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KOÇ UNIVERSITY  
MATH 106 - CALCULUS  
Midterm I      March 6, 2007

**Duration of Exam: 60 minutes**

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**INSTRUCTIONS:** Calculators may not be used on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS)** and **sign your name, and indicate your section below.**

Surname, Name: \_\_\_\_\_

Signature: \_\_\_\_\_

Section (Check One):

Section 1 - 11:30      \_\_\_\_\_

Section 2 - 14:30      \_\_\_\_\_

PROBLEM	POINTS	SCORE
1	27	
2	23	
3	38	
4	12	
<b>TOTAL</b>	<b>100</b>	

1. Evaluate the following limits without using l'Hopital's rule. Specify infinite limits and if the limit does not exist give the reason.

(a) (9 points)  $\lim_{x \rightarrow 1} \frac{\sqrt{5-x} - 2}{x-1}$

(b) (9 points)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(c) (9 points)  $\lim_{x \rightarrow 0^+} \sqrt{x} \cos^2\left(\frac{1}{x}\right)$

2. (12 points) Find the asymptotes (horizontal, vertical, oblique, if exist) of the function

$$f(x) = \frac{4x^3 + 5}{-x^2 - 7x} .$$

(b) (11 points) Prove that  $\lim_{x \rightarrow 1} \frac{2x^2 + 2x - 4}{x - 1} = 6$ .

3. (a) (8 points) Find  $\frac{dy}{dx}$  for  $y = (2x + 2)^3(x^2 - 1)$ .

(b) (8 points) Find  $f'(x)$  for  $f(x) = \frac{\sin x^2}{x^2}$ .

(c) (10 points) The limit  $\lim_{h \rightarrow 0} \frac{(h-1)^2 - 1}{h}$  equals  $f'(a)$  for some function  $f(x)$  and some constant  $a$ . Determine  $f(x)$  and  $a$ .

(d) (12 points) Find an equation for the line tangent to the curve  $x = \sin t$ ,  $y = \sqrt{3} \cos t$  at the point defined by  $t = 2\pi/3$ .

4. (12 points) Suppose that  $f(x)$  is a continuous function with consecutive zeros at  $x = a$  and  $x = b$ ; that is,  $f(a) = f(b) = 0$  and  $f(x) \neq 0$  for  $a < x < b$ . Further, suppose that  $f(c) \geq 0$  for some number  $c$  between  $a$  and  $b$ . Use the Intermediate Value Theorem to argue that  $f(x) > 0$  for all  $a < x < b$ .