Math 200 (midterm 3).

09.05. 2008

1. (10 pts.) Given the function $f(x,y) = 2x^2 + 3xy + y^2$.

a) find the directional derivative of f at the point $P(1,1)$ in direction of the vector $\mathbf{v} = \mathbf{i}+2\mathbf{j}$,

b) find the maximum directional derivative of f at the point $P(1,1)$ and the direction at which it occurs.

2. (15 pts.) Give the definition of a point of local minimum and find points of local minimum for the following functions:

$$a)\ \ f(x,y) = x^2 + y^4 + 2, \quad b)\ \ g(x,y) = 3x - x^3 - 3xy^2.$$

3.(10 pts.) Find the points on the hyperbola $xy = 1$ that are closest to the origin $(0,0)$.

4. (15 pts.) Show that $\{t, t+1, t^2-1\}$ is a basis of the space P_2 of polynomials of order ≤ 2 .

5.(10 pts.) Determine whether the vectors $\{\langle 1,1,1\rangle, \langle 1,-1,1\rangle, \langle 1,0,1\rangle, \langle 0,1,0\rangle\}$ form a basis of $\mathbb{R}^3$ and determine the dimension of the space spanned by these vectors.

6. (10 pts.) Show that the set of all triples $\langle 5a, 3a, -2a\rangle$ form a subspace of R^3.

7. (15 pts.) Given an Euclidean space E and two orthogonal vectors $u, v \in E$, such that $\|u\| = 2$ and $\|v\| = 5$. Show that the u and v are linearly independent and find $(3u+v, u-v)$.

8. (15 pts.) Given the operators

$$A\langle x_1, x_2, x_3\rangle = \langle x_1 + 3x_2 - x_3, x_2 - 2x_3\rangle, \quad T\langle x_1, x_2, x_3\rangle = \langle x_1 + x_2, x_2 - x_1 + x_3, x_2 - x_3\rangle.$$

a) Show that A is a linear operator,
b) Find the null space of the operator T,
c) Are the operators A and T one-to-one? Why?

9. (20 pts.) Given the matrices

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 3 & 1 \end{bmatrix}.$$

a) Find AB, b) $A^T + A$, c) A^{-1}.

10.(10 pts.) Let X be a linear space and $\theta \in X$ is its zero element. Show that $0x = \theta$, $\alpha\theta = \theta$ for each $x \in X$ and $\alpha \in \mathbb{R}$