.

KOÇ UNIVERSITY

MATH 200 - Multivariable Calculus and Matrix Algebra

Exam 1 March 23, 2009

Duration of Exam: 80 minutes

INSTRUCTIONS: Calculators are not allowed. No books, no notes, no questions and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: ________________________________

Signature: ________________________________

Section (Check One):

Section 1: S. Küçükçifçi (Mon-Wed 11:00) ___
Section 2: S. Küçükçifçi (Mon-Wed 15:30) ___

PROBLEM	POINTS	SCORE
1	24	
2	33	
3	23	
4	20	
TOTAL	**100**	

1. (a) (8 points) Determine whether or not the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$, $v_3 = (3, 2, 1)$ in $\mathbb{R}^3$ are linearly independent.

(b) (8 points) Determine the span of v_1, v_2, v_3.

(c) (8 points) Find a basis for $\mathbb{R}^3$ containing the vectors v_1 and v_2.

2. (a) (18 points) Using the Cramer's rule determine the variable x_1 of the following system.

$$\begin{cases} 2x_1 - 3x_2 + 5x_3 &= 1 \\ x_2 - 3x_3 &= -3 \\ -x_2 + 2x_3 &= 2 \end{cases}$$

(b) (15 points) For the augmented matrix below, determine whether the system has solution(s) by discussing the value of k.

$$\left[\begin{array}{cc|c} -1 & 2 & 1 \\ 3 & -6 & k \end{array}\right]$$

3. Consider the matrix $M = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$.

(a) (15 points) Find all eigenvalues and associated eigenvectors for the matrix M.

(b) (8 points) For each eigenvalue λ find the rank of the matrix $M - \lambda I$.

4. (20 points) Let $T((x_1, x_2)) = (x_1 - x_2, x_1 + 3x_2, x_1 + x_2)$. Determine the domain, target space, range and null space of T. Determine a basis set for the range space of T.