Math 201 Fall 2013 Final Exam Solutions

Question 1. **(20 points)** The time it takes for a certain pain reliever (=ağrı kesici) to reduce symptoms is assumed to be normally distributed.

a) In a random sample of 25 patients, the mean and the standard deviation of the time for reducing the pain are found to be 28.3 min. and 5.1 minutes, respectively. Is there sufficient evidence that the mean time to reduce the pain for this pain reliever is less than 30 minutes at 5% level of significance?

$H_0: \mu = 30$

$H_a: \mu < 30$

$\alpha = 0.05 \Rightarrow t_{0.05,24} = 1.711$

$n = 25$

$$t^* = \frac{28.3 - 30}{5.1/\sqrt{25}} = -1.667$$

Since $-1.667 > -1.711$, do not reject H_0.

The mean time to reduce the pain is not significantly less than 30 min.

b) Suppose the true mean to reduce the pain for this pain reliever is 30 minutes and the true standard deviation is 4 minutes. What is the probability that it will take the medication between 35 and 40 minutes to begin to work?

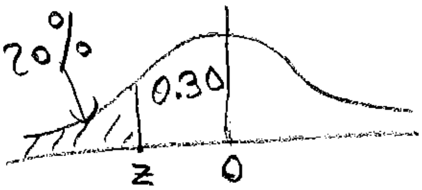
$$P(35 < X < 40) = P\left(\frac{35-30}{4} < Z < \frac{40-30}{4}\right)$$

$$= P(1.25 < Z < 2.5)$$

$$= 0.4938 - 0.3944$$

$$= 0.0994$$

c) Find the 20th percentile of the distribution of the time it takes for this pain reliever to reduce the symptoms, assuming the parameter values given in part b).



$\Rightarrow z = -0.84$

$$\frac{x - 30}{4} = -0.84$$

$\Rightarrow x = 26.64$ minutes

Question 2. **(20 points)** The percentage of times a person wins on slot machines currently available in a casino is 8%.

a) If a person plays in the slot machines 300 times independently, find approximately the probability of winning at most 20 times.

$X \sim Bin(300, 0.08) \Rightarrow np = 24, \; npq = 22.08$

$$P(X \leq 20) \cong P\left(Z \leq \frac{20.5 - 24}{\sqrt{22.08}}\right)$$

$$= P(Z \leq -0.74)$$

$$= 0.5 - 0.2704$$

$$= 0.2296$$

b) A new machine is purchased for the casino. To estimate the probability of winning, the casino manager has played it 500 times and found out that she won 37 times. Construct a 99% confidence interval for the probability of winning for the new machine.

$$\hat{p} = \frac{37}{500} = 0.074 \qquad z_{0.005} = 2.58$$

$$0.074 \mp 2.58 \sqrt{\frac{(0.074)(0.926)}{500}}$$

$$\Rightarrow [0.043, 0.104] \Rightarrow [4.3\%, 10.4\%]$$

c) The new machine is claimed to have a smaller probability of winning than the currently available machines in the casino. Test this claim at α=0.01.

$H_0: p = 0.08$

$H_a: p < 0.08$

$$z^* = \frac{0.074 - 0.08}{\sqrt{\frac{(0.08)(0.92)}{500}}} = -0.49$$

Since $-0.49 > -z_{0.01} = -2.33$, do not reject H_0.

There is not sufficient evidence to prove that the new machine has smaller probability.

Question 3. (20 points) [redacted] The owner of an appliance store produced the following joint probability distribution ~~of~~ for the number of refrigerators and stoves sold daily in her store.

X		Y: Refrigerators			
	Stoves	**0**	**1**	**2**	$p(x)$
	0	0.08	0.14	0.12	0.34
	1	0.09	0.17	0.13	0.39
	2	0.05	0.18	0.04	0.27
	$p(y)$	0.22	0.49	0.29	

a) In a given day, what is the probability that at least one stove is sold in this store?

$$p(1)+p(2) = 0.39+0.27 = 0.66$$

b) What is the approximate probability that at least 100 stoves are sold in six months? Assume that each month has 20 working days. $\Rightarrow 20\times 6 = 120$ days.

$$P(X_1+\dots+X_{120} \geq 100) = ?$$

$$\sim N(120\mu, \sqrt{120}\,\sigma)$$

$$\mu = (0.39)1 + (0.27)2 = 0.93$$

$$\sigma^2 = 1^2(0.39) + 2^2(0.27) - (0.93)^2 = 0.6051 \Rightarrow \sigma \cong 0.778$$

$$\sim N(111.6,\ 8.52)$$

$$\Rightarrow P\left(Z \geq \frac{100-0.5-111.6}{8.52}\right) = P(Z \geq -1.42)$$

$$= 0.5 + 0.4222$$

$$= 0.9222$$

c) Are the number of stoves and number of refrigerators sold daily in this store independent?

$$P(X=0, Y=0) \stackrel{?}{=} P(X=0)\,P(Y=0)$$

$$0.08 \stackrel{?}{=} (0.34)(0.22) \quad \text{No.}$$

They are not independent.

d) What is the probability that $\underbrace{\text{at least one stove is sold}}_{A}$, or $\underbrace{\text{at most one refrigerator is sold}}_{B}$ in a given day?

$$P(A\cup B) = (0.39+0.27) + (0.22+0.49) - (0.09+0.17+0.05+0.18)$$

$$= 0.88$$

e) The store has 7 stoves and 12 refrigerators in display. If 5 appliances are randomly selected, what is the probability that 2 will be stoves and 3 will be refrigerators? $7+12=19$ total

$$\frac{\binom{7}{2}\binom{12}{3}}{\binom{19}{5}} = \frac{\frac{7\cdot 6}{2\cdot 1}\cdot\frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}}{\frac{19\cdot 18\cdot 17\cdot 16\cdot 15}{5\cdot 4\cdot 3\cdot 2\cdot 1}} \cong 0.397$$

Question4. (13 points) The effect an advertisement shown on TV for the products of a company is analyzed for two different age groups: "Under 30" and "30 to 49". In a random sample of 100 respondents from the first group, 49% liked the advertisement. In a random sample of 150 respondents from the second group, 36% liked it. Is there a significant difference between the population proportions for the two age groups? Conduct a test of hypothesis using P-value.

$H_0: p_1 = p_2$

$H_a: p_1 \neq p_2$

$$\hat{p} = \frac{100(0.49) + 150(0.36)}{250} = \frac{49+54}{250} = 0.412$$

$$z^* = \frac{0.49 - 0.36 - 0}{\sqrt{(0.412)(0.588)\left(\frac{1}{100} + \frac{1}{150}\right)}} = \frac{0.13}{0.06354}$$

$$\cong 2.05$$

$$\text{P-value} = 2\,P(z > 2.05)$$
$$= 2\,(0.5 - 0.4798) = 0.0404$$

Since P-value < 0.05 we reject H_0.

There is a significant difference between the two age groups.

Question 5. (15 points) A researcher suggests that male nurses earn more than female nurses. A survey of independent samples of 15 male and 15 female nurses produced the following summary statistics for their yearly incomes given as MINITAB (statistics software) output.

[redacted]

Variable	N	Mean	StDev	SE Mean
MALE	15	23800	300	77.46
FEMALE	15	23650	250	64.55

a) Is there enough evidence to support the claim of the researcher? Assume that the standard deviations of income are different from each other for the populations of male and female nurses. Report the P-value.

Since $\sigma_1 \neq \sigma_2$ and $n_1 = n_2$, standard error of $\bar{x}_1 - \bar{x}_2$ is $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ and $d.f. = n_1 + n_2 - 2$

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

$$t^* = \frac{23800 - 23650}{\sqrt{\frac{300^2}{15} + \frac{250^2}{15}}} \cong 1.488$$

Since $\underbrace{1.313}_{t_{0.1}} < 1.488 < \underbrace{1.701}_{t_{0.05}}$

$\Rightarrow 0.05 < \text{P-value} < 0.1$!

Since P-value is large, we do not reject Ho.

There is not sufficient evidence to support the claim that male nurses earn more.

b) Using the sample statistics given in MINITAB output to estimate the expectation μ and variance σ^2 of the income distributions for male and female nurses, separately, estimate the expectation and variance of the family income of a married couple of nurses by assuming that their incomes are independent.

$\hat{\mu}_1 = 23800 \quad \hat{\mu}_2 = 23650 \quad \hat{\sigma}_1 = 300 \quad \hat{\sigma}_2 = 250$

$E(X_1 + X_2) = 23800 + 23650 = 47450$ \$

$V(X_1 + X_2) = V(X_1) + V(X_2) = 300^2 + 250^2$

$= 152500$ \$

Question 6. (20 points) Investment analysts generally believe that the interest rate on bonds is inversely related to the prime interest rate for loans. The aim is to predict the bond rates by the prime interest rate. The following data are collected for this purpose.

X = Prime Interest Rate (%)	Y = Bond Rate (%)	X^2	XY
5	16	25	80
12	6	144	72
9	8	81	72
15	4	225	60
$\Sigma x = 41$	$\Sigma y = 34$	$\Sigma x^2 = 475$	$\Sigma xy = 284$

a) Find the regression equation for the given data set.

$$\hat{\beta}_1 = \frac{SS_{XY}}{SS_{XX}} = \frac{284 - \frac{(41)(34)}{4}}{475 - \frac{(41)^2}{4}} = \frac{-64.5}{54.75}$$

$$\cong -1.18$$

$$\hat{\beta}_0 = \frac{34}{4} - (-1.18)\frac{41}{4} \cong 20.6$$

$$\hat{y} = -1.18x + 20.6$$

b) Estimate the bond rate when the prime interest rate is 7%.

$$\hat{y} = -1.18(7) + 20.6 = 12.34\ \%$$

c) Construct a 90% confidence interval for the mean of the prime interest rate by using the given data set and assuming that the distribution of the prime interest rate is normal.

$$\bar{x} = \frac{41}{4} = 10.25 \qquad s^2 = \frac{\Sigma x^2 - n\bar{x}^2}{n-1} = \frac{475 - 4(10.25)^2}{3}$$

$$= 18.25 \Rightarrow s \cong 4.27$$

$$\Rightarrow \bar{x} \mp t_{0.05,3}\frac{s}{\sqrt{4}} \qquad 10.25 \mp 2.353\,\frac{(4.27)}{2}$$

$$\Rightarrow [5.23, 15.27]$$