<u>Question 1: (10 Points)</u>

Let f be a function which is thrice differentiable in the interval I. ($f^{'}, f^{''}, f^{'''}$ exist) . Prove that ($(c, f(c))$ is an inflection point for some $c \in$ I if $f^{''}(c) = 0$ and $f^{'''}(c) > 0$.

Question 2: (20Points)

(a) Show that the circle with radius 1 and center (2,3) will be intersected by the line $y = mx + 1$ at two points if $0 < m < 0.75$.

(b) Using implicit differentiation, show that the line $y = mx + 1$ intersects the circle at (2.4, 2.8) for $m = 0.75$.

Question 3: (20 Points)

$$f(x)=\begin{cases} 0 & for \quad x<-2 \\ x+2 & for \quad -2\le x\le 1 \\ (x-3)^2-1 & for \quad 1<x\le 4 \\ 0 & for \quad x>4 \end{cases}$$

a) Sketch $y(x)=2f(2(x+2))+2$

b) Is the function $f(x)$ continuous in [-3 , 5] ? Why?

c) Is the function $f(x)$ differentiable in [-2 , 4] ? Why?

d) Find $\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}$ and $\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}$

Question 4: (15 Points)

Find the following limits:

(a) $\lim\limits_{h\to 0}\dfrac{\left[\dfrac{3}{\pi+h}+\tan(\pi+h)\right]-\left[\dfrac{3}{\pi}+\tan(\pi)\right]}{h}$

(b) $\lim\limits_{x\to 0}\dfrac{\sec(x-\pi)}{\tan^2(2x)}\dfrac{\sin(7x)}{\csc(3x)}$

(c) $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2-4}-\sqrt{2-x}}{x-2}$

Question 5: (20 Points)

Find the derivative f' at (x_0, y_0) if

(a) $f(x) = \cos^2\sqrt{1+x^2}$, $x_0 = 0$

(b) $f(x) = \dfrac{\sqrt{x}\sin(x)}{\tan(x)}$, $x_0 = \pi/4$

(c) $f(x) = \sin(x - \sin(x))$, $x_0 = 0$

(d) $x\cos(y) + y\sin^2(x) = 0$, $x_0 = \pi/2 = f(x_0)$, where $y = f(x)$

Question 6: (15 Points)

The function f is continuous on [-1,10]. f' does not exist at $x=1$ and $x=7$.

x	(-1,1)	(1,5)	5	(5,7)	(7,10)
$f'(x)$	>0	<0	0	>0	<0

x	(-1,1)	(1,2)	2	(2,4)	4	(4,7)	(7,10)
$f''(x)$	<0	>0	0	<0	0	>0	>0

(a) Find all extrema

(b) At $x=1$, the right-sided and left-sided derivatives exist. At $x=7$, they don't. What kind of a point is $(1, f(1))$? What kind of a point is $(7, f(7))$?

(b) If $f(1)=8$ and $f(7)=10$, prove that there is only one point on the graph of $f(x)$ in the interval (5,7) whose tangent has the slope $\frac{1}{3}$.