Math 201 Spring 2013 MT 1 Solutions

Question 1

a) Consider the following stem and leaf display for a data set of exam scores. Find the 60^{th} percentile, the range, and the interquartile range. Also find the relative frequency of students with scores above 90.

3	8 9
4	2 3 3 5 6 7
5	0 1 1 2 3 4 5 6
6	1 2 2 2 4 5 5 6 7 8 8
7	0 0 2 6 6 6 7 8
8	2 3 5 8 9 9
9	4 6 7 8
10	0

$n = 46$ Range $= 100 - 38 = 62$

$46(0.6) = 27.6 \nearrow 28$

$\Rightarrow$ 60^{th} percentile $= 70$

$Q_L = 25^{th}$ percentile $46(0.25) = 11.5 \nearrow 12$

$\Rightarrow Q_L = 52$

$Q_U = 75^{th}$ percentile $= 46(0.75) = 34.5 \nearrow 35$

$\Rightarrow Q_U = 78$ $\Rightarrow$ IQR $= 78 - 52 = 26$

Freq. of 90 or above : 5

$\Rightarrow$ Rel. freq. $= \frac{5}{46} \cong 0.109$

b) The z-scores of 64 and 85 in the data set given in part a) are -0.16 and 1.04, respectively. Find the mean and the standard deviation, and give an interval that contains approximately 95% of the observations using empirical rule. Is using empirical rule justified in view of the shape of the distribution?

$\frac{64 - \bar{x}}{s} = -0.16$ $\frac{85 - \bar{x}}{s} = 1.04$

$64 = \bar{x} - 0.16s$ $85 = \bar{x} + 1.04s$

$-\ 64 = \bar{x} - 0.16s$

$21 = 1.2s$

$\Rightarrow s = 17.5 \Rightarrow \bar{x} = 66.8$

$\bar{x} \mp 2s \Rightarrow 66.8 \mp 2(17.5) \Rightarrow [31.8, 101.8]$ contains app. 95% of scores

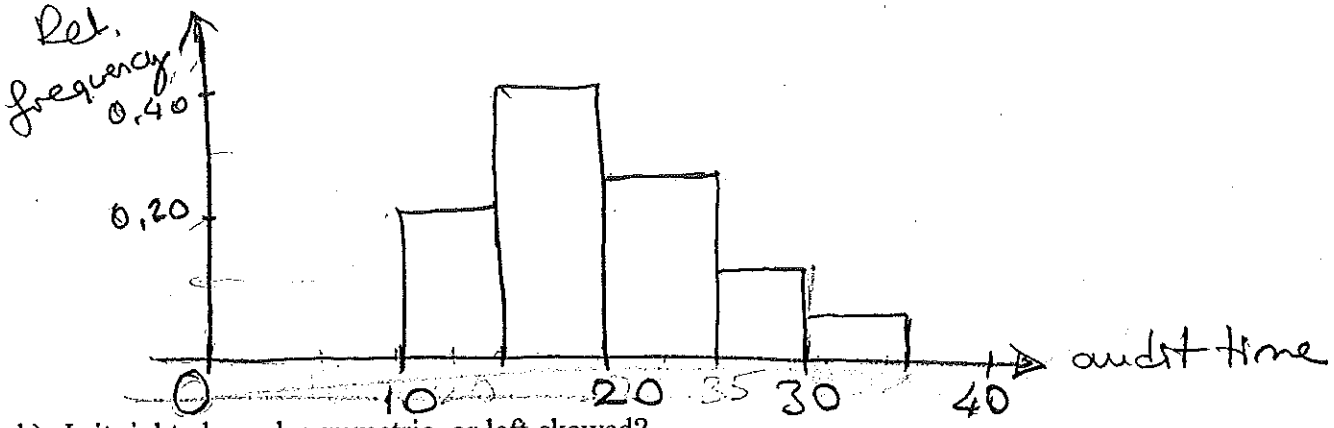
The distribution is approximately bell-shaped (see the stem-and-leaf display), so it is justified.

Question 2 The frequency table for the audit (=teftiş) time in a company is given below.

Audit time (days)	10-15	15-20	20-25	25-30	30-35	total
Frequency	4	8	5	2	1	20

Rel. freq. $4/20 = 0.20$ $8/20 = 0.40$ $5/20 = 0.25$ $2/20 = 0.10$ $1/20 = 0.05$

a) Draw a relative frequency histogram.



b) Is it right-skewed, symmetric, or left-skewed?

It is right-skewed as it has a right tail.

c) In which class is the median? ~~Which is the modal class?~~

It's in 15-20, because $0,20+0,40 > 0,50$

d) The mean and the standard deviation of the audit times are 19.1 and 4.2, repectively. Use Chebyshev's rule to estimate the percentage of observations within the interval [10.7, 27.5].

$19.1 \mp 2(4.2) \Rightarrow [10,7;\ 27.5]$

So this interval contains at least 75% of the observations according to Chebyshev's rule.

e) The following audit times are obtained in another company:

14.8, 12.2, 30.1, 20.7, 23.4, 18.8, 15.6

Find the standard deviation.

$$\bar{x} = \frac{14.8+12.2+\dots+15.6}{7} \cong 19.37$$

$$s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{n-1} = \frac{(14.8)^2+\dots+(15.6)^2 - 7(19.37)^2}{6}$$

$$= \frac{2846.74 - 2626.3783}{6} \cong 36.73 \Rightarrow s \cong 6.06$$

Question 3 Two cards are drawn in succession without replacement from a deck of cards. A face card is Jack, Queen, or King, and the others are number cards (the Ace is counted as the number 1). 3

a) How many elements does the sample space have?

$$52 \times 51 = 2652$$

b) What is the probability of getting a face card on first draw?

$4 \times 3 = 12$ face cards $\Rightarrow \frac{12}{52} \cong 0.23$

c) How many elements does the event "face card on second draw" have?

$52 - 12 = 40$

$\underline{F}\ \underline{F}$ or $\underline{N}\ \underline{F}$

$12 \times 11 + 40 \times 12 = 612$

d) How many elements does the event "face card on first draw and number card on second draw" have?

$\underline{F_1}\ \underline{N_2}$

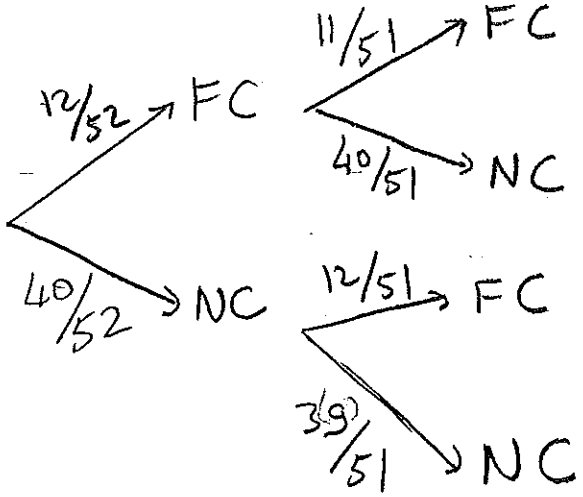
12×40

e) What is the probability of getting a face card on first draw or getting a number card on second draw?

$P(F) = 0.23$ $\quad P(N_2) = \frac{40 \times 39 + 12 \times 40}{52 \times 51} \cong 0.77$

$P(F \cap N_2) = \frac{12 \times 40}{52 \times 51} \cong 0.18$ $\quad \Rightarrow P(F \cup N_2) = 0.23 + 0.77 - 0.18 = 0.82$

f) Draw a probability tree for this experiment. Use the symbols FC and NC. Don't forget the probabilities!



g) What is the probability that a face card will appear before a number card?

$$\frac{12}{52} \times \frac{40}{51} \cong 0.18$$

Question 4 A manager investigates the relationship between usage of a computerized equipment and maintanence (=bakım) cost. The data are as follows.

Weekly usage (hours)	13	10	20	28	32	17	24	31	40	38
Maintenance cost (in 100$)	17	22	30	37	47	30.5	32.5	39	51.5	40

Some statistics: $\sum x_i^2 = 7347$ $\Sigma xy = 9668.5$ $\Sigma x = 253$ $\Sigma y = 346.5$

$\bar{x} = 25.3$ $\bar{y} = 34.65$

a) Find the regression equation.

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{9668.5 - \frac{(253)(346.5)}{10}}{7347 - \frac{(253)^2}{10}} = \frac{902.05}{946.1} \cong 0.95$$

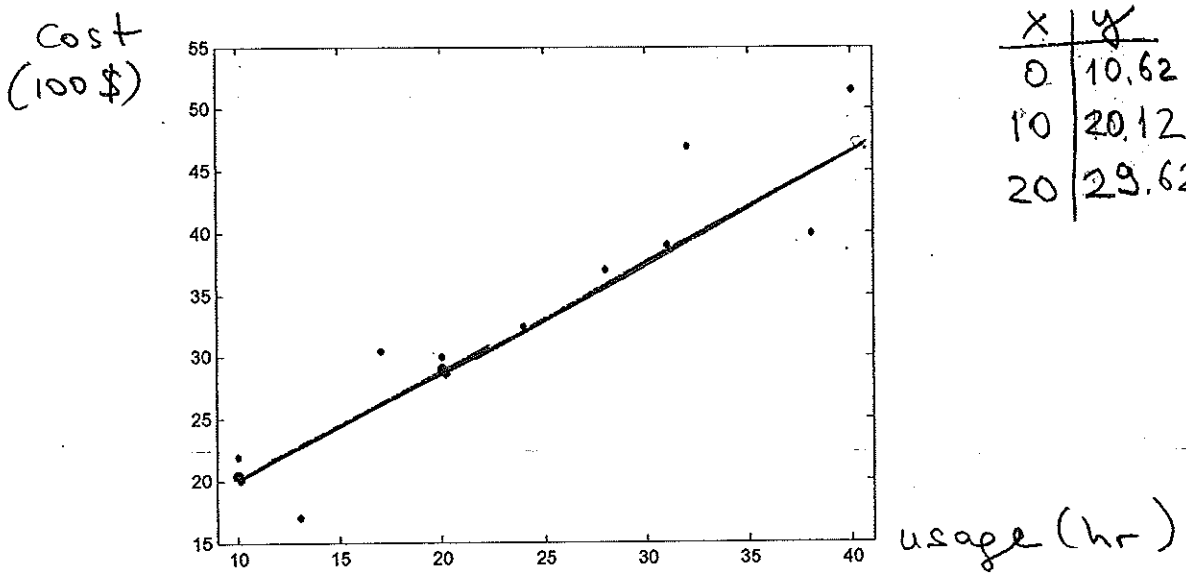
$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 34.65 - (0.95)25.3 \cong 10.62$$

$$\hat{y} = 10.62 + 0.95x$$

b) Plot the regression equation on the following scatterplot.

$\hat{y} = 10.62 + 0.95x$

x	y
0	10.62
10	20.12
20	29.62



c) If the equipment will be used for 30 hours per week, what is the estimated cost?

$$\hat{y} = 10.62 + 0.95(30) = 39.12$$

$$= 3912\ \$$$

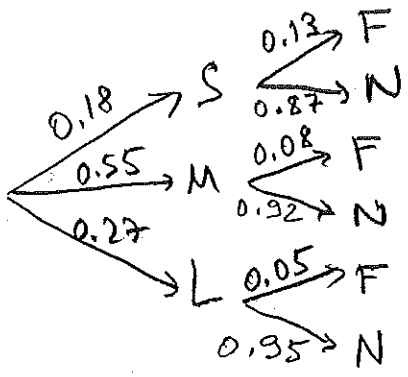
d) Find the vertical error for the data point (40, 51.5).

$$\hat{y} = 10.62 + 0.95(40) = 48.62$$

$$\text{error} = y - \hat{y} = 51.5 - 48.62 = 2.88$$

Question 5 An investigation of the vehicles on the road showed that 18% are small, 55% are medium size and, 27% are bigger cars. If a small car is involved in an accident, the fatality (=death) probability is 0.13. The fatality probability is 0.08 if the car is medium size, and it is 0.05 for a bigger car.

a) If an accident is reported to be fatal, what is the probability that a small car was involved?



$$P(S|F) = \frac{P(S \cap F)}{P(F)}$$

$$= \frac{(0.18)(0.13)}{(0.18)(0.13)+(0.55)(0.08)+(0.27)(0.05)}$$

$$\cong \frac{0.0234}{0.0809}$$

$$\cong 0.289$$

b) Are the events ~~A~~ S="small car is involved in an accident" and ~~B~~ F="an accident is fatal" independent from each other?

a) $\Rightarrow \begin{cases} P(S) = 0.18 \\ P(F) = 0.0809 \\ P(S \cap F) = 0.234 \end{cases}$ $\Rightarrow$ $P(S) \cdot P(F) \overset{?}{=} P(S \cap F)$

$(0.18)(0.0809) \neq 0.234 \Rightarrow$ not independent

S and F are

OR

a) $\Rightarrow P(S|F) = 0.289$, but $P(S) = 0.18 \neq P(S|F)$

So, S and F are not independent

c) Find the probability that a medium size car is involved in the accident or the accident is not fatal.

$$P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

$P(M) \cong 0.55$ $\quad P(N) = (0.18)(0.87)+(0.55)(0.92)+(0.27)(0.95)$

$\overset{OR}{=} 1 - P(F) = 1 - 0.0809 = 0.9191$

$P(M \cap N) = (0.55)(0.92) = 0.506$

$P(M \cup N) = 0.55 + 0.9191 - 0.506 = 0.9631$