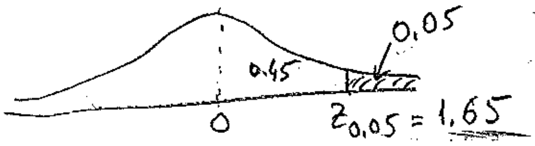
Question 1. (20 points) A trader in stock exchange market reports that the returns (=net profit) in the transactions (= buying or selling) of a certain stock is normally distributed with a mean of 2.8 TL and a standard deviation of 5.3 TL.

a) What is the 95th percentile of the returns?
b) What is the probability that the return will be less than 0, that is, a loss will occur? Round your answer to one digit after the decimal point.
c) In the next 60 next transactions of the trader, what is the probability that at least 15 of them will result in a loss? Use normal approximation to solve this problem and assume that all transactions are independent from each other.

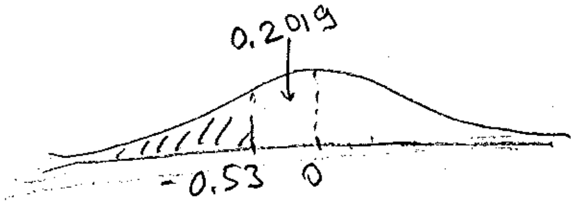
a) $P(X \leq x) = 0.95 \Rightarrow P\left(Z \leq \frac{x-2.8}{5.3}\right) = 0.95$



$1.65 = \frac{x-2.8}{5.3} \Rightarrow x = 11.545\,TL$

b) $P(X<0) = P\left(Z < \frac{0-2.8}{5.3}\right) \cong P(Z<-0.53)$

$= 0.5 - 0.2019 = 0.2981 \cong 0.3$



c) $Y \sim Bin(60, 0.3) \Rightarrow \begin{cases} E(Y) = 60(0.3) = 18 \\ V(Y) = 18(0.7) = 12.6 \end{cases}$

$P(Y \geq 15) \cong P\left(Z \geq \frac{14.5-18}{\sqrt{12.6}}\right)$

$= P(Z \geq -0.99)$

$= 0.3389 + 0.5$

$= 0.8389$

Question 2. **(15 points)** (a) What are the mean and variance of the probability distribution function for the number appearing on a roll of a fair (hilesiz) die?

(b) Prof. Şüpheci wonders whether the die he holds in his hand is fair or loaded (hileli). To test this, he rolls the die 40 times. If the die is fair, what is the probability that the sum of 40 rolls is less than 115?

(c) 50 fair dice are rolled at the same time and the average of the 50 numbers appearing on those dice is calculated. This experiment is repeated many times. Construct an interval which is centered at 3.5 and contains 68.26% of all averages in many tosses of 50 dice.

a) Probability distribution function: $p(x) = \frac{1}{6} \quad x = 1, 2, \ldots, 6$

$$E(X) = \frac{1}{6}1 + \frac{1}{6}2 + \cdots + \frac{1}{6}6 = \frac{1}{6}(1 + \cdots + 6) = 3.5$$

$$V(X) = E(X^2) - (E(X))^2$$
$$= 1^2\frac{1}{6} + 2^2\frac{1}{6} + \cdots + 6^2\frac{1}{6} - (3.5)^2$$
$$= \frac{91}{6} - 12.25 = 2.92$$

b) $E(X_1 + \cdots + X_{40}) = 40\,E(X) = 40(3.5) = 140$

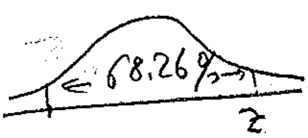
$V(X_1 + \cdots + X_{40}) = 40\,V(X)$

$\qquad\qquad\qquad = 40(2.92) = 116.8$

$$P(X_1 + \cdots + X_{40} < 115) = P\left(Z < \frac{114.5 - 140}{\sqrt{116.8}}\right)$$
(≤ 114)

$$= P(Z < -2.36)$$
$$= 0.5 - 0.4909 = 0.0091$$

c) $n = 50$



$\frac{68.26\%}{2} = 34.13\% = 0.3413$

$\Rightarrow z = 1$

$\Rightarrow \quad 3.5 \mp 1\frac{\sigma}{\sqrt{50}} \qquad \sigma = \sqrt{2.92} \cong 1.71$

$3.5 \mp 0.24 \quad \Rightarrow \quad [3.26, 3.74]$

Question 3. (15 points) To determine how the number of housing starts is affected by mortgage interest rates, an economist recorded the mortgage interest rates and the number of housing starts for the past 10 years as follows. (housing start = beginning of building new housing)

										Sum
Rate	8.5	7.8	7.6	7.5	8.0	8.4	8.8	8.9	8.5	8.0	82
Starts	115	111	185	201	206	167	155	117	133	150	1540

Here are some useful statistics: $\sum x_i^2 = 674.\overset{5}{6}$ $\sum x_i y_i = 12543.4$

a) Determine the regression line.
b) What do the coefficients of the regression line tell you about the relationship between mortgage rates and housing starts?

a) $$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{12543.4 - \frac{(82)(1540)}{10}}{674.56 - \frac{(8.2)^2}{10}}$$

$$= \frac{-84.6}{2.16} \cong -39.17$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \frac{1540}{10} + (39.17)\frac{82}{10} \cong 475.19$$

$$\boxed{\hat{y} = 475.19 - 39.17x}$$

b) Slope $\Rightarrow$ For every unit of increase in the interest rates, we expect an average decrease of 39.17 housing starts.

Intercept $\Rightarrow$ If it were possible to decrease the interest rates to 0 (no interest!) then we would expect an average of 475.19 housing starts.
(This is clearly only hypothetical, as 0 is out of range of the interest rate data)

Question 4. (20 points) Prof. Mina Sprudelt, who teaches mathematical methods for science students, suspects that some students have hacked her computer and have stolen the final exam questions. Based on the results of the final, she thinks that nine students are involved. She calculates the z-sores of all previous math and science exams for these nine students, and obtains a mean of −0.76, and also notices that they are normally distributed. She thinks that if they did not steal the questions, they will have performed as in the past; but if they did obtain the questions, their performance will be significantly better. The average z-score on her final for these nine students is −0.32 with a standard deviation of 0.46 .

(a) Is she justified in her suspicion? Test with $\alpha = 0.01$.

(b) If you know not only the population mean $\mu = -0.76$, but also the population standard deviation, $\sigma = 0.55$, how would you do the test of part (a)? What is the result?

(c) Mina wakes up in the middle of the night with the horrifying thought that these nine may have given some of the questions to other students. She calculates the mean of all grades of all her 95 students in all the math and science exams before her final: it is 58 (out of 100). The average on her final is 66, and the standard deviation is 27. If the students received some questions before the exam, she thinks, they must have done considerably better than their usual performance. Calculate the P-value and decide whether she has reason to worry.

(a) $H_0: \mu = -0.76$ $\quad n = 9$

$H_a: \mu > -0.76$ $\quad t^* = \dfrac{-0.32 - (-0.76)}{0.46/\sqrt{9}} = 2.86$

$t_{\alpha, n-1} = t_{0.01, 8} = 2.896$

Since $2.86 < 2.896$, do not reject Ho

⇒ No justification for her suspicion

(b) Use z-test since σ is known.

$H_0: \mu = -0.76$ $\quad z^* = \dfrac{-0.32 - (-0.76)}{0.55/3} = 2.4$

$H_a: \mu > -0.76$

$z_{0.01} = 2.33$ $\quad z^* > z_{0.01}$ ⇒ Reject Ho.

In this case, we conclude that she is justified in her suspicion.

(c) $H_0: \mu = 58$ $\quad n = 95$

$H_a: \mu > 58$ $\quad z^* = \dfrac{66 - 58}{27/\sqrt{95}} = 2.89$

Normal table ⇒

$P(z > 2.89)$

$= 0.5 - 0.4981$

$= 0.0019$

⇒ P-value = 0.0019

very small (less than 0.05), so reject Ho.

Yes, she has reason to worry as the students' final exam average is significantly greater than their other average 58.

Question 5. (20 points) A store sells extended warranties (=ek garanti süresi) for its products. To learn more about who buys these warranties, a random sample of size 407 is drawn. Each respondent reported whether they paid the regular price or sale price and whether they purchased an extended warranty. Accordingly, 229 customers bought at the regular price and 47 of those bought also extended warranty. The remaining customers bought at sale price and 25 of those bought also extended warranty.

a) Find the 90% confidence interval for the proportion of customers who buy extended warranty for the products of this store.

b) Does the proportion of customers who buy extended warranty for the products of this store significantly differ from [illegible]? Make an inference at 10% level of significance and using your answer to part a).
0.20

c) Can we conclude at $\alpha = 0.05$ that those who paid the regular price are more likely to buy an extended warranty?

	Extended Warranty	Sample size
Reg. price	47	229
Sale price	25	178

$407 - 229 = 178$

a) $\hat{p} = \dfrac{47+25}{407} \cong 0.177$ $\qquad z_{0.05} = 1.65$

$$0.177 \mp 1.65 \underbrace{\sqrt{\frac{(0.177)(0.823)}{407}}}_{0.0189186}$$

$[0.146 \, , \, 0.208]$

b) Since 0.20 is inside the 90% CI, we conclude that p does not significantly differ from 0.20 at $\alpha = 0.10$.

c) $\hat{p}_1 = \dfrac{47}{229} = 0.205 \qquad \hat{p}_2 = \dfrac{25}{178} = 0.14$

$H_0: p_1 - p_2 = 0$

$H_a: p_1 - p_2 > 0$

$$z^* = \frac{0.205 - 0.14}{\sqrt{(0.177)(0.823)\left(\frac{1}{229} + \frac{1}{178}\right)}} \cong 1.70$$

$z_{0.05} = 1.65$ $\qquad$ Since $1.70 > 1.65$, we reject H_0.

Yes, those who paid the regular price are more likely to buy an extended warranty.

Question 6. (20 points) (a) A sample of size 88 taken from population A has an average of 88 and a standard deviation of 8.8. A sample of size 77 taken from population B has an average of $\bar{X}$ and a standard deviation of 7.7. The P-value of the test $H_0: \mu_A - \mu_B = 0$ versus $H_1: \mu_A - \mu_B \neq 0$ is 0.0142. What is $\bar{X}$?
(b) A sample of size 16 taken from population C has an average of 16 and a standard deviation of 1.6. A sample taken from population D of size 15 has an average of 14.86 and a standard deviation of 1.5. Is the mean of population C greater than that of population D? Test with significance 0.025. (Assume that both populations are normal and have approximately the same population standard deviation)

a) $\bar{X}_A = 88 \quad S_A = 8.8 \quad n_A = 88$

$\bar{X}_B \equiv \bar{X} = ? \quad S_B = 7.7 \quad n_B = 77$

$H_0: \mu_A - \mu_B = 0$

$H_1: \mu_A - \mu_B \neq 0$

$\frac{0.0142}{2} = 0.0071$

$0.5 - 0.0071 = 0.4929 \Rightarrow z^* = 2.45$

$$z^* = \frac{88 - \bar{X}}{\sqrt{\frac{(8.8)^2}{88} + \frac{(7.7)^2}{77}}} = 2.45 \Rightarrow \bar{X} \cong 88 - 3.3 = 84.85$$

or $z^* = \frac{\bar{X} - 88}{\dots} = 2.45 \Rightarrow \bar{X} = 91.15$

b) $H_0: \mu_C - \mu_D = 0$

$H_a: \mu_C - \mu_D > 0$

$$s_p^2 = \frac{15(1.6)^2 + 14(1.5)^2}{16+15-2} \cong 2.41$$

$$t^* = \frac{16 - 14.86}{\sqrt{2.41\left(\frac{1}{16} + \frac{1}{15}\right)}} = 2.043$$

$$t_{0.025,\,16+15-2} = 2.045$$

$t^* < 2.045 \Rightarrow$ do not reject Ho.

The means of population C and D are not significantly different from each other.