Question 1: (15)

(a) Find the first four terms of the Taylor series generated by $f(x) = (1 - x^2)^{-1/2}$ at $x = 0$ by using the Taylor series generated by $f(x) = (1 + x)^{-1/2}$ at $x = 0$.

(b) Find the Taylor series of $\sin^{-1}(x)$ using the result of (a).

Question 2: (10)

Find

(a) $\lim_{n\to\infty}(1+\frac{2}{n})^{n/3}$

(b) the fourth term of a series whose sequence of partial sums is $\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},..\}$

Question 3: (15)

(a) Use the integral test to show that the series $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)}$ converges.

(b) To what value does the series in part (a) converge?

Question 4: (10)

For what values of x does the Taylor series $-1+\frac{x-1}{2}-\frac{(x-1)^2}{4}+\frac{(x-1)^3}{8}-...$ converge?

(Hint: What kind of a series is this?)

Question 5: (10)

Evaluate the integral $\int_0^1 (-\ln(x))dx$. Explain fully each step, including any limits that you take.

Question 6: (20)

Evaluate the following integrals:

(a) $\int \frac{dx}{x^2 + x + 2}$

(b) $\int \frac{5x^3 + x^2 - x - 1}{(x^2 - 1)(x^2 + 1)} dx$

(c) $\int e^{5x} \cos(2x) dx$

Question 7: (10 Points)

The graph of $f(x) = x^3 + bx^2 + cx + d$ is increasing in the interval x < 1, decreasing in the interval 1 < x < 3, and increasing in the interval x > 3. The inflection point is on the x-axis. Find the constants b, c, and d.

Question 8: (10 Points)

It is given that $f(x)$ is a positive function with domain $[1,\infty)$. That is, $f(x) > 0$ for $1 \le x < \infty$. We also know that for any $a \ge 1$, the volume of the solid generated by revolving the region bounded by $y = f(x)$, $y = 0$, $x = 1$, and $x = a$ about the x-axis, is $\pi(4a^9 - 4)$. Find $f(x)$.

Question 9: (10 Points)

Find the derivative of $f(x) = (\tan x)^{\cos x}$ by taking the logarithm of both sides and using implicit differentiation.

Question 1: (20 Points)

Evaluate the following integrals :

(a) $\int \sin^3(\frac{x}{2})dx$ (b) $\int \sin(2x)\cos(2x)dx$ (c) $\int (\tan^2(x) + \tan(x))dx$

Question 2: (15 Points)

Evaluate

$$\int \frac{-x^3+2x^2-3x+6}{(x^2+1)(x-1)^2}dx$$

Question 3: (10 Points + 5 Points):

Show that $\int e^{-\sqrt{x}}\,dx = -2\sqrt{x}e^{-\sqrt{x}} - 2e^{-\sqrt{x}} + c$ by

(a) integrating the left side using a substitution and/or Integration by Parts.
(b) using any other method

Question 4: (10 Points)

Find the derivative of $f(x) = (\sin x)^{\tan x}$

(Hint: Express the function $f(x)$ as $e^{h(x)}$, or take the logarithm of both sides and use implicit differentiation)

Question 5: (10 Points)

Evaluate the following integral:

$$\int \frac{2dx}{\sqrt{6x - 2x^2 + 7}}$$

Question 6: (10 Points)

The graph of $f(x) = x^3 + bx^2 + cx + d$ is increasing in the interval $x < -1$, decreasing in the interval $-1 < x < 3$, and increasing in the interval $x > 3$. The graph is concave down for $x < 1$, and concave up for $x > 1$. The inflection point is on the x-axis. Find the constants b, c, and d.

Question 7: (10 Points)

Find an expression for the volume of the solid generated by revolving the region bounded by $y = \tan(x)$, $y = -1$, $x = 0$, and $x = \pi/4$ about the line $y = -1$. Do NOT evaluate the expression.

Question 8: (5 Points)

Find $\lim_{x\to\infty} \frac{x^3}{3^x}$

Question 9: (5 Points)

Find $f'(3)$ for any function f whose domain is R satisfying the inequality $\left|f(x)+x^2-2x+4\right| \le \sin^2(x-3)$ for all real numbers x.

Question 10: (10 Points)

Find the length of the curve $y = f(x)$ between $-3 \le x \le -2$, if $f'(x) = \sqrt{x^2 - 1}$.