(1)

# KOÇ UNIVERSITY

# MATH 203 - MULTIVARIABLE CALCULUS

## Midterm III May 02, 2016

**Duration of Exam: 75 minutes**

**INSTRUCTIONS:** You can use calculators in the exam. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

**Name:** ______________________________

**Surname:** ______________________________

**Signature:** ____________________________________

**Section (Check One):**

Section 1: Ayberk Zeytin Tu - Th (10:00) ___
Section 2: Ayberk Zeytin Tu - Th (11:30) ___
Section 3: Attila Aşkar M - W (8.30) ___
Section 4: Attila Aşkar M - W (14:30) ___

| PROBLEM | POINTS | SCORE |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 25 | |
| 4 | 25 | |
| TOTAL | 100 | |

**2.** (25 points) Evaluate the following integrals:

(a) $\oint_C z\, ds$; where $C$ is the curve obtained by intersecting $\frac{x^2}{4} + y^2 + z^2 = 1$ with the plane $x = 0$.

Intersection curve: $x = 0 \rightarrow y^2 + z^2 = 1$

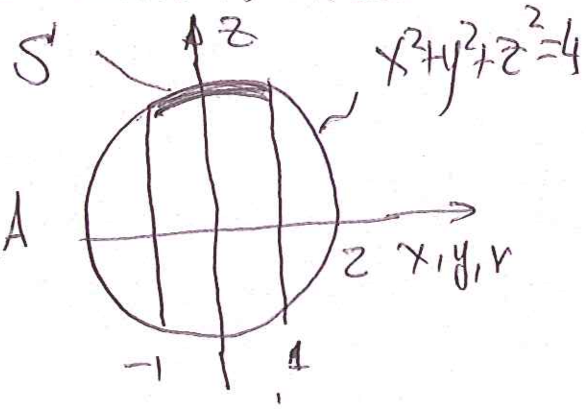
Parametrization: $\vec{r} = (0, \cos t, \sin t)$ (eg)

$d\vec{r} = (0, -\sin t, \cos t)\, dt$

$I = \oint z\, ds \rightarrow$ $ds = |d\vec{r}| = dt$

$$I = \int_{t=0}^{2\pi} \sin t\, dt = 0$$

(b) $\iint_S z\, dS$; where $S$ is the part of the surface $x^2 + y^2 + z^2 = 4$ inside $x^2 + y^2 = 1$, with $z > 0$.




Side view

Method 1:

$$dS = \left(\sqrt{F_x^2 + F_y^2 + F_z^2} \,/\, F_z\right) dA$$

Method 2:

$$dS = a^2 \sin\phi\, d\phi\, d\theta$$

Any method is acceptable

# Calculations for Prob. #2

**Method 1:** $F(x,y,z) = x^2+y^2+z^2-4=0$

$$\frac{\sqrt{F_x^2+F_y^2+F_z^2}}{F_z} = \frac{\sqrt{4(x^2+y^2+z^2)}}{\cancel{z}\, 2z} = \frac{\sqrt{4}}{z} = \frac{2}{z}$$

$$z = \sqrt{4-r^2} = (4-r^2)^{1/2}$$

$$S = 2\int_{\theta=0}^{2\pi}\int_{r=0}^{1} \frac{r\,dr\,d\theta}{(4-r^2)^{1/2}}$$

$$S = \cancel{2}\left(-\frac{1}{\cancel{2}}\right)2\pi \int_{u=4}^{u=3} (u^{-1/2})\,du$$

$$= -2\pi \left.\frac{u^{1/2}}{(1/2)}\right|_{u=4}^{3} = 4\pi u^{1/2}\Big|_{u=3}^{4}$$

$$= 4\pi(2-\sqrt{3})$$

Change of variable for integration

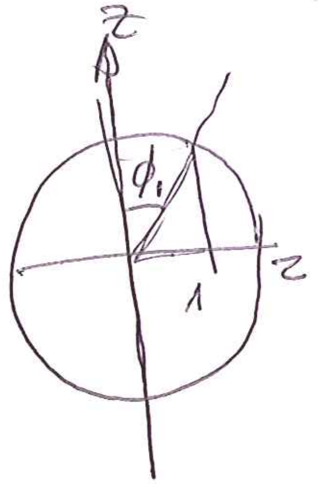
$u = 4-r^2 \quad du = -2r\,dr$

$r\,dr = -\frac{1}{2}\,du$

$r=0 \rightarrow u=4$

$r=1 \rightarrow u=3$

# Calculations for Prob #2 1+




## Method 2

$$S = a^2 \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\phi_1} \sin\phi \, d\phi \, d\theta$$

$a = 2$

$\sin\phi_1 = \frac{1}{2} \rightarrow \cos\phi_1 = \frac{\sqrt{3}}{2}$

$$S = 4 \cdot 2\pi \Big[-\cos\phi\Big]_{\phi=0}^{\phi_1}$$

$$= 8\pi \Big[\cos\phi\Big]_{\phi_1}^{\phi=0} = 8\pi\left[1 - \frac{\sqrt{3}}{2}\right] \Rightarrow$$

$$S = 4\pi\left(2 - \sqrt{3}\right)$$

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3. (25 points) Given

$$\mathbf{F} = \langle 1+\sin(y)e^z,\ 2y+x\cos(y)e^z,\ x\sin(y)e^z\rangle = \left(1+\sin(y)e^z\right)\mathbf{i} + \left(2y+x\cos(y)e^z\right)\mathbf{j} + x\sin(y)e^z\,\mathbf{k}$$

(a) Decide whether **F** is conservative or not. (Give reason.)

Is $\vec{F} = \nabla\phi$? $\Rightarrow$ $\vec{F} = (F, G, H)$

$\qquad\qquad\qquad\qquad x \quad y \quad z$

$\frac{\partial F}{\partial y} = \cos(y)e^z \qquad \frac{\partial G}{\partial x} = \cos(y)e^z$

$\frac{\partial F}{\partial z} = \sin(y)e^z \qquad \frac{\partial H}{\partial x} = \sin y = e^z$

$\frac{\partial G}{\partial z} = x\cos(y)e^z \qquad \frac{\partial H}{\partial y} = x\cos(y)e^z \Rightarrow \vec{F}$: conservative

(b) Hence or otherwise, evaluate

$$I = \oint_C \left(1+\sin(y)e^z\right)dx + \left(2y+x\cos(y)e^z\right)dy + x\sin(y)e^z\,dz;$$

$C$ is the curve obtained by intersecting the surface $x^2+y^2=z$ with the plane $x+y+z=1$

By theorem, because $\vec{F}$ is conservative

$$\oint \vec{F}\cdot d\vec{x} = 0$$

So $I = 0$

**4.** (25 points) Find the field lines ("Integral curves, Trajectories, Streamlines" are equivalent terms) of the following vector fields. Draw the streamlines passing through each of the points $A: (1,0)$; $B: (1,1)$ and the vectors of the field at these points.

(Note for drawings. We do not necessarily expect "artsitic" figures, but we except "reasonable" scale and shape. Use the provided grid.)

(a) Show that $\mathbf{F} = \langle 1,\ 2x\rangle = \mathbf{i} + 2x\mathbf{j}$ are given by the equation $y = x^2 + C$ for all $C$ and do the drawings as requested;

$\vec{F} = (F, G)$

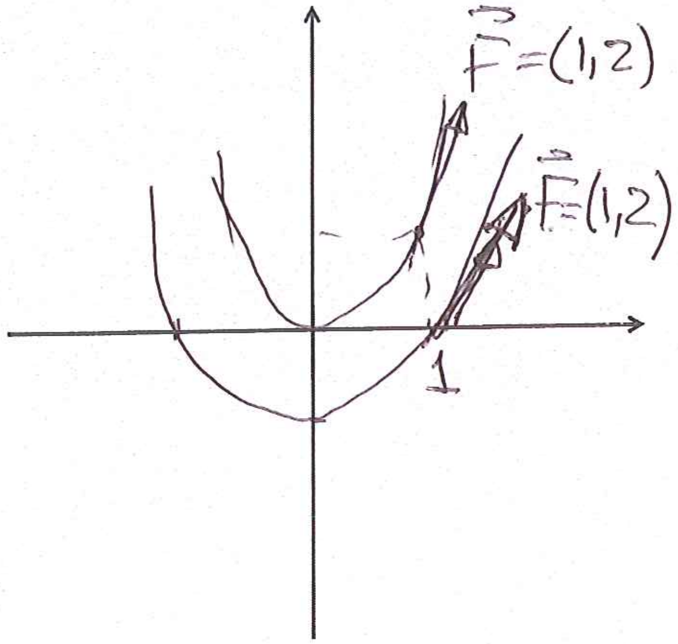
Eq. Field lines: $\dfrac{dx}{F} = \dfrac{dy}{G} \to$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(x^2+C) = 2x = \dfrac{G}{F} = \dfrac{2x}{1}$

Plots:

For $A: (1,0) \to 0 = 1 + C \to y = x^2 - 1$

For $B = (1,1) \to 1 = 1 + C \to y = x^2$




(b) For $\mathbf{F} = \langle -y,\ x\rangle = -y\mathbf{i} + x\mathbf{j}$ find the field lines and do the drawings as requested.

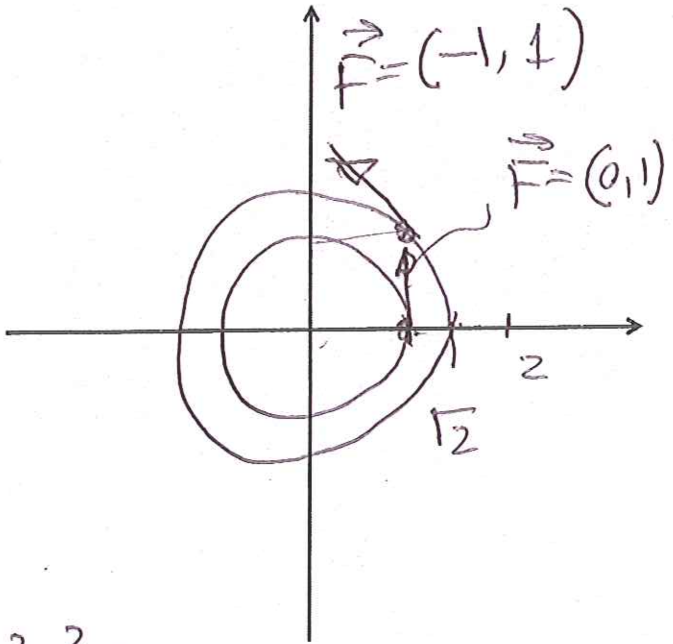
Eq. of field lines

$\dfrac{dx}{-y} = \dfrac{dy}{x} \to$

$x\,dx = -y\,dy \to$

$\dfrac{x^2}{2} = -\dfrac{y^2}{2} = C' \to$

$\boxed{x^2 + y^2 = C}$




For plots: $A: (1,0) \to 1 = C \to x^2 + y^2 = 1$

$B: (1,1) \to 1 + 1 = C \to x^2 + y^2 = 2$