

Selda Küçükçifçi  
Math 200: Multivariable Calculus and Matrix Algebra  
FINAL EXAM

June 10, 2005

2 hr & 15 mn, no calculator, no questions

NAME: \_\_\_\_\_ SECTION: \_\_\_\_\_

1	/16
2	/22
3	/18
4	/14
5	/14
6	/16
Total:	/100

WRITE ALL ANSWERS CLEARLY, AND SHOW ALL WORK TO GET CREDIT

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1. a) (12 points) Determine the domain, range and null space of the linear transformation  $T((x_1, x_2, x_3)) = (x_1 - x_2, x_2 - x_1, x_1 - x_3)$ . Determine a basis set for each of these spaces. State if the given transformation is one-to-one.

b) (4 points) Consider the linear transformation  $T$  in the part (c), find the matrix representing  $T$  with respect to the basis  $B = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$ .

2. a) (5 points) Evaluate the determinant of the matrix  $M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$  by elementary row operations.

b) (8 points) Solve the following system using the Cramer's rule

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= 2 \\ 2x_1 + 5x_2 + 3x_3 &= 1 \\ x_1 + 8x_3 &= 7 \end{cases}$$

c) (9 points) Find all eigenvalues and associated eigenvectors for the matrix  $N = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ .

3. a) (10 points) Let  $f(x, y)$  be a function having the first order partial derivatives  $f_x = 2x + 2y$ ,  $f_y = 2x - \sin y$ .

a.i) Determine  $f(x, y)$ .

a.ii) Determine the mixed second order partial derivatives of  $f(x, y)$ .

b) (8 points) Let  $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ .

b.i) Determine the limit  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  if it exists.

b.ii) Is it possible to define  $f(0, 0)$  in a way that makes  $f$  continuous at the origin? Explain your answer.

4. a) (7 points) Find all the local maxima, local minima, and saddle points of the function  $f(x, y) = xy - x^2y - xy^2$ .

b) (7 points) Find the absolute maximum and absolute minimum values of the function given in part (a) over the triangle with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(0, 1)$ .

5. a) (6 points) Assume that the equation

$$y^2 - 3xy + \ln y + 4e^{x+y} = 0$$

define  $y$  as a differentiable function of  $x$ . Find the value of  $dy/dx$  at the point  $P(0, 1)$ .

b) (8 points) The total daily profit (in dollars) realized by the Country Workshop in manufacturing and selling its rolltop desks is given by the profit function

$$P(x, y) = -0.2x^2 - 0.25y^2 - 0.2xy + 100x + 90y - 4000$$

where  $x$  stands for the number of finished units and  $y$  denotes the number of unfinished units manufactured and sold per week. The company's management decides to restrict the manufacture of these desks to a total of exactly 200 units/week. How many finished units and how many unfinished units should be manufactured per week to maximize the company's weekly profit **by using the Lagrange multipliers method**.

6. Evaluate the following double integrals.

a) (4 points) Evaluate the given double integral:  $\int_0^1 \int_0^y (xy + y^2) dx dy$

b) (4 points) Evaluate the given double integral:  $\int_R \int xy^2 dA$ , where  $R$  is the finite region in the first quadrant bounded by the curves  $y = x^2$  and  $x = y^2$ .

c) (4 points)  $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} dy dx$

d) (4 points)  $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$