Selda Küçükçifçi March 21, 2005

Math 200: Multivariable Calculus and Matrix Algebra

EXAM I

NAME: ______________________________

1	/16
2	/22
3	/22
4	/24
5	/16
Total:	/100

WRITE ALL ANSWERS CLEARLY, AND SHOW ALL WORK TO GET CREDIT
NO QUESTIONS, NO CALCULATOR, 90 MINUTES

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1. Let M be the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

(8 points) a) Evaluate the determinant of M by elementary row operations.

(8 points) b) Evaluate the determinant by cofactor expansions (that is by using the formula).

2. (16 points.) a) Using the Cramer's rule solve the following system

$$\begin{cases} x_1 - 3x_2 + x_3 &= 4 \\ 2x_1 - x_2 &= -2 \\ 4x_1 - 3x_3 &= 0 \end{cases}$$

(6 points) b) For the matrix and right hand side below, determine the rank and whether the system has solution(s).

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 2 & -1 & 0 & -2 \\ 4 & 0 & -3 & 0 \end{array}\right]$$

3. Consider the matrix $M = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$,

(10 points) a) Find all eigenvalues and associated eigenvectors for the matrix M.

(6 points) b) Is M diagonalizable? Explain your conclusion.

(6 points) c) Find M^{10} for the matrix M.

4. (4 points) a) Determine whether or not the vectors $v_1 = (1, -1, 0), v_2 = (-1, 1, 0)$, $v_3 = (1, 0, 1)$ in $\mathbb{R}^3$ are linearly independent.

(4 points) b) Determine the span of v_1, v_2, v_3.

(4 points) c) Is the vector $v_4 = (3, -1, 2)$ in the span of v_1, v_2, v_3. Explain your answer.

(4 points) d) Find a basis for $\mathbb{R}^3$ containing the vector v_3.

(8 points) e) Find the orthogonal projection of v_1 on v_3 as well as the vector component of v_1 orthogonal to v_3.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by
$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_1, x_1 - x_3)$.

(6 points) a) Find the matrix representing T with respect to the basis $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$.

(4 points) b) Determine the null space of T.

(6 points) c) Determine whether T is one-to-one.