Part I. (20 points) The level of lead in the blood of a person is an important indicator of air pollution. In a metropolitan area, the following data are obtained for the level of lead in the blood of a random sample of people:

16
10, 23, 45, 8, 9, 16, 12, 24, 10, 8, ~~4~~, 19

The mean of the data set is 16.7 and its standard deviation is 10.5

1. (4 points) The mean level of lead in the population is estimated to be 16.7 give or take (that is, plus/minus) 3.03. $\left(\frac{10.5}{\sqrt{12}}\right)$

2. (6 points) Find a 95% confidence interval for the mean level of lead in the population.

$\bar{x} = 16.7$ For the confidence coef. 0.95, $\alpha = 0.05$ and $\frac{\alpha}{2} = 0.025$
$s = 10.5$

$df = n-1 = 12-1 = 11$, $t_{0.025} = 2.201$

The 95 % C.I. is $\bar{x} \mp t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \Rightarrow 16.7 \mp 2.201 \cdot \frac{10.5}{\sqrt{12}} \cong 16.7 \mp 6.6716$

$= [10.03, 23.37]$

3. (10 points) Test the hypothesis that the mean level of lead in the population is larger than 15.

To determine whether the mean level of lead in the population is larger than 15, we test $H_0: \mu = 15$
$H_a: \mu > 15$.

Test stat. : $t = \frac{\mu - \mu_0}{s/\sqrt{n}} \Rightarrow t = \frac{16.7 - 15}{10.5/\sqrt{12}} = \frac{\sqrt{12}(16.7 - 15)}{10.5} \doteq 0.56$

$df = 12 - 11 = 11$ degrees of freedom
Ho is rejected if $t > 1.796$ (for 5%) $\Rightarrow 0.56 < 1.796$ $\Rightarrow$ Do not reject Ho.

OR P-value for $t = 0.56$ is greater than 25%

$\Rightarrow$ P-value > 5% $\Rightarrow$ Do not reject Ho.

Therefore we conclude that the mean level of lead in the blood of the population is not larger than 15.

Part II. (20 points) It is argued that in an auction (=açık artırma), the winning bid (= son teklif edilen fiyat) is usually above the real value of the item (=eşya). Two groups of bidders (=katılımcılar) in an auction were compared: 1) more experienced group and 2) less experienced group. In the more experienced group, 32 of 200 winning bids were above the item's real value; and in the less experienced group, 30 of 150 winning bids were above the item's real value.

1. (12 points) Does "experience in auctions" affect the occurrence of buying an item above its real value, significantly?

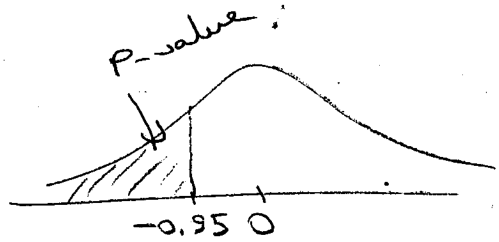
$H_0: p_1 = p_2$

$H_a: p_1 < p_2$ (more experienced ⇒ less likely to be above real value)

$\hat{p}_1 = \frac{32}{200} = 0.16 = 16\%$ $\hat{p}_2 = \frac{30}{150} = 0,20 = 20\%$

$SE_{difference} = \sqrt{\frac{(0.16)(0.84)}{200} + \frac{(0.20)(0.80)}{150}} \cong 0.042 = 4.2\%$

$z = \frac{16\% - 20\%}{4.2\%} = -0.95$



$p\text{-value} = \frac{100 - 65.79}{2}\% \cong 17.1\%$

p-value > 5% ⇒ Do not reject Ho.

Experience does not seem to affect buying an item over its value

2. (8 points) Construct a 90% confidence interval for the difference of the true percentages in the two groups of bidders.

$(16\% - 20\%) \mp 1.65(4.2\%)$

$90\% \xrightarrow{z\text{-table}} 1.65$ $-4\% \mp 6.9\%$

$\Rightarrow [-10.9\%, 2.9\%]$

Part III. (20 points) Two different diets are considered for weight loss, regular diet (but low in calories) and low fat diet (also low in calories). Summary statistics of weight loss is given below for the treatment and control groups, as obtained from SPSS which is a statistical software.

	Diet	N	Mean	Std Deviation	Std Error
1 →	Low Fat	100	9.3 kg	4.7 kg	0.47
2 →	Regular	100	7.4 kg	4.0 kg	0.40

1. (11 points) Is there a significant difference between the two types of diet for weight loss?

$H_o: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 > 0$

$$z = \frac{9.3 - 7.4}{\sqrt{(0.47)^2 + (0.40)^2}} = 3.08$$

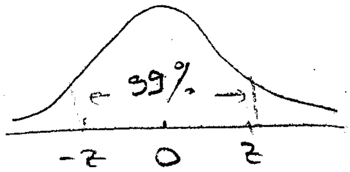
$$\text{P-value} = \frac{100\% - 99.806\%}{2} = 0.097\% < 5\%$$

$\Rightarrow$ Reject H_o

Yes, there is a significant difference between the diets, low fat diet causes more weight loss.

2. (7 points) Find a 99% confidence interval for the difference in the mean weight loss due to the two types of diet.

$(\bar{X}_1 - \bar{X}_2) \mp z\, SE_{difference}$



$(9.3 - 7.4) \mp \underbrace{2.6\ (0.617)}_{0.16}$

$\left(SE_{diff} = \sqrt{(0.47)^2 + (0.40)^2} = 0.617 \right) \Rightarrow [1.74\text{ kg}, 2.06\text{ kg}]$

3. (2 points) Which type of diet would you call the "treatment" and which one "control"?

Low fat → treatment

Regular → control

Part IV. (20 points) In a random sample of 60 smokers, it is found that only 5% has ever entered into a treatment program to help them quit smoking.

1. (8 points) The Minister of Health formerly estimated the percentage of smokers who ever enter a treatment program as 7%. Has the minister overestimated this percentage? Conduct a hypothesis test at α=1%.

$H_0 : p = 7\%$

$H_a : p < 7\%$

$$z = \frac{0.05 - 0.07}{\sqrt{\frac{0.07 \cdot 0.93}{60}}} = \frac{-0.02}{0.033} = -0.61 \qquad p\text{-value} = \frac{100\% - 45.15\%}{2}$$

$$= 27.425\%$$

$$> 1\%$$

So, don't reject Ho. The minister's estimation is reasonable.

2. (7 points) True or false:

F The 98% confidence interval for the population percentage of smokers who ever enter into a treatment program to quit smoking, as obtained from the given sample is [2.4%, 3.1%].

Justify your answer by finding this interval.

$$CI : \hat{p} \mp z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.05 \mp \underbrace{2.35 \cdot \sqrt{\frac{0.05 \cdot 0.95}{60}}}_{0.0661}$$

$$\Rightarrow [-1.6\%, 11.6\%] \Rightarrow \sim [0\%, 11.6\%]$$

3. (2 points) Fill in the blanks by using two of the following: 5%, 7%, parameter, statistic.

The observed value for the percentage of smokers who ever enter into a treatment program to quit smoking is 5% which is a statistic.

4. (3 points) In repeated sampling, suppose 98% confidence intervals are constructed from 50 different samples. How many of these intervals do you expect to cover the true percentage of smokers who ever enter into a treatment program to quit smoking?

$50 \cdot 0.98 = 49$ intervals

Part V. (8 points) A psychologist investigates personality style versus blood pressure by classifying personality type as A or B and the blood pressure level as normal, high and low. The following table gives the percentages of each category in the population.

	Blood Pressure		
Personality	Normal	High	Low
A	55%	8%	12%
B	13%	5%	7%

Find the probability distribution of blood pressure for type A personality.

$P(\text{Normal}) = \dfrac{0.55}{0.75} \cong 0.73$

$P(\text{High}) = \dfrac{0.08}{0.75} \cong 0.106$

$P(\text{Low}) = \dfrac{0.12}{0.75} \cong 0.16$

Part VI. (12 points) The psychologist mentioned in Part V concludes that 25% of the population has type B personality. Suppose a simple random sample of 12 subjects is drawn from this population.

1. (2 points) What is the expected number of type B personality subjects in this sample?

$12 \cdot 0.25 = 3$

2. (10 points) What is the probability that at most 2 subjects will have type B personality?

$$\binom{12}{0}(0.25)^0(0.75)^{12} + \binom{12}{1}(0.25)^1 \cdot (0.75)^{11} + \binom{12}{2}(0.25)^2(0.75)^{10}$$

$\cong 0.0317 + 0.1267 + 0.232$

$= 0.3904$