

Math 200: Final Exam

23 May 2008

1. (10 pts) Find the equation of the line that passes through the point $P(0, 1, -1)$ and is parallel to the line of intersection of the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$.

2. (5 pts) Find $f_{xzy} - f_{yzz}$ where $f(x, y, z) = x^3 + y^3 - 2xy \cos z$.

3. (10 pts) If x thousand dollars is spent on labor, and y thousand dollars is spent on equipment, it is estimated that the output of certain factory will be $Q(x, y) = 50x^{2/5}y^{3/5}$ units. If 150,000 dollars is available, how should this capital be allocated between labor and equipment to generate the largest possible output?

4. (10 pts) Find and classify critical points of the functions

a) $f(x, y) = 2x^2 + 2xy + y^2 + 4x - 2y + 1$, b) $g(x, y, z) = 5 - x^2 - y^2 - 2z^2 + 2y$.

5. (10 pts) Is the system of vectors $v_1 = \langle 1, -1, 1 \rangle, v_2 = \langle 1, 1, -1 \rangle, v_3 = \langle 2, 1, 3 \rangle$ a basis of \mathbb{R}^3 ? Why?

6. (15 pts) Given the operators $A\langle x_1, x_2, x_3 \rangle = \langle x_1 + x_2 + x_3, x_2 - x_3, x_1 \rangle$ and

$T\langle x_1, x_2 \rangle = \langle x_1 - x_2, x_1 + x_2, 2x_2 - 2x_1 \rangle$.

a) Find $A^3\langle 1, 0, 2 \rangle$, b) the null space of T .

7. (15 pts) Given the system of equations

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 - x_2 - bx_3 = 0 \\ x_1 + 3x_2 - x_3 = b \end{cases}$$

Use the Cramer's rule to find the values of b for which:

a) the system has a unique solution, b) has not a solution.

8 (20 pts) Given the matrices

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}.$$

a) Find the eigenvalues and eigenvectors of the matrix A . b) Diagonalize B and find B^{50} .

9. (10 pts) Let u and v be two elements of an Euclidean space E . Show that $\|u + v\| \leq \|u\| + \|v\|$.