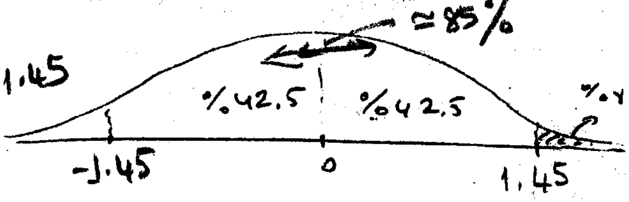
- **Part I. (20 points)** Waiting time at a doctor's office is normally distributed with mean 20 minutes and standard deviation 7 minutes.

1. (6 points) What is the chance that an arbitrary patient will wait more than half an hour?

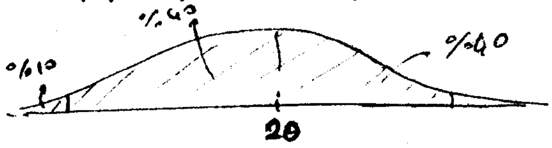
$z = \frac{30-20}{7} = \frac{10}{7} = 1.43 \to 1.45$



area ≅ %85

$\frac{100-85}{2} = \boxed{\%7.5}$

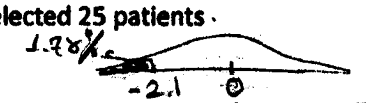
2. (4 points) Find the 90th percentile of the waiting time.



%80 → $z \cong 1.30$

$20 + 1.30 \times 7 = 20 + 9.1$
$= 29.1$

3. (6 points) What is the chance that the average waiting time of randomly selected 25 patients will be less than 17 minutes?



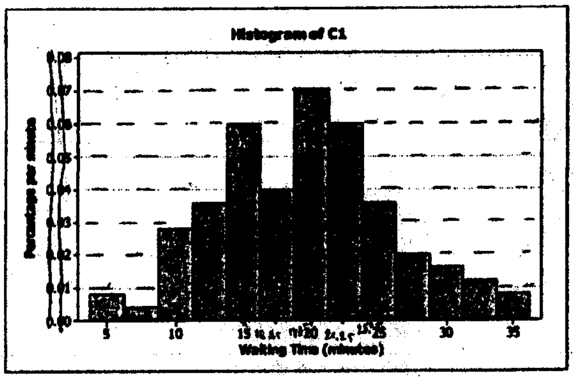
$z = \frac{17-20}{7/\sqrt{25}} = \frac{-3}{7/5} = \frac{-15}{7} = -2.1$ ⟹ 96.43 — 17 — 20

$100 - 96.43 = 3.57$

$\frac{3.57}{2} = 1.78$

%1.78

4. (4 points) Here is a density scale histogram from a sample of patients of the same doctor.



What percent of the patients in the sample have waited between 15 and 20 minutes?

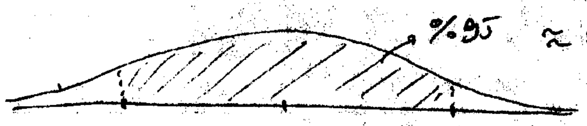
$1.25 \times 6 + 2.5 \times 4 + 1.25 \times 7 = 7.5 + 10 + 8.75 = 26.25$

Part II. (15 points) Any sentence that contains a noun as a direct object (=nesne) and another noun as a second object is called a "Double-Object Dative", shortly a DOD. For example in the sentence "Sue offered Ann a cookie", "Ann" is the direct object and "cookie" is the second object.

A study about DOD was conducted in an introductory English composition course. 35 native speakers were randomly chosen and they each made a sentence using the verb "buy". Of these sentences, 10 had a DOD structure.

1. (7 points) Construct a 95% confidence interval for the true percentage of sentences with the verb "buy" that have a DOD structure.

$\frac{10}{35} = 0.28$



%95 $\approx z=2$

$SE = \sqrt{\frac{0.28 \cdot 0.72}{35}}$

$= 0.075$

$\Rightarrow \%7.5$

$28 \mp 2\times 7.5 \Rightarrow 28 \mp 15 \Rightarrow [13, 43]$

2. (8 points) Perform a test of significance to determine whether the true fraction of sentences with the verb "buy" that have a DOD structure is less than 33%.

Ho : $p = \%33$

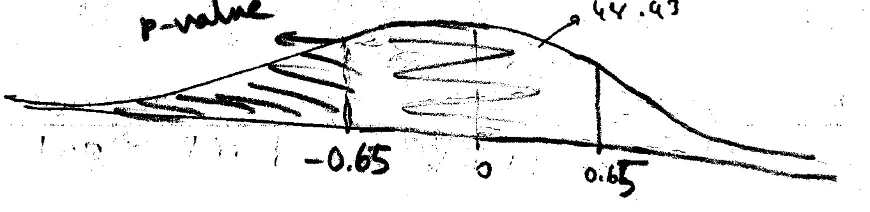
Ha : $p < \%33$

$z = \frac{28-33}{7.95} = \frac{-5}{7.95} = -0.63 \rightarrow -0.65$

$SE = \sqrt{\frac{0.33\,(1-0.33)}{35}}$

$= 7.95\%$

p-value



$\text{pvalue} = \frac{100 - 48.43}{2} = 25.78\% > \%5$

Do not reject Ho.

The true fraction of sentences with the verb "buy" that have a DOD structure is not less than % 33.

Part III: (20 points) The following frequency table shows the type of movie borrowed from the library versus the age group of the borrower in a random sample of 55 people.

	Mystery	Comedy	Action	
Young	7	12	15	34
Old	10	6	5	21
	17	18	20	55

1. (15 points) Does the type of movie borrowed depend on age?

expected frequency

	Mystery	Comedy	Action
Young	10.50	11.12	12.36
Old	6.50	6.87	7.63

Ho: movie borrowed is independ on age.
Ha: movie borrowed depend on age

$$\chi^2 = \frac{(7-10.5)^2}{10.50} + \frac{(12-11.12)^2}{11.12} + \frac{(15-12.36)^2}{12.36} + \frac{(10-6.5)^2}{6.50} + \frac{(6-6.87)^2}{6.87} + \frac{(5-7.63)^2}{7.63}$$

$$\Rightarrow \chi^2 = 1.16 + 1.85 + 0.065 + 0.11 + 0.56 + 0.50 = 4.68$$

Degrees of freedom $(2-1)\cdot(3-1) = 1.2 = 2$.

$0.05 < p < 0.1 \Rightarrow$ Do not reject Ho.
(or $4.68 < 5.99 = \chi^2_{0.05,2}$)

Hence, type of movie borrowed and age are independent.

2. (5 points) If a subject is chosen in particular from <u>the young group of subjects</u> randomly, what is the probability that s/he rents comedy movies?

$$\frac{12}{34} = 0.35 \quad (\text{or } 35\%)$$

$$17 \times \frac{34}{55} = 17 \times 0.62 \cong 10.50$$

Part IV. (15 points) Sociologists state that one in every four women has been a victim of domestic abuse in the population.

1. (10 points) In a random sample of 15 women, find the probability that at most 3 of them have been a victim of domestic abuse.

$P = 1/4 = 0.25$

$\binom{15}{0}(0.25)^0(0.75)^{15} + \binom{15}{1}(0.25)(0.75)^{14} + \binom{15}{2}(0.25)^2(0.75)^{13}$

$0.013 \qquad 0.25 \quad 0.017 \qquad 0.0625 \quad 0.02$

$+\binom{15}{3}(0.25)^3(0.75)^{12}$

$0.015 \qquad 0.031$

$= 0.013 + 0.063 + 0.15 + 0.21 \cong 0.436$ or 43.6%

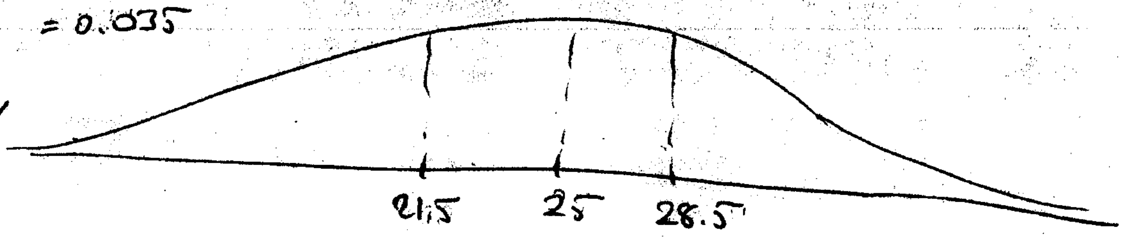
2. Now consider a random sample of 150 women.

   a. (1 points) What is the expected value of the percentage of women who have been a victim of domestic abuse in the sample?

   $1/4 = 25\%$

   b. (4 points) Sketch an approximate histogram for all sample percentages with this sample size. Show at least three points on the x-axis.

$SE = \sqrt{\frac{0.25 \cdot 0.75}{150}} = 0.035$ or 3.5%



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Part V. (15 points) The following statistics for the number of books required for randomly selected courses in three departments at a large university is given below.

	Philosophy	English	History
Sample size	6	7	7
Mean	4.2	5.1	4.3
SD	0.8	0.9	1.1

1. (3 points) What is the variable under study?

The number of books required by a course.

2. (12 points) Test the claim that the mean number of books required are equal in the three departments by using $\alpha = 0.05$

$H_0: \mu_1 = \mu_2 = \mu_3$

H_a: at least one μ is different

$k = 3 \quad N = 20$

$$\bar{X}_{GM} = \frac{6(4.2) + 7(5.1) + 7(4.3)}{20} = 4.55$$

$$s_B^2 = \frac{6(4.2-4.55)^2 + 7(5.1-4.55)^2 + 7(4.3-4.55)^2}{3-1}$$

$$= 1.645$$

$$s_W^2 = \frac{(6-1)(0.8)^2 + (7-1)(0.9)^2 + (7-1)(1.1)^2}{20-3}$$

$$= 0.901$$

$$F = \frac{1.645}{0.901} = 1.826$$

$$F_{2,17,0.05} = 3.59$$

Since $1.826 < 3.59$, we do not reject H_0.

The mean number of books required is the same in the three departments.

Part VI. (15 points) According to a recent census, 68% of all families in a country have two parents present at home, 23% have only mother present, 5% have only a father present, and 4% have no parents present. A random sample of 200 families from the same country revealed the following results

	Two parents	Mother only	Father only	No parent
observed:	120	40	30	10
Expected:	136 = 200×%68	46 = 200×%23	10 = 200×%5	8 = 200·%4

1. (9 points) Is there sufficient evidence to conclude that the percentages of families by type of parent(s) present at home differ from those reported by the census?

$H_0: p_1 = 68\%, \; p_2 = 23\%, \; p_3 = 5\%, \; p_4 = 4\%$

H_a: At least one percentage is different

$$\chi^2 = \frac{(120-136)^2}{136} + \frac{(40-46)^2}{46} + \frac{(30-10)^2}{10} + \frac{(10-8)^2}{8} = 43.16$$

$d.f. = 4-1 = 3 \qquad \chi^2_{critical,0.05} = 7.82$

Since $43.16 > 7.82$, we reject H_0.

There is a significant evidence that the percentages of families by type of parents have changed recently.

2. (6 points) The average number of parents in a family as found in this sample will be compared with the average number of parents in a family as found from a sample of 250 families taken in 1970's.

a) Fill in the blank:

The test to be used for comparison is two sample z-test.

b) Which sample can be considered as the control group in this study?

The sample taken in 1970's.

c) Is this a historical or contemporaneous control group?

Historical because there is no experiment conducted at the same time.