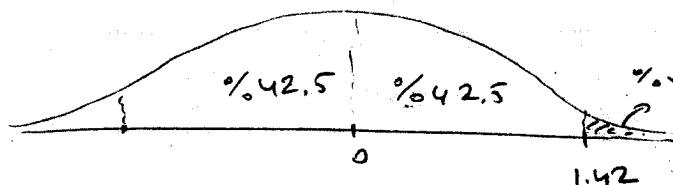


- **Part I. (20 points)** Waiting time at a doctor's office is normally distributed with mean 20 minutes and standard deviation 7 minutes.

1. (6 points) What is the chance that an arbitrary patient will wait more than half an hour?

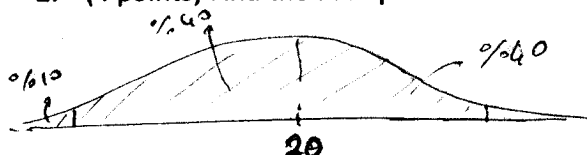
$$z = \frac{30 - 20}{7} = \frac{10}{7} = 1.43$$

area = %85



$$\frac{100 - 85}{2} = \boxed{\%7.5}$$

2. (4 points) Find the 90th percentile of the waiting time.



$$\%80 \rightarrow z = 1.27$$

$$1.27 = \frac{x - 20}{7} \Rightarrow x = 28.89$$

3. (6 points) What is the chance that the average waiting time of randomly selected 25 patients will be less than 17 minutes?

$$z = \frac{17 - 20}{7/\sqrt{25}} = \frac{-3}{1.4} = -2.1$$

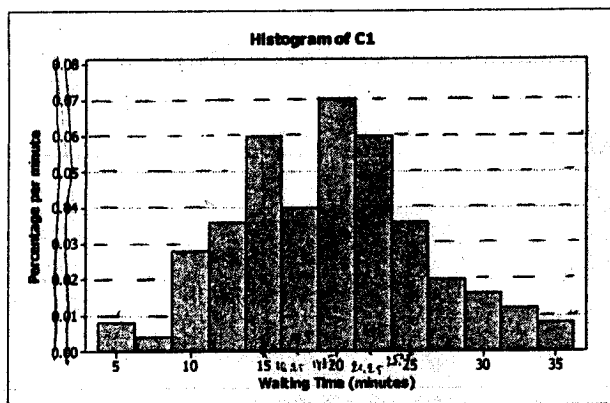
$$\Rightarrow 96.43$$

$$100 - 96.43 = 3.57$$

$$\frac{3.57}{2} = 1.78$$

%1.78

4. (4 points) Here is a density scale histogram from a sample of patients of the same doctor.



What percent of the patients in the sample have waited between 15 and 20 minutes?

$$1.25 \times 6 + 2.5 \times 4 + 1.25 \times 7 = 7.5 + 10 + 8.75 = 26.25$$

Part II. (15 points) Any sentence that contains a noun as a direct object (=nesne) and another noun as a second object is called a "Double-Object Dative", shortly a DOD. For example in the sentence "Sue offered Ann a cookie", "Ann" is the direct object and "cookie" is the second object.

A study about DOD was conducted in an introductory English composition course. 35 native speakers were randomly chosen and they each made a sentence using the verb "buy". Of these sentences, 10 had a DOD structure.

1. (7 points) Construct a 95% confidence interval for the true percentage of sentences with the verb "buy" that have a DOD structure.



$$\frac{10}{35} = 0.28$$

$$SE = \sqrt{\frac{0.28 \cdot 0.72}{35}}$$

$$= 0.075$$

$$\Rightarrow \%7.5$$

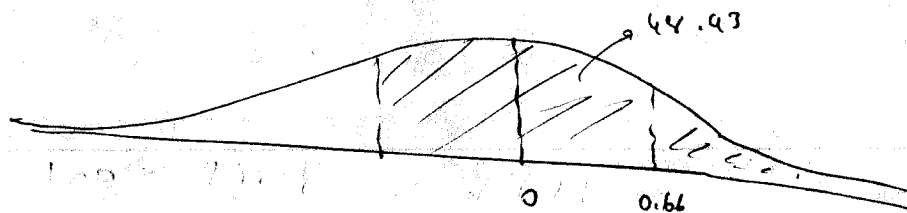
$$28 \pm 7.5 \cdot 2 = 28 \pm 15 \Rightarrow [13, 43]$$

2. (8 points) Perform a test of significance to determine whether the true fraction of sentences with the verb "buy" that have a DOD structure is less than 33%.

$$H_0 : p = \%33$$

$$H_a : p < \%33$$

$$\frac{33 - 28}{7.5} = \frac{5}{7.5} = \frac{2}{3} = 0.66$$



$$\frac{100 - 44.43}{2} = 25.78\% > \%5$$

Do not reject H_0 .

The true fraction of sentences with the verb "buy" that have a DOD structure is not less than %33.

- Part III. (20 points) The following frequency table shows the type of movie borrowed from the library versus the age group of the borrower in a random sample of 55 people.

	Mystery	Comedy	Action	
Young	7	12	15	34
Old	10	6	5	21
	17	18	20	

1. (15 points) Does the type of movie borrowed depend on age?

	expected frequency		
Young	Mystery 10.50	Comedy 11.12	Action 12.36
Old	6.50	6.87	7.63

H_0 : movie borrowed is independent on age.

H_a : movie borrowed depends on age.

$$\chi^2 = \frac{(10.50 - 7)^2}{10.50} + \frac{(11.12 - 12)^2}{11.12} + \frac{(12.36 - 15)^2}{12.36} + \frac{(6.50 - 10)^2}{6.50} + \frac{(6.87 - 6)^2}{6.87} + \frac{(7.63 - 5)^2}{7.63}$$

$$\Rightarrow \chi^2 = 1.16 + 1.85 + 0.065 + 0.11 + 0.56 + 0.50 = 4.68$$

Degrees of freedom $(2-1) \cdot (3-1) = 1 \cdot 2 = 2$.

$\% 0.05 < p < 0.1 \Rightarrow$ Do not reject H_0 .
(or $4.68 < 5.99 = \chi^2_{0.05,2}$)

Hence, type of movie borrowed depends on age.

2. (5 points) If a subject is chosen in particular from the young group of subjects randomly, what is the probability that s/he rents comedy movies?

$$\frac{12}{34} = 0.35 \quad \left(\% 35 \right)$$

- Part IV. (15 points) Sociologists state that one in every four women has been a victim of domestic abuse in the population.

1. (10 points) In a random sample of 15 women, find the probability that at most 3 of them have been a victim of domestic abuse.

$$p = \frac{1}{4} = 0.25$$

$$\begin{aligned} & \binom{15}{0} (0.25)^0 (0.75)^{15} + \binom{15}{1} (0.25)^1 (0.75)^{14} + \binom{15}{2} (0.25)^2 (0.75)^{13} \\ & \quad 0.013 + 0.25 \cdot 0.017 + 0.0625 \cdot 0.021 \\ & \quad + \binom{15}{3} (0.25)^3 (0.75)^{12} \\ & \quad 0.015 + 0.031 \end{aligned}$$

$$= 0.013 + 0.063 + 0.15 + 0.21 \approx 0.436$$

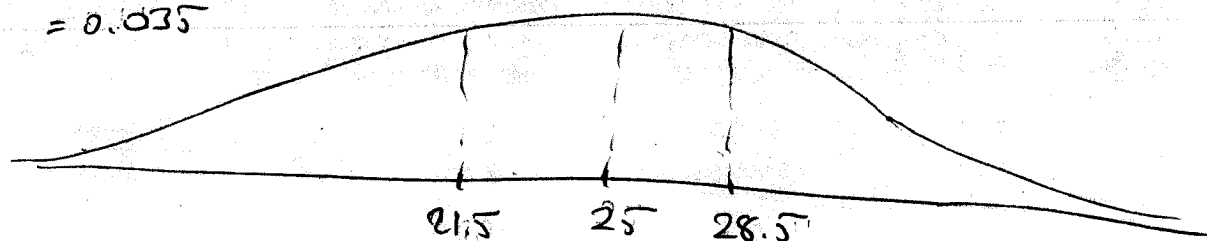
2. Now consider a random sample of 150 women.

- a. (1 points) What is the expected value of the percentage of women who have been a victim of domestic abuse in the sample?

$$\frac{1}{4} = \%25$$

- b. (4 points) Sketch an approximate histogram for all sample percentages with this sample size. Show at least three points on the x-axis.

$$\begin{aligned} \sigma_E &= \sqrt{\frac{0.25 \cdot 0.75}{150}} \\ &= 0.035 \\ &= \%3.5 \end{aligned}$$



Part V. (15 points) The following statistics for the number of books required for randomly selected courses in three departments at a large university is given below.

	Philosophy	English	History
Sample size	6	7	7
Mean	4.2	5.1	4.3
SD	0.8	0.9	1.1

1. (3 points) What is the variable under study?

The number of books required by a department

2. (12 points) Test the claim that the mean number of books required are equal in the three departments by using $\alpha = 0.05$

$$H_0: \mu_1 = \mu_2 = \mu_3$$

H_a : at least one μ is different

$$k=3 \quad N=20$$

$$\bar{X}_{GM} = \frac{6(4.2) + 7(5.1) + 7(4.3)}{20} = 4.55$$

$$S_B^2 = \frac{6(4.2 - 4.55)^2 + 7(5.1 - 4.55)^2 + 7(4.3 - 4.55)^2}{3-1}$$

$$= 1.645$$

$$S_W^2 = \frac{(6-1)(0.8)^2 + (7-1)(0.9)^2 + (7-1)(1.1)^2}{20-3}$$

$$= 0.901$$

$$F = \frac{1.645}{0.901} = 1.826$$

$$F_{2,17,0.05} = 3.59$$

Since $1.826 < 3.59$, we do not reject H_0 .

The mean number of books required is the same in the three departments.

Part VI. (15 points) According to a recent census, 68% of all families in a country have two parents present at home, 23% have only mother present, 5% have only a father present, and 4% have no parents present. A random sample of 200 families from the same country revealed the following results

Two parents	Mother only	Father only	No parent
120	40	30	10
Expected: 136 = $200 \times 68\%$	46 = $200 \times 23\%$	10 = $200 \times 5\%$	8 = $200 \times 4\%$

1. (9 points) Is there sufficient evidence to conclude that the percentages of families by type of parent(s) present at home differ from those reported by the census?

$$H_0: p_1 = 68\%, p_2 = 23\%, p_3 = 5\%, p_4 = 4\%$$

H_a : At least one percentage is different

$$\chi^2 = \frac{(120-136)^2}{136} + \frac{(40-46)^2}{46} + \frac{(30-10)^2}{10} + \frac{(10-8)^2}{8} = 43.16$$

$$d.f. = 4 - 1 = 3 \quad \chi^2_{critical, 0.05} = 7.82$$

Since $43.16 > 7.82$, we reject H_0 .

There is a significant evidence that the percentages of families by type of parents have changed recently.

2. (6 points) The average number of parents in a family as found in this sample will be compared with the average number of parents in a family as found from a sample of 250 families taken in 1970's.

- a) Fill in the blank:

The test to be used for comparison is two sample z-test.

- b) Which sample can be considered as the control group in this study?

The sample taken in 1970's.

- c) Is this a historical or contemporaneous control group?

Historical because there is no experiment conducted at the same time.