Part I. (20 points) The time it takes for a certain pain reliever (=ağrı kesici) to reduce the symptoms has been reported to have a mean of 30 minutes and a standard deviation of 5 minutes, based on extensive clinical trials.

A simple random sample of 33 patients who use this medication yields an average time of 28.3 minutes and a standard deviation of 4.7 minutes.

1. (12 points) Do the data indicate that the mean time for the pain reliever to reduce the symptoms is different from 30 minutes?

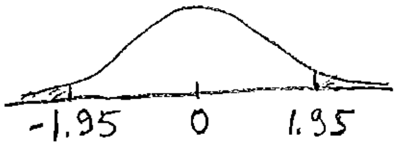
$H_0: \mu = 30$ (SD = 5) $n = 33 \Rightarrow SE = \frac{SD}{\sqrt{n}}$

$H_a: \mu \neq 30$

$\Rightarrow SE = \frac{5}{\sqrt{33}} = 0.87$

$z = \frac{28.3 - 30}{0.87} \cong -1.95$

P-value:



(two-sided test)

$(100 - 94.88)\%$

$= 5.12\% > 5\%$

We (barely) do not reject Ho.

The data do not indicate a significant difference from 30 minutes.

2. (8 points) Using the sample data, construct a 90% confidence interval for the mean time that the pain reliever reduces the symptoms.

$28.3 \mp (1.65) \frac{4.7}{\sqrt{33}}$ $SE \cong 0.82$

28.3 ∓ 1.4

$[26.9, 29.7]$ minutes.

Part II. (20 points) In 1970, 59% of college freshmen thought that there should not be capital punishment (=idam cezası). In 2005, a random sample of 100 freshmen were asked whether there should be capital punishment or not. Among these, 35 of them said no, there should not be.

1. (12 points) Is there a significant ~~difference~~ change in the opinion of students in 2005 ~~from those in~~ since 1970?

$H_0: p = 0.59$

$H_a: p \neq 0.59$

$\frac{35}{100} = 0.35$

$$z = \frac{0.35 - 0.59}{\sqrt{\frac{(0.59)(0.61)}{100}}} = \frac{-0.24}{0.06} = -4$$

P-value $= (100 - 99.9937)\% = 0{,}0063\ \%$

very low P-value ⇒ Reject H_0

So, there is a highly significant decrease in the percentage of freshmen who are against capital punishment from 1970 to 2005.

2. (8 points) Construct a ~~90%~~ 95% confidence interval for the proportion of all college freshmen who are against capital punishment in 2005.

$35\% \mp (1.95)SE$

$SE = \sqrt{\frac{(0.35)(0.65)}{100}} \cong 0.05 = 5\%$

$35\% \mp (1.95)5\%$

$[25.3\%, 44.8\%]$

Part III. (25 points) In a lab, 16 mice were randomly assigned to treatment and control groups and their survival times (given in days) following a test surgery were recorded as below.

Group	Data	Mean	Standard Error
Treatment	94, 38, 23, 197, 99, 16, 141	88.9	25.2
Control	52, 10, 40, 104, 51, 27, 146, 30, 46	56.2	14.1

1. (15 points) Did the treatment prolong the survival time?

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

$SE = \sqrt{(25.2)^2 + (14.1)^2} \cong 28.9$

$t = \dfrac{88.9 - 56.2}{28.9} = 1.13$

$88.9 - 56.2 = 32.7$ mins

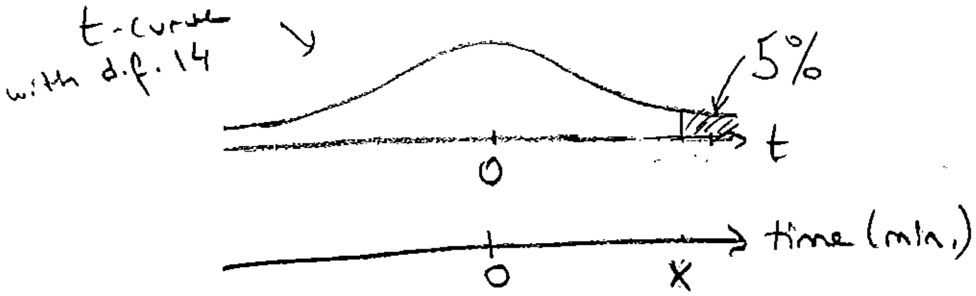
$d.f. = 7-1+9-1 = 14$

So, from t-table $10\% < \text{P-value} < 25\%$

Large P-value ⇒ do not reject Ho.

No, the treatment did not significantly prolong the survival time.

2. (7 points) What is the highest difference in survival time (in days) between the two groups which would not imply the rejection of the null hypothesis in Question 1 at 5% level of significance?



$\dfrac{X - 0}{28.9} = 1.76$

$\Rightarrow X \cong 50.9$ mins

(In the question, only 32.7 mins)

3. (3 points) What is the standard deviation in the treatment group?

$SE = \dfrac{SD}{\sqrt{n}} \Rightarrow SD = (25.2)\sqrt{7} = 66.7$ mins

Part IV. (20 points) The Gallup poll asks respondents how they would rate the honesty and ethical standards of employees in different fields such as advertisement. education, clergy, government offices, etc. The percentage of respondents who rated the employees in a certain field as "very high or high" is found to be 65% in a random sample of 500 respondents in 2010. In 2015, a new poll of 400 respondents showed that this percentage dropped to 60%.

1. (14 points) Can the drop in the ratings be explained as a chance variation?

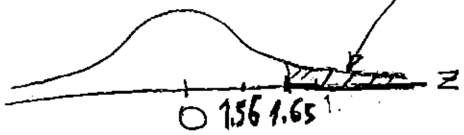
$H_0: p_1 = p_2$

$H_a: p_1 > p_2$

$$S.E. = \sqrt{\frac{(0.65)(0.35)}{500} + \frac{(0.60)(0.40)}{400}}$$

$$\cong 0.032$$

$$z = \frac{0.65 - 0.60}{0.03} = 1.56 < 1.65 \text{ for } z \text{ of } 5\%$$



$\Rightarrow$ Do not reject H_0.

Rejection region: $z > 1.65$

No, the drop in the mean ratings appears to be due to chance.

2. (6 points) Which one(s) of the following is(are) your assumptions for a test of significance, which you conducted in Question 1:

   i) Both groups of respondents in 2010 and 2015 are selected randomly from the population.

   ii) The respondents in 2010 and 2015 are independent from each other.

   iii) The distribution of response scores (1- Very high 2- High 3-Average 4- Low 5-Very low) in the population is approximately normal.

Part V. (25 points) Can deferring (=erteleme) gratification (=ödül/memnuniyet) be an indicator of future success? This is what Walter Mischel of Stanford University sought to determine in his 1972 Marshmallow Experiment. Children ages four to six were taken into a room where a marshmallow was placed on the table in front of them. Before leaving each of the children alone in the room, the examiner told them they would receive a second marshmallow if the first was still on the table after 15 minutes. The examiner recorded how long each child resisted eating the marshmallow and later noted whether it correlated with the child's success in adulthood. A minority of the 600 children ate the marshmallow immediately and one-third deferred gratification long enough to receive the second marshmallow. In follow-up studies, Mischel found that those who deferred gratification were significantly more competent and received higher SAT scores than their peers, meaning that this characteristic is likely to remain with a person for life.

1. (18 points) Do the following statistics indicate the same conclusion with Mishcel?

The SAT scores of 75 students who ate the marshmellow in the first 15 minutes:
Mean: 1~~4~~508 SD = 147

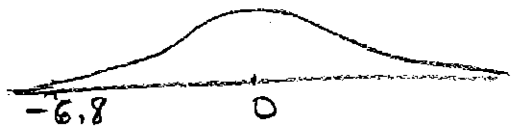
The SAT scores of 525 students who did not eat the marshmellow in the first 15 minutes:
Mean: 1629 SD = 113

$$H_0: \mu_1 = \mu_2 \qquad SE_1 = \frac{147}{\sqrt{75}} = 16.97$$

$$H_a: \mu_1 < \mu_2 \qquad SE_2 = \frac{113}{\sqrt{525}} = 4.93$$

$$SE_{diff} = \sqrt{(16.97)^2 + (4.93)^2} \cong 17.7$$

$$z = \frac{1508 - 1629}{17.7} = -6.8$$ → not even in the table!



P-value ≈ 0

So, Reject H_0. Deferred gratification behavior is an indicator of future success, like Mischel concluded.

2. (4 points) If this experiment is repeated independently 30 times, and 30 different confidence intervals are constructed for the difference between the mean SAT scores at 90% confidence level, how many of these intervals do you expect to cover the true difference?

$$30 \times \frac{90}{100} = 27$$ of them are expected.

3. (3 points) Is a confidence interval for the difference in the samples or the populations of SAT scores of students who can defer gratification and who cannot?

The CI is for the difference in the means of the populations.