Mine Çağlar   December 1, 2017

Math 202: Statistics for Social Sciences

Fall 2017 EXAM 2

Calculator OK, 100 min.

<u>Instructions</u>: There are six parts to this exam I-VI. Please inspect the exam and make sure you have all five pages of questions. Do <u>all</u> your work on these pages. If you use the back of a page, make sure to indicate that.

Remember: ***You <u>must</u> show your work to get proper credit. In hypothesis testing questions, show all steps of the test and state your conclusion in plain English.***

<u>Academic Honesty Code</u>: Koç University Academic Honesty Code stipulates that "copying from others or providing answers or information, written or oral, to others is cheating." By taking this exam, you are assuming full responsibility for observing the Academic Honesty Code.

NAME: _____

Part I:	/25
Part II:	/20
Part III:	/20
Part IV:	/25
Part V:	/20
Total:	/110

10 points bonus

<u>Formulas:</u>

Confidence interval (CI) : $\overline{X} \mp z\ SE$ or $\overline{X} \mp t\ SE$ or $\hat{p} \mp z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$(\hat{p}_1 - \hat{p}_2) \mp z\ SE_{2-sample}$ or $(\overline{X}_1 - \overline{X}_2) \mp z\ SE_{2-sample}$ or $(\overline{X}_1 - \overline{X}_2) \mp t\ SE_{2-sample}$

Standard Error (SE) : $SE_{2-sample} = \sqrt{(SE_1)^2 + (SE_2)^2}$, $SE = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ for percentages,

and $SE = \dfrac{SD}{\sqrt{n}}$ for averages.

Binomial Distribution :

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0,1,2,\ldots,n$$

Part I. **(25 points)** According to several studies, marriage (prevents) dementia (=bunama), as reported in Hürriyet on Nov. 30, 2017. In one of these studies, suppose 95 randomly selected subjects were followed to determine if they developed dementia or not. 38 of these subjects were married, and the remaining 57 were single. Among the married subjects, 10 of them suffered from dementia eventually, whereas this number was 29 within single subjects.

1. (15 points) Test the claim reported in the newspaper at 1% level of significance.

$N_1 = 38$, $P_1 = \frac{10}{38} = 0,263$

$N_2 = 57$, $P_2 = \frac{29}{57} = 0,508$

$H_0: P_1 = P_2$

$H_a: P_1 < P_2$

$SE_1 = \sqrt{\frac{(0,263)(0,737)}{38}} = \sqrt{0,0051} =$

$= 0,071$

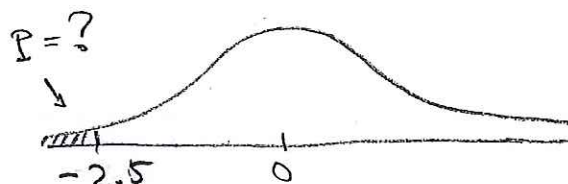$SE_2 = \sqrt{\frac{(0,508)(0,492)}{57}} = \sqrt{0,00438} =$

$= 0,0662$

$SE = \sqrt{SE_1^2 + SE_2^2} =$

$= \sqrt{0,0051 + 0,00438} = 0,097$

$z = \frac{P_1 - P_2}{SE} = \frac{0,263 - 0,508}{0.097} =$

$= -2.52$

z	Area
2.50	98,76

$P = ?$

$P = (100 - 98,76) \cdot \frac{1}{2}\% = 0,62\% < 1\%$.

We reject H_0: We have very significant evidence to support the claim that marriage prevents dementia.

2. (10 points) Find a 90% confidence interval for the difference in the percentage of dementia between married and single people.

$(P_1 - P_2) \pm z \cdot SE$

$(0,263 - 0,508) \pm (1,65)(0,097)$

$- 0,245 \pm 0,1155$

$(-0,3885, -0,0915)$

Part II. (**20 points**) According to a new economic decision, small and mid-sized companies are given loans (=kredi, borç) by the government. The companies make an application for this loan. The government expects 130 thousand TL be requested per company, nationwide. After the introduction of the loan, 24 small and mid-sized companies which form a random sample from Turkey have made an application for 135 thousand TL on the average. The standard deviation of those loan applications is 18 thousand TL.

1. (12 points) Is the expectation of the government supported by the data? Report the P-value.

$H_0: \mu = 130$ (or $130,000$ TL)

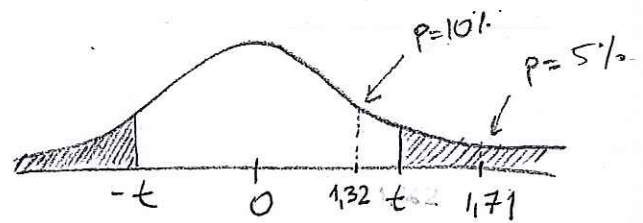$H_a: \mu \neq 130$

$N = 24 \rightarrow$ t-test, deg. of freed $= 23$

$SE = \dfrac{SD}{\sqrt{N}} = \dfrac{18}{\sqrt{24}} = 3.67$

$t = \dfrac{135-130}{3.67} = 1.362$

From the t-table with d.f. $= 23$

$1.32 < t < 1.71$

$2.5\% < P < 2 \cdot 10\%$

$10\% < P < 20\%$

P=10% P=5%

$-t \quad 0 \quad 1.32 \; t \; 1.71$

Since P > 5%, we don't reject H_0, the expectation 130 th.TL is supported by the data.

2. (8 points) Construct a 95% confidence interval for the mean of the loan applications nationwide.

$\bar{\mu} \pm t \cdot SE =$ d.f. $= 23$

$= 135 \pm 2.07 (3.67) =$

$= 135 \pm 7.6 \implies (127.4 \; ; \; 142.6)$

Part III. **(20 points)** Striking research on the relationship between the education level and employment has been published in a book entitled "İşsiz Yaşam" from Koç University Press. The education level is measured on a scale 1,2,3, …, up to 9, where 1 corresponds to "illiterate" and 9 is "PhD (=doktora) graduate".

The data are collected from 25 employed and 41 unemployed subjects to yield
 a mean of 4.86 and a standard deviation of 0.87 for the employed group,
 and a mean of 5.42 and a standard deviation of 1.19 for the unemployed.

1. (13 points) Conduct a test of hypothesis on the relationship between the education level and employment on the basis of the given statistics, and state your result in plain English (as always!).

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

$$z = \frac{\mu_1 - \mu_2}{SE} = \frac{4,86 - 5,42}{0,25} = \frac{-0,56}{0,25} = -2,24$$

$$SE_1 = \frac{0,87}{\sqrt{25}} = 0,174$$

$$SE_2 = \frac{1,19}{\sqrt{41}} = 0,186$$

z	Area
2,24	97,56

$$SE = \sqrt{SE_1^2 + SE_2^2} =$$
$$= \sqrt{0,03 + 0,0345} =$$
$$= 0,25$$

$$P = 100 - 97,56 = 2,44 < 5\%$$

Reject H_0!

The data gives us significant evidence against the null hypothesis

2. (7 points) Fill in the blanks. Options: i) exactly equal to, ii) estimated from the data as

a) The sample mean of the education level of employed persons is ___i)___ 4.86

Explain in one sentence: We have exact information about the sample.

b) The SE for the sample mean of the education level of employed persons is ___ii___ 0.174

Explain in one sentence: We don't know the population SD therefore we estimate it using the sample SD.

Part IV. **(25 points)** As a result of a comprehensive (=wide, general) investigation in Human Resources, it is found that 39% of the CEO's (=managers) cannot adapt to a change in the organization of their corporation.

1. (7 points) In a random sample of 8 CEO's, what is the probability that at least 7 of them cannot adapt to a change in the organization?

$$P = 0,39$$
$$N = 8$$

By the Binomial formula $P(\text{At least } 7) =$
$$= P(7 \text{ can adapt}) + P(8 \text{ can adapt}) =$$
$$= \binom{8}{7}(0,39)^7(0,61) + \binom{8}{8}(0,61)^8 \cdot 1 =$$
$$\doteq 0,00669 + 0,000535 = 0,00723$$

2. (5 points) Suppose exactly 7 CEO's cannot adapt to a change in the organization among 8 CEO's randomly selected from various corporations. Does this observation support that the percentage of CEO's who cannot adapt to a change is 39%?

If $P = 0,39$, then the p-value for this observation is $P(\text{At least } 7 \text{ cannot adapt}) = 0,00723$, (or $0,72\%$)

Since $P = 0,72\% < 1\%$, this observation suggests that the percentage of CEO's who cannot adapt to a change is significantly different from 39%.

3. (13 points) In a larger random sample of 40 CEO's, it has been found that 15 CEO's cannot adapt to a change in the organization. Does this observation support that the percentage of CEO's who cannot adapt to a change is 39%?

$H_0: p = 0,39$
$H_a: p \neq 0,39$

$$SE = \sqrt{\frac{(0,39)(0,61)}{40}} = 0,077$$

$$z = \frac{0,375 - 0,39}{0,077} \approx -0,20$$

z	Area
0,20	15,85

$P = 100 - 15,85 = 84,15\%$

$P > 5\%$, Don't reject H_0.

The observation supports that the % of CEO's who cannot adapt to change is 39%.

Part V. **(20 points)** The following table gives the joint probability distribution of the perception of crime in the city in relation to the city of residence, as estimated from a large survey in the largest cities of Turkey (and expressed as percentages). Based on interviews, crime was categorized as a major concern, a minor concern, or of no concern for each individual interviewed.

	Istanbul	Ankara	Izmir
Major	13%	14%	8%
Minor	21%	11%	10%
None	12%	4%	7%

a) (8 points) Find the conditional probability distribution of the perception of crime in Istanbul.

$$\text{Major: } \frac{13}{13+21+12} = \frac{13}{46} = 0{,}282 \quad (28{,}2\,\%)$$

$$\text{Minor: } \frac{21}{46} = 0{,}456 \quad (45{,}6\,\%)$$

$$\text{None: } \frac{12}{46} = 0{,}2608 \quad (26.08\,\%)$$

b) (4 points) What is the probability that a randomly selected person from Izmir has major concern about crime?

$$\frac{8}{8+10+7} = \frac{8}{25} = 0{,}32 \quad (32\,\%)$$

c) (3 points) What percent of the people are from Ankara?

$$14\% + 11\% + 4\% = 29\,\%$$

d) (5 points) If we select 5 people from the population of the three largest cities, what is the probability that exactly 2 of them has no concern about crime?

$$P = 12\% + 4\% + 7\% = 23\% = 0{,}23$$

$$P(\text{exactly 2 have no concern}) = \binom{5}{2}(0{,}23)^2(0{,}77)^3 =$$

$$10 \qquad\qquad = 0.242$$

$$(24{,}2\,\%)$$