

Question 1: (10 Points)

Find the following derivative : $\frac{d}{dt} \int_{2t-1}^{\ln t} x e^x dx$

Question 2: (10 Points)

Given that the derivative of $e^x(x^2 - 2x - 1)$ is equal to $e^x(x^2 - 3)$, evaluate the following integral:

$$\int_0^1 e^x(x^2 - 3)dx$$

Question 3: (20 Points)

- (a) Write an integral expression for the volume of the solid generated by revolving the region bounded by $y = -x + 2$, and the lines $y = 0$, $x = 0$, and $x = 2$, about the x -axis.
- (b) Write an integral expression for the volume of the solid generated by revolving the region bounded by $y = -x + 2$, and the lines $y = 0$, $x = 0$, and $x = 2$, about $y = 3$.
- (c) Find the volume of the solids in parts (a) and (b) by using the Theorem of Pappus.

Question 4: (10 Points)

Find the area of the region between the curve $y = 2 - \sqrt{x}$ and the y-axis for $0 \leq x \leq 9$.

Question 5: (15 Points)

Find the derivatives of the inverse functions $f^{-1}(x)$ of the following functions $f(x)$:

(a) $f(x) = 2^x$

(b) $f(x) = \log_3(x)$

(c) $f(x) = 2\sin(x)$

Question 6: (15 Points)

Find the integrals of the following functions:

(a) $f(x) = x^\pi$

(b) $f(x) = \pi^x$

(c) $f(x) = 6 \tan(3x)$

Question 7: (20 Points)

Evaluate the following integrals

(a) $\int \cos^3(x) \sin(x) dx$

(b) $\int \frac{dx}{x \log_{10} x}$

(c) $\int \frac{2^{\ln x}}{x} dx$

(d) $\int \frac{4 \sec^2 x}{1 - 4 \tan x} dx$

MIDTERM I PART:

QUESTION 1(15 Points)

Find the following limits:

(a) $\lim_{x \rightarrow 0} \left[\frac{1}{x \cos(a+x)} - \frac{1}{x \cos a} \right]$, given that $\sec a = 2$

(b) $\lim_{x \rightarrow 0} \frac{\csc(\pi/2 - x)}{\csc(x)} \frac{\tan(2x)}{\sin^2(3x)}$

(c) $\lim_{x \rightarrow 11} \frac{121 - x^2}{\sqrt{x-2} - 3}$

QUESTION 2(15 Points)

The graph of $y = f(x)$ is shown below. The x-coordinates of the points A, B ... are x_A, x_B, \dots . The inflection points are I_1, I_2, I_3, I_4 . The tangent to the graph at I_1 is horizontal, and at I_2 is vertical.

- a) What can you say about $f'(x)$ at A, B, ..., L ? Explain !
- b) What can you say about $f''(x)$ at A, B, ..., L ? Explain !
- c) Which are the critical points?
- d) What is $f'''(x)$ at I_1, I_2, I_3 ?