

Question 1: (10 Points)

Find all solutions to $\sin(2x-1) = -0.2$ for $x \in [-\pi, \pi]$.

Question 2: (10Points)

Find the derivative of $\sqrt{x^2 + 1}$ using the definition of the derivative (which involves a limit)

Question 3: (20 Points)

Determine the constants a,b,c,d,e so that $f(x)$ is continuous and differentiable everywhere.

$$f(x) = \begin{cases} a & \text{for } x < -2 \\ -2x^3 + bx^2 + cx + 16 & \text{for } -2 \leq x \leq 2 \\ 12(x-3)^2 & \text{for } 2 < x \leq e \\ d & \text{for } x > e \end{cases}$$

Question 4: (15 Points)

Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\tan(7x) \sec(7x)}{\sin^2(2x) \csc(3x)}$

(b) $\lim_{x \rightarrow \infty} \frac{\sin(3x)}{x}$

(c) $\lim_{x \rightarrow 0^+} \frac{\cos(2x)}{x}$

Question 5: (30 Points)

Find the derivative of

(a) $\sec^{-1}\left(x^2 - \frac{1}{x}\right)$

(b) $\log_3\left(\frac{1}{x}\right)$

(c) $\arccos(\sin x)$

(d) $2^{\ln x}$

(e) $y = f(x)$, where $3^y = x$, using (i) the definition of the logarithm
(ii) implicit differentiation

(f) $f^{-1}(x)$ at $x = 2$, if $f(1) = 2$ and $f'(1) = 3$.

Question 6: (15 Points + Bonus 5 Points)

Given the function $f(x) = \frac{x-1}{(x^2+x-2)(x+1)}$, find

- (a) the domain
- (b) $\lim_{x \rightarrow -1^-} f(x)$
- (c) $\lim_{x \rightarrow 1} f(x)$
- (d) $\lim_{x \rightarrow -2^+} f(x)$
- (e) the critical points of $f(x)$

Bonus: Using the result in (e), find the range.