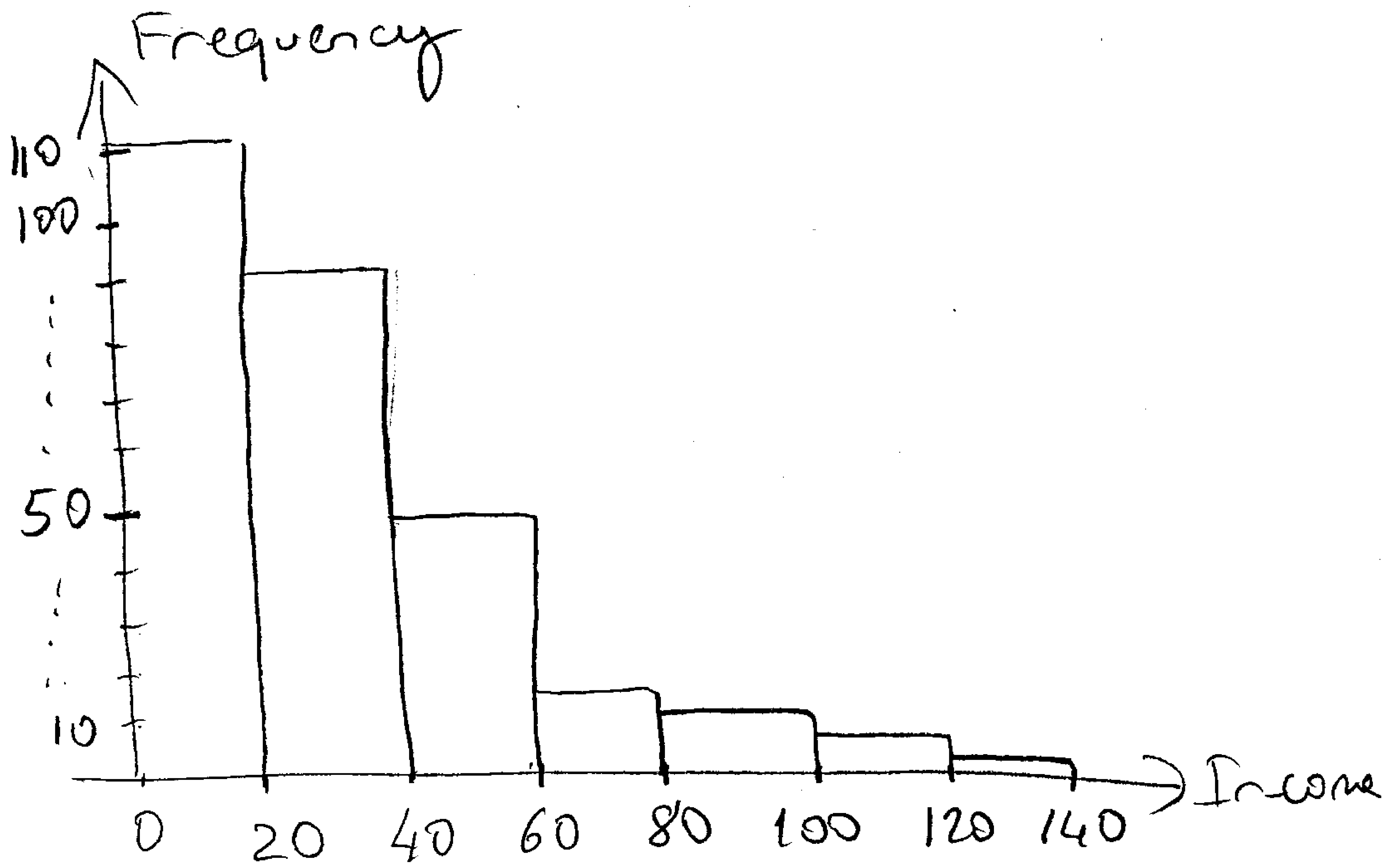
Math 201 Fall 2013, Exam 1 Solutions

Question 1 Consider the following frequency table for the yearly income of a random sample of 285 people. Income is reported in thousand TL.

Income	Frequency
0-20	111
20-40	90
40-60	49
60-80	18
80-100	* 8
100-120	6
120-140	3

\+ 277

$* = 285 - 277 = 8$



a) Find the missing frequency (shown as *). Then draw a frequency histogram of the data set in the space above.

b) In which intervals are the median, upper quartile and lower quartile?

$\frac{285}{2} = 142.5 \nearrow 143^{rd}$ observation is the median $\Rightarrow$ in $[20-40]$ interval. Since $111+90 > 143$

$Q_L = (285)(0.25) = 71.25 \nearrow 72^{nd} \Rightarrow [0,20]$

$Q_U = (285)(0.75) = 213.75 \nearrow 214 \Rightarrow [40,60]$

c) What is the relative frequency of people with income above 80,000 TL?

$\frac{8+6+3}{285} \cong 0.06$

d) Is the mean or the median a better measure of the center in this data set? Explain.

The data set is right skewed. The mean is affected by the right tail. So median is a better measure of the center in this case

1, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7

Question 2 Consider the following data set: 4,1,4,5,6,6,3,4,5,7,3,6,4,5

a) Find the mean and the standard deviation of the sample.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{63}{14} = 4.5$$

$$s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{n-1} = \frac{315-(14)(4.5)^2}{13} = 2.42$$

$s \cong 1.56$

b) If the highest observation 7 is replaced by 9, does the mean increase or decrease? ~~Does the standard deviation increase or decrease~~? What can you say about the median?

Arithmetic average, that is, the mean will increase,

The median will not change.

c) Use Chebyshev's rule to estimate the percentage of observations within $\bar{x} \pm 2s$

indicates that at least 75% are within $\bar{x} \mp 2s$

d) What is the true percentage of observations within $\bar{x} \pm 2s$?

$\bar{x} \mp 2s \Rightarrow [1.38, 7.62] \rightarrow$ 13 out of 14

$4.5 \mp 2(1.56)$

$13/14 \cong 0.93 = 93\%$

e) Find the 90th percentile.

$14(0.9) = 12.6 \rightarrow 13^{th}$ obs^n.

It is 6.

f) How many different orderings is possible for the observations given in the data set above?

$$\frac{14!}{1!\,2!\,4!\,3!\,3!\,1!} = \frac{14.13.\cancel{12}.11.10.9.8.7.\cancel{6}.5}{\cancel{2}.\cancel{3}\cancel{2}.\cancel{3}.\cancel{2}}$$

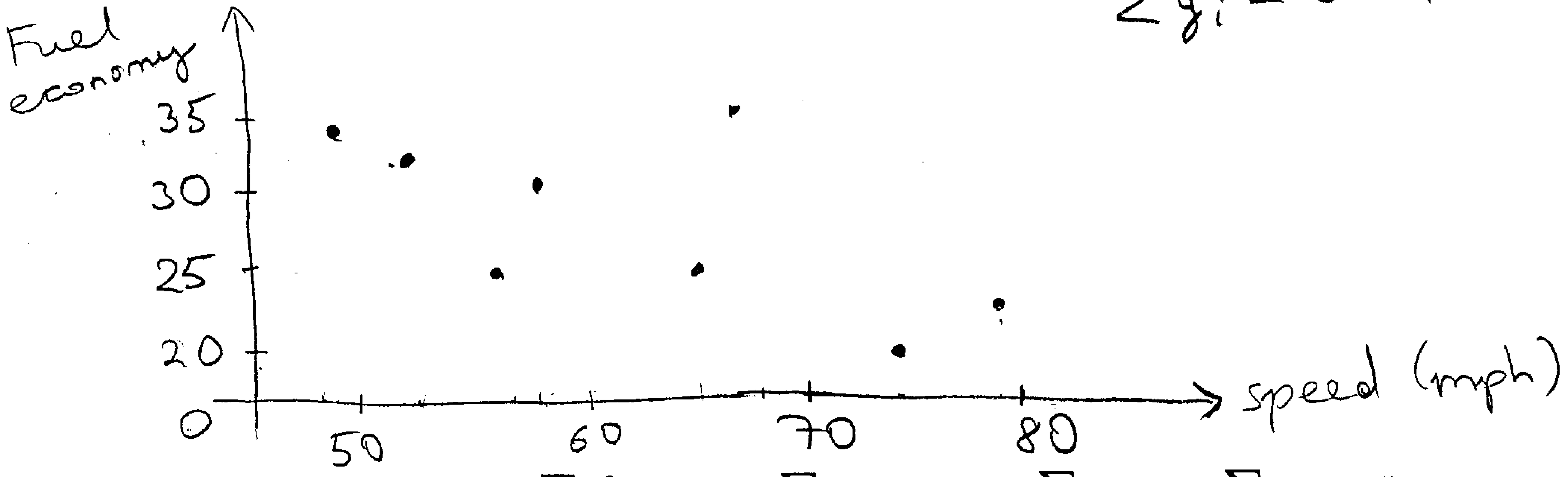
$= 50450400$

Question 3. It is reported in US Government's official web site for fuel economy information that gas mileage (measured in miles per gallon) usually decreases rapidly at speeds above 50 mph (miles per hour). In order to check this information, a data set is obtained as follows. (1 mile = 1.6 km, and 1 gallon = 3.8 litre)

x	Speed (mph)	48	52	58	65	74	79	66	57
y	Fuel economy (mpg)	34	32	30	25	20	23	35	25

a) Draw a scatter plot of the data.

$\Sigma y_i^2 = 6484$



b) Find the regression equation. (Hint: $\sum x_i^2 = 31919$ $\sum x_i y_i = 13693$ $\sum x_i = 499$ $\sum y_i = 224$)

$$\hat{\beta}_1 = \frac{13693 - \frac{(499)(224)}{8}}{31919 - \frac{(499)^2}{8}} = \frac{-279}{793.875} \cong -0.35$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \frac{224}{8} - \frac{499}{8}(-0.35) \cong 28 + 21.83 = 49.83$$

$$\hat{y} = 49.83 - 0.35x$$

c) Interpret the slope.

If you increase your speed 1 mph, you can expect to go 0.35 miles less with 1 gallon of gas.

d) Can you interpret the intercept? If you can, then interpret. If not, explain why not.

Intercept corresponds to x=0. However, 0 is not in the range of x values. So, it does not make sense to interpret it as such (that is, y for x=0)

e) Find the correlation coefficient. What does its value indicate?

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}}\sqrt{SS_{yy}}} \qquad SS_{yy} = 6484 - \frac{(224)^2}{8} = 212$$

$$\Rightarrow r = \frac{-279}{\sqrt{793.875}\sqrt{212}} = -0.68$$

medium negative correlation.

It indicates a negative, medium strength, linear relationship between speed and fuel economy.

Question 4. The following table classifies the computers in an office according to their type and usage.

	Desktop	Laptop	Tablet
New	3	5	7
Used	8	0	2

Total: 25

a) If a randomly selected computer from this office is a desktop, what is the probability that it is new?

A total of 11 desktops $(3+8=11)$

$\Rightarrow P(N|D) = \frac{3}{11}$ (since 3 of them are new)

b) Find the probability that a randomly selected computer from this office is a tablet or a used computer.

$P(T) = \frac{7+2}{25} = \frac{9}{25}$ $\quad P(U) = \frac{8+2}{25} = \frac{10}{25}$

$P(T \cap U) = \frac{2}{25} \Rightarrow P(T \cup U) = \frac{9}{25} + \frac{10}{25} - \frac{2}{25}$

$= \frac{17}{25} = 0.68$

c) Are the events "the selected computer is a laptop" and "the selected computer is a used one" independent?

$P(L \cap U) = 0$, but $P(L) = 5/25 = 1/5$

and $P(U) = 10/25$

Since $P(L \cap U) \neq P(L) \cdot P(U)$, they are not independent.

d) Are the events "the selected computer is a laptop" and "the selected computer is a used one" mutually exclusive?

Yes, because $P(L \cap U) = 0$.

e) 3 computers are randomly selected from this office without replacement. Let X be the number of laptop computers among those 3. Find the probability distribution of X.

A total of 5 laptops, 20 other computers.

X can be 0, 1, 2, 3.

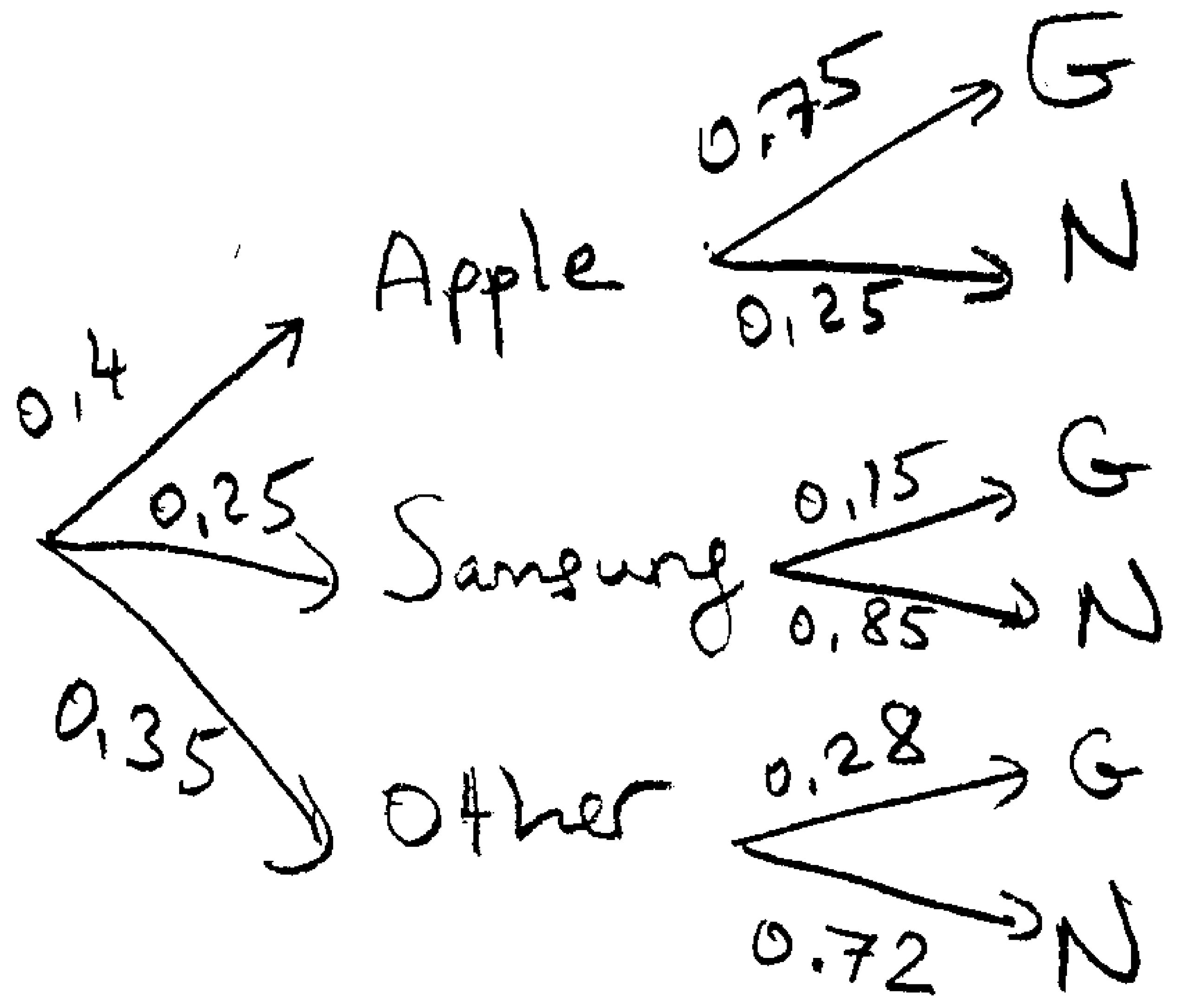
$P(X=0) = \frac{\binom{5}{0}\binom{20}{3}}{\binom{25}{3}} = 0.496$ $\quad P(X=2) = \frac{\binom{5}{2}\binom{20}{1}}{\binom{25}{3}} = 0.087$

$P(X=1) = \frac{\binom{5}{1}\binom{20}{2}}{\binom{25}{3}} = 0.413$ $\quad P(X=3) = \frac{\binom{5}{3}\binom{20}{0}}{\binom{25}{3}} = 0.004$

Question 5. Of the iPhone owners, 75% upgraded their phones to iPhone 5s recently. It is also estimated that 15% of Samsung phone owners has switched to iPhone 5s. This percentage is 28% for the other smart phone owners. Note that the market share of iPhone (by Apple) was 40% and the share of Samsung phones was 25% in the smart phone market, right before iPhone 5s was launched.

a) Find the probability that a current iPhone 5s user had owned a Samsung phone before. (Assume that only smartphone users buy an iPhone 5s, and that those who do not have a smart phone already will not buy it).

G: upgrade to iphone 5S
N: do not upgrade



$$P(S \mid G) = ?$$

$$\frac{P(S \cap G)}{P(G)}$$

$$\Rightarrow \frac{(0.25)(0.15)}{(0.25)(0.15)+(0.4)(0.75)+(0.35)(0.28)}$$

$$= \frac{0.0375}{0.4355} \cong 0.086$$

b) Find the probability that a randomly selected smart phone owner is an iPhone 5s user and was an iPhone user before as well.

$$P(A \cap G) = (0.4)(0.75)$$

$$= 0.3$$

c) If 60% of US people $\underbrace{\text{own a smart phone}}_{S}$, what is the probability that a randomly selected person from US is an $\underbrace{\text{iPhone 5s user}}_{I5}$?

$$P(S \cap I5) = \underbrace{(0.6)}_{P(S)} \underbrace{\left[(0.25)(0.15)+(0.4)(0.75)+(0.35)(0.28)\right]}_{P(I5 \mid S)}$$

$$\cong 0.26$$