Math 201: Statistics June 5, 2011

Spring 2011 Final Exam

Closed book & notes; only a two-sided and handwritten A4 formula sheet and a calculator allowed; 120 minutes. No questions accepted!

Instructions: There are eight pages (one cover and seven pages with questions) in this exam. Please inspect the exam and make sure you have all 8 pages. You may only use your calculator and your formula sheet. Do all your work on these pages. If you use the back of a page, make sure to indicate that. **You may not exchange any kind of material with another student.** *You might get one bonus point for filling the front cover properly!*

Remember: You must show all your work to get proper credit. Round your final answers to **two decimal** places in all exam questions.

Academic Honesty Code: Koç University Academic Honesty Code stipulates that "copying from others or providing answers or information, written or oral, to others is cheating." By taking this exam, you are assuming full responsibility for observing the Academic Honesty Code.

NAME and SURNAME: KEY SIGNATURE: ____________

INSTRUCTOR : ____________ LECTURE TIME : ____________

1	/15
2	/15
3	/13
4	/15
5	/12
6	/15
7	/15
Total:	/100

Question 1: (15 points)

The manager of a supermarket takes a random sample of 64 grocery purchases from his store in a particular week. This sample produced an average of \$54.63 and a standard deviation of \$17.61.

(a) Estimate the true mean dollar amount of grocery purchases with 99% confidence.

$n = 64 > 30$ is large so 99% CI on μ is

$\bar{x} = \$54.63$

$s = \$17.61$

$$\bar{x} \pm (z_{\alpha/2}) \frac{s}{\sqrt{n}}$$

$\alpha = 0.01$

$$= 54.63 \pm (2.575) \frac{17.61}{\sqrt{64}}$$

$\frac{\alpha}{2} = 0.005$

$$= 54.63 \pm 5.668$$

$z_{0.005} = 2.575$

$$= (48.96, 60.30)$$

(b) Test the claim that mean amount of grocery purchases is different from \$60 per week at this supermarket using a critical value or region in your decision. Use α=0.05.

$H_0: \mu = 60 \Rightarrow \mu_0 = 60 \quad \alpha = 0.05$

$H_a: \mu \neq 60$

$$T.S. = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{54.63 - 60}{17.61/\sqrt{64}} = \frac{-5.37}{2.201}$$

$$= -2.44$$

D.R: Reject H_0, if $|T.S.| > z_{\alpha/2} = z_{0.025} = 1.96$

Since $|-2.44| = 2.44 > 1.96$, we reject H_0.

(c) Also calculate the *p*-value for the test in part (b) and make your decision based on the *p*-value. Would the decision be different, if α were equal to 0.01?

$$p\text{-value} = 2P(Z > |T.S.|)$$

$$= 2P(Z > 2.44)$$

$$= 2(0.5 - 0.4927) = 0.0146$$

Since p-value $< \alpha = 0.05$, we reject H_0.

If $\alpha = 0.01$ were the case, then we would not reject since p-value $\not< \alpha = 0.01$.

Question 2: (15 points)
A college student has a part-time job as a waiter (garson). In applying for credit, he must estimate the income he gets through daily tips (günlük bahşiş). He obtains an average of \$25.75 for 19 days with a standard deviation of \$3.85. (He also feels that the distribution of daily tips is normal).

(a) Calculate a 95% confidence interval for the mean daily tips for this student.

$n = 19 < 30$, small sample

$\bar{X} = 25.75$

$s = 3.85$

$t_{18, 0.025} = 2.101$

95% CI on μ is

$$\bar{X} \pm \left(t_{n-1, \alpha/2}\right) \frac{s}{\sqrt{n}}$$

$$= 25.75 \pm (2.101) \frac{3.85}{\sqrt{19}}$$

$$= 25.75 \pm (2.101)(0.8833)$$

$$= 25.75 \pm 1.8557 = (23.89, 27.61)$$

(b) Test the hypothesis that mean of the daily tips is more than \$20 using critical value or region. Use α=0.05.

$H_0: \mu = 20 \Rightarrow \mu_0 = 20 \quad \alpha = 0.05$

$H_a: \mu > 20$

$$T.S. = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{25.75 - 20}{\frac{3.85}{\sqrt{19}}}$$

$$= \frac{5.75}{0.8833} = 6.51$$

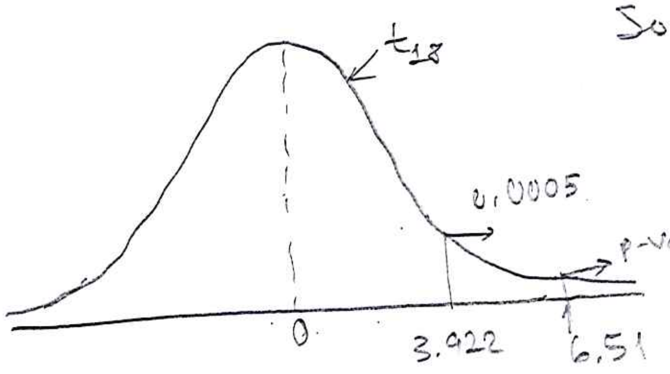
D.R: Reject H_0, if $T.S. > t_{n-1, \alpha} = t_{18, 0.05} = 1.734$

Since $6.51 > 1.734$, we reject H_0.

(c) Also calculate the range of *p*-values for the test in part (b).

p-value $= P(T_{18} > 6.51)$ where $T_{18} \sim t_{18}$

So, p-value < 0.0005.



Question 3: (13 points)

A nursery (fidanlık) sells a certain type of bulb (çiçek soğanı) in packages of 20 bulbs each. The probability is 0.80 that any given bulb in the package will sprout (filizlenmek).

(a) What is the probability that at least 13 but fewer than 17 bulbs will sprout?

$n = 20, \quad p = 0.80 \qquad X \sim BIN(20, 0.80)$

$P(13 \leq X < 17) = P(13 \leq X \leq 16)$

$= P(X \leq 16) - P(X \leq 12)$

$= 0.589 - 0.032$ (from Table II - Binomial table)

$= 0.557$

(b) Calculate the mean and standard deviation of the number of sprouting bulbs in a package.

$\mu = E(X) = np = 20(0.80) = 16$

$Var(X) = np(1-p)$

$= 20(0.80)(0.20)$

$= 3.2$

$SD(X) = \sqrt{3.2} = 1.79$

(c) In the context of the problem, what is a "trial"? And what is a "success"?

a trial = a bulb

success = sprouting of the bulb

Question 4: (15 points)

Neslihan, an amateur runner, decides to try a sports energy drink: She chooses 9 different running trails (yol, patika) and records her finish time for each trail twice: with and without the energy drink. Her finish times (minutes) for the trails are as follows:

	Trail #	1	2	3	4	5	6	7	8	9
X	without	10.2	19.2	5.89	8.9	6.6	20.6	9.1	30.5	40.3
Y	with	10.0	15.4	5.7	8.5	4.8	18.7	8.8	28.7	38.6

(a) Does the energy drink have any effect on her performance? (use α =0.01)

Paired data, use difference $D = X - Y$

$D = \{0.2, 3.8, 0.19, 0.4, 1.8, 1.9, 0.3, 1.8, 1.7\}$ $\bar{D} = 1.34$ $\sigma_D = 1.20$

One-population, two tailed H-test

small sample $n < 30$

$H_0: \mu_D = 0$

d.o.f = 8 $\alpha = 0.01$

$H_a: \mu_D \neq 0$

$t_{0.005, 8} = 3.355$

Reject H_0 if $|t| > 3.355$

$t = \frac{1.34}{1.20/\sqrt{9}} = 3.35 < 3.355$

Do not reject H_0

(b) Does the energy drink improve her performance? (use α =0.01)

One population, one tailed (upper) H-test

$H_0: \mu_D = 0$

$H_a: \mu_D > 0$

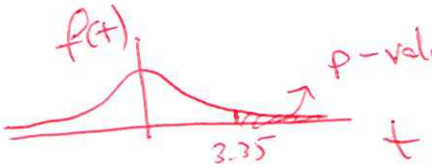
$\alpha = 0.01$ $t_{0.01, 8} = 2.896$

Reject H_0 if $|t| > 2.896$

$t = 3.35 > 2.896$

Reject H_0

(c) Estimate the *p*-value for part (b).



p-value $\simeq$ 0.005

from the table.

Question 5: (12 points)

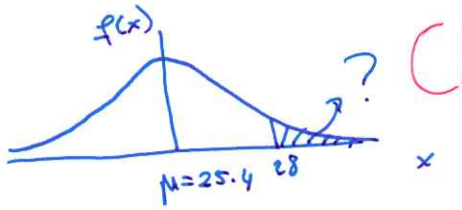
Ali Dayı's water melons (karpuz) have a normal distribution for their diameter (çap) with mean 25.4 cm and standard deviation of 7.4 cm.

(a) What is the probability that a randomly chosen water melon has a diameter larger than 28 cm?

$\mu = 25.4$ cm

$\sigma = 7.4$ cm

x: diameter



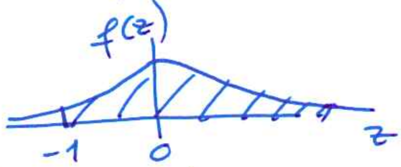
$$z = \frac{28 - 25.4}{7.4} = \frac{2.6}{7.4} = 0.3514 \rightarrow P(x > 28) = P(z > 0.35)$$

$$= 0.5 - 0.1368$$

$$= 0.36$$

(b) Supermarkets reject any water melon with diameter smaller than 18 cm. If you buy one of Ali Dayı's water melons from a supermarket, what is the probability that its diameter is larger than 28 cm?

$$P(x > 18) = P\left(z > \frac{18 - 25.4}{7.4}\right) = P(z > -1) = 0.5 + 0.3413 \cong 0.84$$



$P(x > 28) = 0.36$ (from part a)

$$P(x > 28 \mid x > 18) = \frac{P(x > 28 \text{ and } x > 18)}{P(x > 18)} = \frac{P(x > 28)}{P(x > 18)} = \frac{0.36}{0.84}$$

$$= 0.43$$

Question 6: (15 points)

On Friday nights, long lines of hungry people wait in front of the Bratwurst (mangalda kızarmış dana sosisi) stands. Salim Salam, who owns a Bratwurst stand, thinks that his customers wait less than at Siegfried's, his main competitor. Salim observes that the average waiting time for 14 people who bought a Bratwurst at Siegfried's is 8.4 min, with a standard deviation of 2.1 min. For 12 people who bought Bratwurst from Salim, the average waiting time was 6.8 min. with a standard deviation of 1.8 min. Do people wait less at Salim's stand? Test at $\alpha = 0.025$. Also, state all assumptions that have to be made to be able to do this test.

→ The waiting times for Salim's and for Siegfried's stands are normally distributed

→ The standard deviations of both waiting times are assumed to be equal

$H_0: \mu_1 - \mu_2 = 0 \quad H_1: \mu_1 - \mu_2 < 0$

μ_1: mean waiting time for Salim's stand

μ_2: " " " Siegfried's stand

$$S^2_{pool} = \frac{13(2.1)^2 + (11)(1.8)^2}{14+12-2} = 3.87$$

$$\text{Test Score} = \frac{6.8 - 8.4}{S_{pool}\sqrt{\frac{1}{14} + \frac{1}{12}}} = -2.066$$

$t_{0.025}\ (d.f. = 24) = 2.064$

⇒ Since $-2.066 < 2.064$, we reject H_0.

(there is enough statistical evidence at $\alpha = 0.025$ to conclude

Question 7:(15 points)
(a) Bratwurst lengths in Bayern have a mean of 22.4 cm, with a standard deviation of 2.4 cm. What is the probability that 52 of them have a combined length of more than 12 meters?

$$SUM \sim N\left((22.4)(52),\ 2.4\sqrt{52}\right)$$

$$Prob\,(SUM > 1200) = Pr\left[z > \frac{1200 - 1164.8}{2.4\sqrt{52}}\right]$$

$$= Pr[z > 2.033] = 0.5 - 0.4788 = 0.0212$$

(b) If the Bratwurst lengths in Bayern are distributed normally, what is the approximate probability that more than 5 out of 250 have a length of more than 28 cm? (The mean and standard deviation for Bratwurst lengths are given in (a))

$$Pr[X > 28] = Pr\left[z > \frac{28 - 22.4}{2.4}\right] = Pr[z > 2.33]$$

$\cong 0.01$ (Prob. that one Bratwurst is longer than 28 cm)

Let B be a binomial variable representing the number of successes in 250 trials with $p = 0.01$

$Pr[250 \geq B \geq 6] \cong Pr[C > 5.5]$, where

C is $\sim N(np, \sqrt{npq}) = N(2.5, 1.57)$

$$\Rightarrow Pr\left[z > \frac{5.5 - 2.5}{1.57}\right] = Pr[z > 1.91] = 0.0281$$