
KOÇ UNIVERSITY
MATH 107 - LINEAR ALGEBRA
Midterm I March 18, 2015
Duration of Exam: 75 minutes

INSTRUCTIONS: CALCULATORS ARE ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Name: _____

Surname: _____

Signature: _____

Section (Check One):

- Section 1: E. Şule Yazıcı M-W(10:00) _____
Section 2: E. Şule Yazıcı M-W(13:00) _____
Section 3: Haluk Oral M-W(14:30) _____
Section 3: Haluk Oral M-W(16:00) _____

PROBLEM	POINTS	SCORE
1	20	
2	15	
3	15	
4	30	
5	10	
6	15	
TOTAL	105	

1. (20 points) Solve the following system of linear equations by row reducing the associated augmented matrix. Find the row reduced echelon form of the matrix. Write the solution set and find two particular solutions for the system if there exist more than one solution.

$$\begin{cases} x_1 + 2x_3 - 2x_5 = 11 \\ x_2 - 3x_4 - x_5 = 0 \\ 2x_1 + 5x_3 - 4x_5 = 27 \\ 3x_1 + 2x_2 + 7x_3 - 6x_4 - 8x_5 = 38 \end{cases}$$

2. (15 points) Let V be a vector space and $S = \{v_1, v_2, \dots, v_n\} \subseteq V$. Prove that if S is linearly independent then for every $v \in S$, $v \notin \text{Span}(S \setminus \{v\})$. (Note: In a vector space V with $A \subseteq V$, $\text{Span}(A)$ is the subspace generated by the set A)

3. (15 points) Let $V = \{(2c, a - b, b - 3c, a + 2b) \mid a, b, c \in \mathbb{R}\}$. Find a basis for V and compute its dimension.

4. (a)(15 points) Let $S = \{(1, -2, 3), (2, 3, 1)\}$. Find the coordinates of $(4, -1, 7)$ in the $\text{Span}(S)$ with respect to the basis S .

(b)(15 points) Show that $T = \{(1, -2, 3), (2, 3, 1), (0, 0, 1)\}$ is linearly independent. What is $\text{span}(T)$ =?

5. (10 points) Find the cosine of the angle between the vectors $A = (1, -2, 3)$ and $B = (2, 3, 1)$ in \mathbb{R}^3

6. (15 points) Show that $V = \{(a, a, a) \mid a \in \mathbb{R}\}$ is a subspace of \mathbb{R}^3