Question 1. **(25 points)** a) Evaluate the following expression using the binomial table :

$$\sum_{i=12}^{17} \binom{25}{i} 0.4^{25-i} 0.6^{i} \qquad Bin(25, 0.6)$$

12, 13, 14, 15, 16, 17

in binomial table $p = 0.60$

$k = 17$ and $p = 0.6 \Rightarrow 0.846$

$k = 11$ and $p = 0.60 \Rightarrow 0.078$

$0.846 - 0.078 = 0.768$ //

b) 56% of all Lalala(=a population) have lulu(=a characteristic). What is the probability that among 71 randomly selected Lalala, between 42 and 46 have lulu? (42 and 46 are included). Use the normal approximation.

$n = 71$

$p = 0.56$

Expected value = mean = $\mu = n.p$

$\mu = 71 \times 0.56 = 39{,}76$

Variance = $n.p.(1-p)$

$= 71 \times (0.56) \times (1 - 0.56)$

$= 17{,}4944$

std dev $= \sqrt{n.p.(1-p)}$

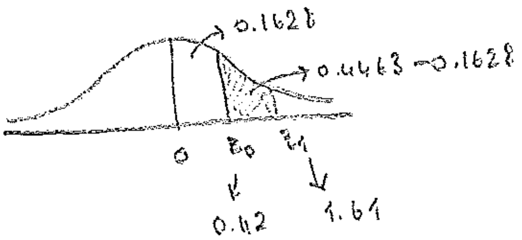
$= \sqrt{17{,}4944}$

$= 4.1826$

$X \overset{A}{\sim} N(39{,}76, 4{,}1826)$ by using CLT

$z_0 = \dfrac{41.5 - 39.76}{4.1826} = 0.4160$

$z_1 = \dfrac{46.5 - 39.76}{4.1826} = 1.6114$



$0.4463 - 0.1628 = 0.2835$ //

Question 2. (25 points) The following joint probability distribution table indicates the length of stay Y in the hospital after a surgery, in days, and the difficulty of the surgery, denoted by X, where X=0 is low difficulty and X=1 is highly difficult.

X \ Y	1	2	3	4	
0	0.30	0.20	0.10	0	0.60
1	0.02	0.08	0.25	0.05	0.40
	0.32	0.28	0.35	0.05	

a) What is the probability that a surgery is highly difficult and the length of stay is less than 3 days?

$0.02 + 0.08 = 0.10$

b) Find the covariance of X and Y.

$E[X] = 0.40$ $\quad E[Y] = 0.32 + 2.(0.18) + 3.(0.35) + 4.(0.05) = 2.13$

$E[XY] = 1.(0.02) + 2.(0.08) + 3.(0.25) + 4.(0.05) = 1.63$

$0.16 \quad 0.75 \quad 0.2$

$Cov(X,Y) = E[XY] - E[X] \cdot E[Y]$

$= 1,13 - (0.4)(2.13)$

$= 1.13 - 0.852$

$= 0.278$

c) Are X and Y independent? Why or why not?

Since covariance is different from 0, they cannot be independent.

d) If 50 surgeries are selected at random in this hospital, how many of them are expected to be highly difficult?

$(50).(0.40) = 20$ of them //

Question 3. (25 points) The service manager of a car service shop at Atatürk Oto Sanayi (=Car Service District) wishes to determine the cost of performing car service on a typical family car.
At the time of service, the kilometers on a typical family car is normally distributed with a mean of 10000 kilometers (km) and a standard deviation of 2000 km.

a) Find the probability that a typical family car has
   - 8000 km or less
   - between 8000 km and 15000 km
   - more than 15000 km

when serviced.

$X \sim N(10.000, 2000)$

$P(X \leq 8000) \Rightarrow \frac{8000-10000}{2000} = -1 \rightarrow 0.5 - 0.3413 = 0.1587$

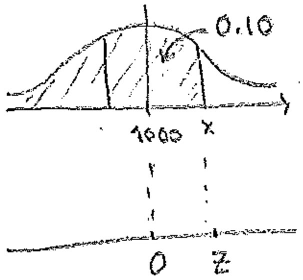
$P(8000 \leq X \leq 15000) = P\left(\frac{8000-10000}{2000} \leq x \leq \frac{15000-10000}{2000}\right)$

$= P(-1 \leq x \leq 2.5)$

$= 0.3413 + 0.4938 = 0.8351$

$P(x > 15000) = 1 - 0.1587 - 0.8351 = 0.0062$ //

b) Find the kilometers on the car which has more kilometers than 60% of all family cars.



$z = 0.25 \Rightarrow \frac{x-10.000}{2.000} = 0.25$

$x = 10.500$ km //

c) The cost of service will be 200 TL if a typical family car has 8000 km or less, 400 TL if it has between 8000 km and 15000 km, and 750 TL if it has more than 15000 km when serviced. Find the expected cost of a service for a typical family car.

Y = cost

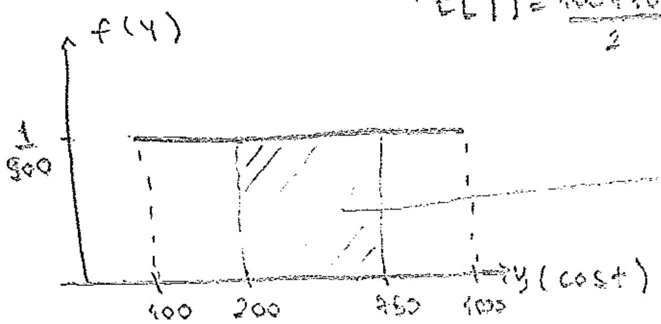
y	200	400	750
P(y)	0.1587	0.8351	0,0062

← part a)

$E[Y] = 200(0.1587) + 400(0.8351) + 750(0.0062)$

$= 370.43$ TL

d) Suppose the cost of service is not as described in part c), but it has a uniform distribution between 100 TL and 1000 TL.
   - What is the probability that the cost is between 200 TL and 750 TL? $\frac{750-200}{900} = 0.67$
   - What is the expected cost? $\rightarrow E[Y] = \frac{100+1000}{2} = 550$ TL



Question 4. **(25 points)** The time spent in traffic on the way to work by the employees of a large firm has a mean of 60 minutes and a standard deviation of 15 minutes.

a) If a random sample of 40 employees is selected, what is the probability that the average time they spend in traffic will be over 67 minutes?

CLT : $P(\bar{X} > 67) \cong P\left(Z > \frac{67-60}{15/\sqrt{40}}\right)$

$= P\left(Z > \frac{7}{2.37}\right)$

$= P(Z > 2.95)$

$= 0.5 - 0.4984$

$= 0.0016$ //

b) The following table shows the probability distribution of the $\underbrace{\text{number of traffic lights}}_{X}$ encountered (=seen) on the way to work by an employee.

6	7	8	9	10
0.2	0.15	0.3	0.25	0.1

Find the probability that 300 traffic lights are encountered by 40 employees in total.

$\mu = E[X] = 6.(0.2) + 7.(0.15) + 8.(0.3) + 9.(0.25) + 10.(0.1) = 7.9$

$\sigma^2 = Var(X) = E(X^2) - \mu^2 = 6^2.(0.2) + 7^2(0.15) + 8^2(0.3) + 9^2(0.25)$
$+ 10^2(0.1) - (7.9)^2 = 1.59$

$P(X_1 + \dots + X_{40} = 300) \cong P\left(\frac{299.5 - 40\cdot(7.9)}{\sqrt{40}\cdot\sqrt{1.59}} < Z < \frac{300.5 - 40.(7.9)}{\sqrt{40}.\sqrt{1.59}}\right)$

$= P(-2.07 < Z < -1.94)$

$= 0.4808 - 0.4738$

$= 0.007$ //

Question 5. **(10 points)** Researchers want to construct a 97% confidence interval for the unknown mean height of a newly discovered tribe (group of people). If the confidence interval should have a width (length of the interval) of at most 2 cm, what must the sample size be? (Assume that the population standard deviation in that part of the world is roughly constant, and about 6 cm)

$\bar{X} \pm z_{\frac{\alpha}{2}} \cdot \sigma_{\bar{X}} \Rightarrow [m, k]$ $\qquad \alpha = 1 - 0{,}97 = 0{,}03$

$\Rightarrow k - m \leq 2\text{cm}$

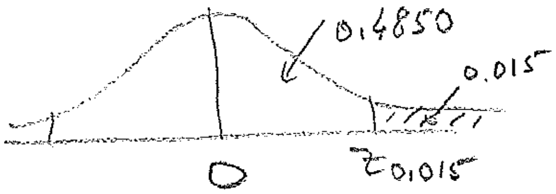
$\Rightarrow \bar{X} + z_{0{,}015}\, \sigma_{\bar{X}} - \left(\bar{X} - \left(z_{\frac{\alpha}{2}} \cdot \sigma_{\bar{X}} \right) \right)$

$\Rightarrow 2 \cdot \left[z_{0{,}015}\, \sigma_{\bar{X}} \right] \leq 2$

$z_{0.015} = 2.17$

$\Rightarrow 2 (2.17) \dfrac{6}{\sqrt{n}} \leq 2 \Rightarrow 13{,}02 \leq \sqrt{n}$

$n \geq 169.5 \rightarrow n = 170$



$\Rightarrow z_{0{,}015} = 2{,}17$