\sqrt{n}}$

12. (5 points) Find the volume of the region enclosed by $y = \sqrt{x}$ and $y = x^2$ rotated around the x-axis.

13. (7 points) a) Show that if a series is absolutely convergent then it is convergent.

b) (3 points) Give an example of a series that is convergent but not absolutely convergent.

Bonus Question (5 points) Show that

$$\lim_{n\to\infty} \int_n^{2n} \frac{\cos^2 x}{x^2} = 0$$