Math 200:Calculus with Linear Algebra
Koç University
Midterm 1
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Name :_____

Problem	Points	Score
1	15 points	
2	15 points	
3	15 points	
4	15 points	
5	20 points	
6	20 points	
Total	100 points	

Good Luck

1. **(15 points).** Let

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & -3 \\ 4 & 5 & -2 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}.$$

a. Compute Ab, $b^T A$, $A^2 = AA$.

b. Compute $det(A)$.

c. Compute A^{-1} by Gaussian elimination.

d. Solve the equation $Ax = b$.

2. **(15 points).** Consider the following linear system of equations

$$\begin{aligned} 2x_1 + x_2 - 2x_3 &= 4 \\ x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 + 3x_2 - x_3 &= c, \end{aligned}$$

where c is a parameter. Using Gauss elimination method, determine the value of c for which the above system of equations has at least one solution. For this value of c, find the general form of the solution.

3. **(15 points).** Let

$$\mathbf{v}_1 = (2, 0, 1), \quad \mathbf{v}_2 = (5, -1, 2), \quad \mathbf{v}_3 = (1, -1, 0).$$

(a) Show whether or not $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $\Re^3$.

(b) Determine whether $\mathbf{u} = (-7, 3, -1)$ is in the span of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$. Explain your answer.

4. **(15 points).** Let $W = \{(x, y, z, w) : x - y + 2z + w = 0, 3x + y - z + 2w = 0, 5x - y + 3z + 4w = 0\}$

   (a) Find a basis for the set W.

   (b) Express (-1, 7, 4, 0) with respect to the above basis.

5. **(20 points).** Show that the polynomials

$$\begin{aligned} p_0(x) &= \frac{(x-1)(x-2)}{2} \\ p_1(x) &= \frac{x(x-2)}{-1} \\ p_2(x) &= \frac{x(x-1)}{2} \end{aligned}$$

form a basis for the set of polynomials of degree 2, $P_2(x)$.

6. **(20 points).** Let the linear transformation T be defined as

$$T(\vec{x}) = T(x_1, x_2, x_3) = (x_1 - 2x_2 + x_3, x_2 + 5x_3)$$

(a) Show that T is a linear transformation

(b) Determine the transformation matrix $\mathbf{A}$ such that $T(\vec{x}) = \mathbf{A}\vec{x}$.

(c) Find the range and null space of T

(d) Is the transformation one-to-one? Explain.