
KOÇ UNIVERSITY

MATH 107

Final May 25, 2012

Duration of Exam: 120 minutes

INSTRUCTIONS: Calculators are not allowed on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS)** and **sign your name, and indicate your section below.**

Surname, Name: _____

Signature: _____

Section (Check One):

- | | |
|--|------|
| Section 1: E. Mengi (13:30, MWF) | ____ |
| Section 2: S. Ünver (12:30, TTF) | ____ |
| Section 3: S. Ünver (14:30, TTF) | ____ |
| Section 4: A. Mostafazadeh (9:30, TTF) | ____ |

PROBLEM	POINTS	SCORE
1	15	
2	5	
3	15	
4	15	
5	20	
6	10	
7	10	
8	10	
TOTAL	100	

1. Find the following limit

$$\lim_{t \rightarrow 0} \int_{-t}^t \frac{\sin(x^2)}{x^2} dx.$$

- 2.** State what it means for the series $\sum_{n=1}^{\infty} a_n$ to be absolutely convergent.

3. Find the inverse of the following matrix using row operations:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}.$$

4. Consider the following system of equations (depending on the variable α) where x_1, x_2 , and x_3 are the unknowns:

$$x_1 + 2x_2 - x_3 = 1$$

$$3x_1 + x_2 - 2x_3 = \alpha$$

$$x_1 - 3x_2 = -6$$

- (i) (10 pts.) Determine for which α the above system has a solution.
(ii) (5 pts.) For each α that you found in (i) above, determine whether the solution of the corresponding equation is unique.

5. Let $A := \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(i) (10 pts.) Find the eigenvalues of A .

(ii) (10 pts.) Find a matrix S such that $S^{-1}AS$ is a diagonal matrix.

6. Let W_1 and W_2 are two subspaces of a vector space V . Prove that $W_1 \cap W_2$ is a subspace of V .

7. Let $\mathcal{C}^\infty(\mathbb{R}, \mathbb{R})$ be the vector space of functions $f : \mathbb{R} \rightarrow \mathbb{R}$ that are infinitely many times differentiable on \mathbb{R} (that is they are differentiable and their derivatives are differentiable and so on). Let $f_1, f_2, f_3 \in \mathcal{C}^\infty(\mathbb{R}, \mathbb{R})$ such that $\{f'_1, f'_2, f'_3\}$ is a linearly independent subset of $\mathcal{C}^\infty(\mathbb{R}, \mathbb{R})$. Prove that $\{f_1, f_2, f_3\}$ is also linearly independent.

8. Let V be a finite dimensional vector space and $L : V \rightarrow V$ be a linear operator. Show that if $(L + I)(L - I) = 0$ then L is an invertible operator.