Math 201: Statistics April 27, 2012

Spring 2012 Midterm #2

Closed book & notes; only a single-sided and handwritten A4 formula sheet and a calculator allowed; 90 minutes. No questions accepted!

Instructions: There are six pages (one cover and five pages with questions) in this exam. Please inspect the exam and make sure you have all 6 pages. You may only use your calculator and your formula sheet. Do all your work on these pages. If you use the back of a page, make sure to indicate that. **You may not exchange any kind of material with another student.** *You might get one bonus point for filling the front cover properly!*

Remember: ***You must show all your work to get proper credit.***

Academic Honesty Code: Koç University Academic Honesty Code stipulates that "copying from others or providing answers or information, written or oral, to others is cheating." By taking this exam, you are assuming full responsibility for observing the Academic Honesty Code.

NAME and SURNAME:____________________ SIGNATURE:____________________

INSTRUCTOR: ____________________ LECTURE TIME: ____________________

1	/20
2	/20
3	/20
4	/20
5	/20
Total:	/100

Show all your work to be eligible for partial credit and round your **final** answers to **two decimal** places in **all of the exam questions**.

Question 1 (20 points) A random variable X has the following distribution

x	-2	-1	0	1	2
$p(x)$	0.10	0.15	0.40	0.30	0.05

(a) Find $P(X > -1)$.

$P(X > -1) = P(X=0) + P(X=1) + P(X=2) = 0.40 + 0.30 + 0.05 = 0.75$

(b) Calculate μ and σ^2.

$\mu = \sum x\, p(x) = -2(0.10) + (-1)(0.15) + 0(0.40) + 1(0.30) + 2(0.05) = 0.05$

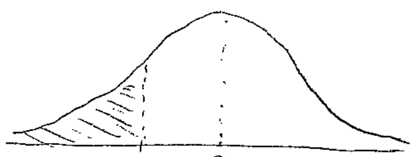
$\sigma^2 = \sum x^2 p(x) - \mu^2 = (-2)^2(0.10) + (-1)^2(0.15) + 0^2(0.40) + 1^2(0.30) + 2^2(0.05) - (0.05)^2$

$\simeq 1.05$

$\sigma \simeq 1.02$

(c) If a random sample of size 64 is chosen from this distribution, find the probability that their sum is smaller than or equal to 1.

$P(X_1 + \cdots + X_{64} \leq 1) = P(\bar{X} \leq \frac{1}{64}) \simeq P\left(Z \leq \frac{\frac{1}{64} - 0.05}{1.02/\sqrt{64}}\right)$



$= P(Z \leq -0.27)$

$= 0.5 - 0.1064$

$= 0.3936$

$= 0.39$

Question 2 (20 points): According to the Dental Association, 60% of all dentists use nitrous oxide (laughing gas) in their dental practice.

(a) In a random sample of 7 dentists, find the probability that the number of dentists using laughing gas is more than 5.

$$P(x>5) = P(x=6)+P(x=7) = \binom{7}{6}\cdot(0.60)^6(0.40)+\binom{7}{7}(0.60)^7 \cong 0.1586 = 0.16$$

(b) In a random sample of 20 dentists, find the probability that at least 10 of them use laughing gas. Answer by using a Binomial table.

$$P(x \geq 10) = 1 - P(x \leq 9) = 1 - 0.128 = 0.872 = 0.87$$

(c) What is the expected number of dentists using laughing gas in a random sample of 100 dentists? What is the variance?

$$\mu = np = 100\,(0.6) = 60$$

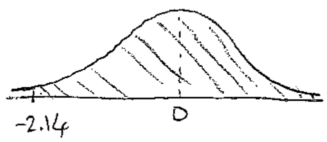
$$\sigma^2 = npq = 100\,(0.6)\,(0.4) = 24$$

(d) Find the probability that at least half of the dentists in a random sample of 100 dentists use laughing gas.

$$P(X \geq 50) \cong P\left(z \geq \frac{(50-0.5)-60}{\sqrt{24}}\right) = P(z \geq -2.14) = 0.5+0.4838$$

$$= 0.9838$$

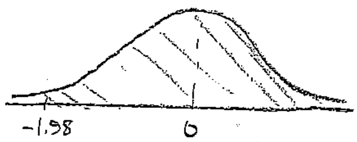
$$= 0.98$$



Question 3: **(20 points)**

(a) Find $P(Z > -1.98)$.

$P(Z > -1.98) = 0.5 + 0.4761 = 0.9761$
≈ 0.98



(b) From the mid-1960s to the early 1990s, there was a slow but steady decline (sürekli düşüş) in the Scholastic Aptitude Test (SAT) scores. For the Verbal SAT, the average in 1967 was about 466; by the 1994 the average was down to about 423. However, the standard deviation stayed close to 110. Assume that the histograms for the 1967 and the 1994 Verbal SAT scores follow the normal curve. Obtain the percentage of students scoring over 600 on the Verbal SAT in 1967. Also find the percentage of students scoring over 600 on the Verbal SAT in 1994.

$$P(X > 600) = P\left(Z > \frac{600-466}{110}\right) = P(Z > 1.22) = 0.5 - 0.3888 = 0.1112$$

So 11.12 % of the students scored over 600 in 1967.

$$P(Y > 600) = P\left(Z > \frac{600-423}{110}\right) = P(Z > 1.61) = 0.5 - 0.4463 = 0.0537$$

So 5.37 % of the students scored over 600 in 1994

0.675

(c) The histogram of heights for a certain group of women follows the normal curve. The 25th percentile of the height distribution is 158 cm and the 75th percentile is 167 cm. Find the 90th percentile of the height distribution. (Hint: first you need to find the mean and standard deviation of the height of women.)

$$\left.\begin{array}{l} P(X \leq 158) = 0.25 \Rightarrow \dfrac{158-\mu}{\sigma} = -0.675 \\ P(X \leq 167) = 0.75 \Rightarrow \dfrac{167-\mu}{\sigma} = 0.675 \end{array}\right\} \begin{array}{l} 158-\mu = -0.675\sigma \\ + \; 167-\mu = 0.675\sigma \\ \hline 325 - 2\mu = 0 \Rightarrow \mu = 162.5 \end{array}$$

So $\dfrac{167-162.5}{\sigma} = 0.675 \Rightarrow \sigma = 6.67$

$$P(X \leq a) = 0.9 \Rightarrow P\left(Z \leq \frac{a-162.5}{6.67}\right) = 0.9$$

$$\Rightarrow \frac{a-162.5}{6.67} = 1.28 \Rightarrow a = 171.04$$

So, 171.04 is the 90th percentile.

Question 4: (20 points)

(a) A study of college students nationwide found that the mean hours of sleep students get the night before an exam is 6 hours, with standard deviation 1.9 hours. The distribution is bell-shaped (i.e., you can assume it has a normal distribution). What is the probability that a randomly selected student gets between 2 and 5 hours of sleep before an exam?

X = hours of sleep students get the night before an exam has $N(6, 1.9)$.

$$P(2 < X < 5) = P\left(\frac{2-6}{1.9} < \frac{X-6}{1.9} < \frac{5-6}{1.9}\right)$$

$$= P(-2.11 < Z < -0.53)$$

$$= 0.4826 - 0.2019 = 0.2807 \text{ or } \underline{\underline{0.28}}$$

(b) At Koç University (KU), a random sample of 40 students is taken. The mean hours of sleep these students get the night before an exam is 5.5 hours. The standard deviation is found to be same as the nationwide college student population. What is the 95% CI on the mean sleep hours for KU students? Do KU students sleep less than the nationwide average? Explain.

$n = 40$, $\bar{X} = 5.5$, $s = 6$

$n \geq 30$, so 95% CI for μ is $\bar{X} \mp z_{0.025} \frac{s}{\sqrt{n}}$

$$= 5.5 \mp 1.96 \frac{1.9}{\sqrt{40}} = 5.5 \mp 0.59 = (4.91, 6.09)$$

We are 95% confident that average sleep hours is between 4.91 and 6.09 hours, which includes 6 hours as a plausible value. So KU students sleep about the same as nationwide average.

Question 5 (20 points): The following table shows the joint distribution of happiness Y and income X where the values are Y=0 (not happy), Y=1 (fairly happy), Y=2 (very happy), and the income is in thousand dollars per year.

		Y		
		0	1	2
X	15	0.04	0.07	0.05
	30	0.03	0.13	0.11
	50	0.02	0.14	0.15
	90	0.01	0.09	0.16

(a) Find the marginal probability distribution of income.

X	15	30	50	90
p(x)	0.16	0.27	0.31	0.26

(b) Are X and Y independent? According to your answer to this question, comment (=interpret(yorumla)) if money makes people happier or not.

$P(X=15)\cdot P(Y=0) = (0.16)(0.1) = 0.016 \neq 0.04 = P(X=15, Y=0)$

So X and Y are not independent.

According to data in the table, money makes people happier.

(c) What percent of people in the population are happy (fairly or very happy)?

$P(Y=1)+P(Y=2) = 1-P(Y=0) = 1-0.1 = 0.9$ So 90% of people are happy.

(d) Find Cov(X,Y).

$\mu_X = \sum x p(x) = 15(0.16)+30(0.27)+50(0.31)+90(0.26) = 49.4$

$\mu_Y = \sum y p(y) = 0(0.1)+1(0.43)+2(0.47) = 1.37$

$E(XY) = \sum xy\, p(x,y) = 15\cdot 1(0.07)+15\cdot 2(0.05)+30\cdot 1\cdot(0.13)+30\cdot 2(0.11)+50\cdot 1(0.14)+50\cdot 2\cdot(0.15)$
$+90\cdot 1(0.09)+90\cdot 2\cdot(0.16)$

$= 71.95$

$Cov(X,Y) = E(XY) - \mu_X \mu_Y = 71.95-(49.4)(1.37) = 4.272$

$= 4.27$