Part I. (20 points) Each time you pick a card from a well shuffled deck (karıştırılmış deste) (52 cards in total), there is a 25% chance that it is one of hearts (same goes for spades, diamonds and clubs). Consider the experiment where you are given a deck of 52 cards to choose from repeatedly, and each time you pick one, you check whether it is hearts or not, and put it back in the deck for a reshuffle (=karıştırma)..

1. (5 points) What is the chance that in 7 picks you will get at most 3 hearts.

$p = 0{,}25 \quad (25\%)$

$1-p = 0{,}75$

$$P(\text{at most 3 hearts}) = \underbrace{(0{,}75)^7}_{\text{chance for no hearts}} + \underbrace{\binom{7}{1}(0{,}25)(0{,}75)^6}_{\#1\text{-heart}} + \underbrace{\binom{7}{2}(0{,}25)^2(0{,}75)^5}_{\#2\text{-hearts}} + \underbrace{\binom{7}{3}(0{,}25)^3(0{,}75)^4}_{\#3\text{-hearts}} = 0.928$$

2. (10 points) Suppose that you repeatedly pick cards one by one by the routine described above 100 times, and in this sample of data you obtain the hearts only 18 times. Could this mean that the deck that was given to you is a rigged (=hileli) deck that contains fewer hearts?

$H_0: p = 0{,}25$

$H_a: p < 0{,}25$

$n = 100$

$$SE = \sqrt{\frac{p\cdot(1-p)}{n}} = \sqrt{\frac{(0{,}25)(0{,}75)}{100}} = 0{,}0187$$

$$z = \frac{0{,}18 - 0{,}25}{0{,}0187} = \frac{-0{,}07}{0{,}0187} = -3{,}74$$

z	Area
3,74	99,982

$$P\text{-value} = \frac{100 - 99{,}982}{2} < 1\%$$

P-value < 1%, we therefore have significant evidence to reject the null hypothesis, and think that the deck is rigged.

3. (5 points) Construct a 90% confidence interval for the percentage of hearts in your deck in view of the data from the sample in Question 2.

$p = 0{,}18 \quad (18\%)$

$z = 1{,}65$

$$SE = \sqrt{\frac{(0{,}18)(0{,}82)}{100}} = 0{,}038 \quad (3{,}8\%)$$

$$p \pm z\cdot SE = 0{,}18 \pm (1{,}65)(0{,}038) = (0{,}1173 \,,\, 0{,}2427)$$

Part II. (15 points) Lipton, a company primarily known for tea, considered using coupons to stimulate sales of its packaged dinner entrees. The company was particularly interested whether there was a difference in the effect of coupons on singles versus married couples. Random samples of single people and married people (with sample sizes 31 and 60, respectively) were asked to respond to the question "Do you use coupons regularly?" by using a numerical scale, where 1 stands for agree strongly, 2 for agree, 3 for neutral, 4 for disagree, and 5 for disagree strongly. The results of the poll are given in the following table:

	Mean	Standard Deviation
Singles	3.10	1.44
Married	2.43	1.35

1. (12 points) Is the observed difference significant at $\alpha = 0.01$ level of significance?

$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 \neq \mu_2$

$SE_1 = \frac{1{,}44}{\sqrt{31}} = 0{,}258$ $SE_2 = \frac{1{,}35}{\sqrt{60}} = 0{,}174$

$SE_{diff} = \sqrt{SE_1^2 + SE_2^2} = 0{,}31$

z	Area
2,16	96,84

$z = \frac{(\mu_1 - \mu_2) - 0}{SE_{diff}} = \frac{3{,}10 - 2{,}43}{0{,}31} = \frac{0{,}67}{0{,}31} = 2{,}16$

$p = 100\% - 96.84\% = 3{,}16\% > 1\%$, do not reject H_0

The difference is not significant at the $\alpha = 0{,}01$ level.

2. (3 points) How would you state your null and alternative hypothesis if "according to Lipton's previous surveys, "single people used coupons more frequently and therefore scored on average higher by 1 point on the numerical scale," and you are curious whether it stayed this way"?

$H_0: \mu_1 = \mu_2 + 1$
$H_a: \mu_1 \neq \mu_2 + 1$

or

$H_0: \mu_1 - \mu_2 = 1$
$H_a: \mu_1 - \mu_2 \neq 1$

Part III. (20 points) A study involving a random sample from the largest cities of Turkey was conducted about the perception of crime in the city. Based on interviews, crime was categorized as a major concern, a minor concern, or of no concern for each individual interviewed. The following table shows the number of observations in each category. H_0: Level of concern for crime is independent from city
H_a: The two variables are not independent.

	Istanbul	Ankara	Izmir	
Major	42	40	25	107
Minor	69	38	31	138
None	37	14	26	77
	148	92	82	

1. (15 points) Is the level of concern regarding crime independent from the city of residence?

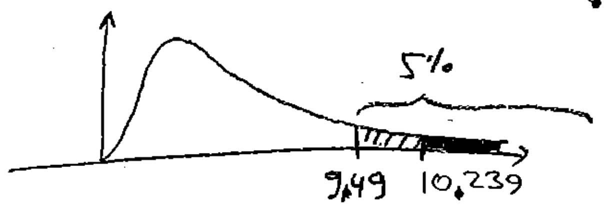
EXP. VALUES TABLE.

$\frac{148\cdot 107}{322} = 49{,}2$	$\frac{92\cdot 107}{322} = 30{,}57$	$\frac{82\cdot 107}{322} = 27{,}25$
$\frac{148\cdot 138}{322} = 63{,}42$	$\frac{92\cdot 138}{322} = 39{,}43$	$\frac{82\cdot 138}{322} = 35{,}14$
$\frac{148\cdot 77}{322} = 35{,}39$	$\frac{92\cdot 77}{322} = 22$	$\frac{82\cdot 77}{322} = 19{,}61$

$$\chi^2 = \sum_{i=1}^{9} \frac{(\text{obs.}-\text{exp})^2}{\text{EXP}} = \frac{(42-49{,}2)^2}{49{,}2} + \frac{(40-30{,}57)^2}{30{,}57} + \frac{(25-27{,}25)^2}{27{,}25} + \frac{(69-63{,}42)^2}{63{,}42} + \frac{(38-39{,}43)^2}{39{,}43} + \frac{(31-35{,}14)^2}{35{,}14} + \frac{(37-35{,}39)^2}{35{,}39} + \frac{(14-22)^2}{22} + \frac{(19{,}61-26)^2}{19{,}61} = 10{,}239$$

deg. of fr. $= (3-1)(3-1) = 4$

$10{,}239 > 9{,}49 \Rightarrow$ P-value $< 5\%$



We have significant evidence that the level of concern is not independent from the city of residence.

2. (2 points) If a subject is randomly selected from the given sample, what is the probability that s/he is from Ankara and has no concern about crime?

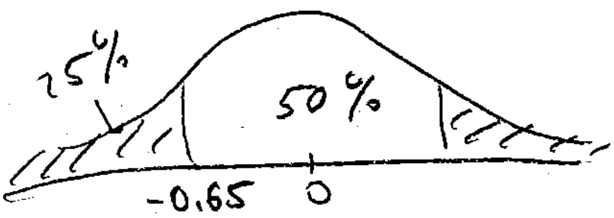
$$\frac{14}{322} = 0{,}043 \quad (4{,}3\%)$$

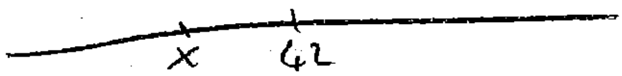
3. (3 points) What is the probability that a person has major concern about crime if s/he is from Istanbul?

$$\frac{42}{148} = 0{,}283 \quad (28.3\%)$$

Part IV. (15 points) The number of times a person gets his/her car washed over a year is normally distributed with a mean of 42 and a SD of 13 in a certain city.

1. (5 points) Some cars get cleaned very rarely. About how many times a car gets cleaned over a year if this number is the 25th percentile of the distribution?





$$\frac{x-42}{13} = -0.65$$

$$\Rightarrow x \cong 33.5$$

2. (10 points) In a more developed region of the city, the cars are claimed to be washed more often on the average than the region mentioned above. A random sample of 23 cars is taken from the developed region and it is found that the mean number of times these cars get washed is 55 over a year. Is the claim supported by the data?

$H_0: \mu = 42$

$H_a: \mu > 42$

$$t = \frac{55-42}{13/\sqrt{23}} \cong 4.8$$

d.f. $= 23 - 1 = 22$

P-value $< 0.5\%$

Since P-value $< 5\%$, we reject Ho.

The claim is supported by data.

The cars get washed more often in the developed region.

Part V. (15points) The following table gives the percentage of mostly preferred investment instruments by the population last year.

	Gold	Bank-Interest	Stock Market	Dollar/Euro
Percentage	32%	14%	25%	29%

A recent sample of 200 randomly selected individuals shows that 29% prefer Gold, 20% prefer the Bank, 21% prefer Stock Market, and the rest prefer Dollar/Euro.

$H_0: p_1 = 0.32, p_2 = 0.14, p_3 = 0.25, p_4 = 0.29$ H_a: At least one p is different.

Has the investment preference of people changed significantly from last year to this year?

Frequency \ Investment type	Gold	Bank	Stock.	Dollar/Euro
Expected	64	28	50	58
Observed	58	40	42	60

$200 \times 29\% = 58$, etc.

$$\chi^2 = \frac{(58-64)^2}{64} + \frac{(40-28)^2}{28} + \frac{(42-50)^2}{50} + \frac{(60-58)^2}{58}$$

$$\left.\begin{array}{l} = 7.05 \\ d.f. = 4-1 = 3 \end{array}\right\} \Rightarrow 5\% < \text{P-value} < 10\%$$

Since P-value > 5%, we do not reject Ho.

There is no significant change in the investment preference from last year to this year.

Part VI. (20 points) A Human Resource specialist can choose to use three different methods of judgement for the performance evaluation of the employees in the company. S/he can decide on the basis of her direct experience, indirect experience, or the combination of the two. The specialists were asked "Which method do you use?" to identify their method, and then they received a score on the basis of correct judgment. The following table gives the scores, their means and variances for the three methods that specialists have used. Higher scores indicate more accurate (better) judgement for performance evaluation of employees.

	Direct	Indirect	Combination
	17	16.6	25.2
	18.5	22.2	24.0
	15.8	20.5	21.5
	18.2	18.3	23.8
	20.2	24.2	
Mean	17.9	20.4	23.6
Variance	2.7	9.1	2.4

$$\bar{X}_{GM} = \frac{5(17.9)+5(20.4)+4(23.6)}{14} = 20.4$$

1. (5 points) Is this an observational study or an experiment? Explain in two sentences at most.

Specialists were free to choose the method they use, that is, their group (direct, indirect or combination). Therefore, this is an observational study.

2. (15 points) Is there a significant difference between the accuracy of the judgement methods?

$H_0: \mu_1 = \mu_2 = \mu_3$

H_a: at least one mean is different

$$s_B^2 = \frac{5(17.9-20.4)^2 + 5\underbrace{(20.4-20.4)^2}_{0} + 4(23.6-20.4)^2}{3-1} =$$

$$s_W^2 = \frac{4(2.7)+4(9.1)+3(2.4)}{5-1+5-1+4-1} = 4.95$$

$$F = \frac{36.1}{4.95} = 7.3$$

$F_{critical} = 3.98$ d.f.N. = 2 d.f.D. = 14 − 3 = 11

$F > F_{critical} \Rightarrow$ Reject H_0

At least one method has different accuracy than the other(s).