HW10

1. (Problem 10.16 in the book) The turbine of a steam power plant operating on a simple Rankine cycle produces 1750 kW of power when the boiler is operated at 6 MPa, the condenser at 20 kPa, and the temperature at the turbine entrance is 500°C. Determine the rate of heat supply in the boiler, the rate of heat rejection in the condenser, and the thermal efficiency of the cycle.

10-16 A simple ideal Rankine cycle with water as the working fluid operates between the specified pressure limits. The rates of heat addition and rejection, and the thermal efficiency of the cycle are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

Analysis From the steam tables (Tables A-4, A-5, and A-6).

\[ h_1 = h_f @ 201 \text{Pa} = 251.42 \text{kJ/kg} \]
\[ \nu_1 = \nu_f @ 201 \text{Pa} = 0.001017 m^3/kg \]
\[ w_{p,in} = \nu_1 (P_2 - P_1) = (0.001017 m^3/kg)(0000 - 30) \text{kPa} \left( \frac{1 \text{kJ}}{1 \text{kJ} \cdot m^3} \right) = 6.08 \text{kJ/kg} \]
\[ h_2 = h_1 + w_{p,in} = 251.42 + 6.08 = 257.50 \text{kJ/kg} \]

\[ P_3 = 6 \text{MPa} \quad h_3 = 3423.1 \text{kJ/kg} \]
\[ T_3 = 500^\circ \text{C} \quad s_3 = 6.8826 \text{kJ/kg} \cdot \text{K} \]
\[ P_4 = 30 \text{kPa} \quad x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.8826 - 0.8320}{7.0752} = 0.8552 \]
\[ s_4 = s_3 \]
\[ h_4 = h_f + x_4 h_{fg} = 251.42 + (0.8552)(2357.5) = 2267.5 \text{kJ/kg} \]

Knowing the power output from the turbine the mass flow rate of steam in the cycle is determined from

\[ \dot{W}_{T,out} = \dot{m}(h_3 - h_4) \rightarrow \dot{m} = \frac{\dot{W}_{T,out}}{h_3 - h_4} = \frac{1750 \text{kJ/s}}{(3423.1 - 2267.5) \text{kJ/kg}} = 1.514 \text{kg/s} \]

The rates of heat addition and rejection are

\[ \dot{Q}_{in} = \dot{m}(h_3 - h_2) = (1.514 \text{ kg/s})(3423.1 - 257.50) \text{kJ/kg} = 4794 \text{ kW} \]
\[ \dot{Q}_{out} = \dot{m}(h_4 - h_1) = (1.514 \text{ kg/s})(2267.5 - 251.42) \text{kJ/kg} = 3053 \text{ kW} \]

and the thermal efficiency of the cycle is

\[ \eta_{th} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{3053}{4794} = 0.363 = 36.3\% \]
2. (Problem 10.20 in the book) A simple Rankine cycle uses water as the working fluid. The boiler operates at 6000 kPa and the condenser at 50 kPa. At the entrance to the turbine, the temperature is 450°C. The isentropic efficiency of the turbine is 94%, pressure and pump losses are negligible, and the water leaving the condenser is subcooled by 6.3°C. The boiler is sized for a mass flow rate of 20 kg/s. Determine the rate at which heat is added in the boiler, the power required to operate the pumps, the net power produced by the cycle, and the thermal efficiency. (Answers: 59660 kW, 122 kW, 18050 kW, 30.3%)

10-20 A simple Rankine cycle with water as the working fluid operates between the specified pressure limits. The rate of heat addition in the boiler, the power input to the pumps, the net power, and the thermal efficiency of the cycle are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

**Analysis** From the steam tables (Tables A-4, A-5, and A-6),

\[
\begin{align*}
P_1 &= 50 \text{ kPa} \\
T_1 &= T_{\text{sat}(50\text{ kPa})} - 6.3 = 81.3 - 6.3 = 75^\circ\text{C} \\
\eta_1 &= \eta_{f_{75^\circ\text{C}}} = 0.001026 \text{ m}^3/\text{kg} \\
\dot{w}_{\text{p,in}} &= \eta_1 (P_2 - P_1) \\
&= (0.001026 \text{ m}^3/\text{kg}) (6000 - 50) \text{ kPa} \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\
&= 6.10 \text{ kJ/kg} \\
\dot{h}_2 &= \dot{h}_1 + \dot{w}_{\text{p,in}} = 314.03 + 6.10 = 320.13 \text{ kJ/kg} \\
P_3 &= 6000 \text{ kPa} \quad \dot{h}_3 = 3302.9 \text{ kJ/kg} \\
T_3 &= 450^\circ\text{C} \quad s_3 = 6.7219 \text{ kJ/kg K} \\
P_4 &= 50 \text{ kPa} \quad x_4 = \frac{s_4 - s_f}{s_{fg}} = -6.7219 - 1.0912 = -0.8660 \\
s_4 = s_3 \\
\dot{h}_4 = \dot{h}_3 + x_4 \dot{h}_f = 340.54 + (0.8660)(2304.7) = 2336.4 \text{ kJ/kg} \\
\eta_T &= \frac{\dot{h}_3 - \dot{h}_4}{\dot{h}_3 - \dot{h}_4} = h_3 - \eta_T (h_3 - h_4) = 3302.9 - (0.94)(3302.9 - 2336.4) = 2394.4 \text{ kJ/kg} \\
\end{align*}
\]

Thus,

\[
\begin{align*}
\dot{Q}_\text{in} &= \dot{m} (h_3 - h_2) = (20 \text{ kg/s})(3302.9 - 320.13) = 59,660 \text{ kW} \\
\dot{W}_\text{T,net} &= \dot{m} (h_1 - h_4) = (20 \text{ kg/s})(3302.9 - 2394.4) = 18,170 \text{ kW} \\
\dot{W}_\text{p,in} &= \dot{m} \dot{w}_{\text{p,in}} = (20 \text{ kg/s})(6.10 \text{ kJ/kg}) = 122 \text{ kW} \\
\dot{W}_\text{act} &= \dot{W}_\text{T,net} - \dot{W}_\text{p,in} = 18,170 - 122 = 18,050 \text{ kW} \\
\end{align*}
\]

and

\[
\eta_\text{net} = \frac{\dot{W}_\text{act}}{\dot{Q}_\text{in}} = \frac{18,050}{59,660} = 0.3025
\]
3. (Problem 10.36 in the book) Consider a steam power plant that operates on the ideal reheat Rankine cycle. The plant maintains the boiler at 7000 kPa, the reheat section at 800 kPa, and the condenser at 10 kPa. The mixture quality at the exit of both turbines is 93%. Determine the temperature at the inlet of each turbine and the cycle's thermal efficiency. (Answers: 373°C, 416°C, 37.6%)

10-36 An ideal reheat Rankine with water as the working fluid is considered. The temperatures at the inlet of both turbines, and the thermal efficiency of the cycle are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

**Analysis** From the steam tables (Tables A-4, A-5, and A-6),

\[
\begin{align*}
 h_1 &= h_{f_{10kPa}} - 191.81 \text{kJ/kg} \\
 \nu_1 &= \nu_{f_{10kPa}} = 0.001010 \text{ m}^3/\text{kg} \\
 w_{p, in} &= \nu_1 (P_2 - P_1) \\
 &= (0.001010 \text{ m}^3/\text{kg})(7000 - 10)\text{kPa} \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{ m}^3} \right) \\
 &= 7.06 \text{kJ/kg} \\
 h_2 &= h_1 + w_{p, in} = 191.81 + 7.06 = 198.87 \text{kJ/kg} \\
 P_3 &= 800 \text{kPa} \quad \begin{cases} 
 h_3 &= h_f + x_4 h_{f_{g}} = 720.87 + (0.93)(2047.5) = 2625.0 \text{kJ/kg} \\
 x_4 &= 0.93 \\
 s_3 &= s_f + x_4 s_{f_{g}} = 2.0457 + (0.93)(4.6160) = 6.3385 \text{kJ/kg \cdot K} \\
 P_3 &= 7000 \text{kPa} \quad h_4 = 3085.5 \text{kJ/kg} \\
 s_4 &= s_d \quad T_3 = 373.3^\circ \text{C} \\
 P_4 &= 10 \text{kPa} \quad h_5 &= h_f + x_4 h_{f_{g}} = 191.81 + (0.93)(2392.1) = 2416.4 \text{kJ/kg} \\
 x_5 &= 0.90 \\
 s_5 &= s_f + x_4 s_{f_{g}} = 0.6492 + (0.93)(7.4996) = 7.6239 \text{kJ/kg \cdot K} \\
 P_5 &= 800 \text{kPa} \quad h_6 = 3302.0 \text{kJ/kg} \\
 s_5 &= s_d \quad T_5 = 416.2^\circ \text{C} \\
\end{cases}
\]

Thus,

\[
\begin{align*}
 q_{in} &= (h_3 - h_2) + (h_3 - h_4) - 3085.5 - 198.87 + 3302.0 - 2625.0 - 3563.6 \text{kJ/kg} \\
 q_{out} &= h_8 - h_1 - 2416.4 - 191.81 - 2224.6 \text{kJ/kg} \\
\end{align*}
\]

and

\[
\eta_{in} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2224.6}{3563.6} = 0.3757 = 37.6% 
\]

4. (Problem 10.38 in the book) A steam power plant operates on the reheat Rankine cycle. Steam enters the high-pressure turbine at 12.5 MPa and 550oC at a rate of 7.7 kg/s and leaves at 2 MPa. Steam is then reheated at constant pressure to 450oC before it expands in the low-pressure turbine. The isentropic efficiencies of the turbine and pump are 85% and 90%, respectively. Steam leaves the condenser as a saturated liquid. If the moisture content of the steam at the exit of the turbine is not to exceed 5%, determine (a) the condenser pressure, (b) the net power output, and (c) the thermal efficiency. (Answers: (a) 9.73 kPa, (b) 10.2 MW, (c) 36.9%)
10-38 A steam power plant that operates on a reheat Rankine cycle is considered. The condenser pressure, the net power output, and the thermal efficiency are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

**Analysis (a)** From the steam tables (Tables A-4, A-5, and A-6).

\[
\begin{align*}
P_3 &= 12.5 \text{ MPa} \quad h_3 = 3476.5 \text{ kJ/kg} \\
T_3 &= 550^{\circ}\text{C} \quad \tau_3 = 6.8317 \text{ kJ/kg K} \\
P_4 &= 2 \text{ MPa} \quad h_{h_4} = 2948.1 \text{ kJ/kg} \\
\eta_T &= \frac{h_3 - h_4}{h_3 - h_{h_4}} \\
\rightarrow h_4 &= h_3 - \eta_T (h_3 - h_{h_4}) \\
&= 3476.5 - (0.85)(3476.5 - 2948.1) \\
&= 3027.3 \text{ kJ/kg} \\
P_5 &= 2 \text{ MPa} \quad h_5 = 3385.2 \text{ kJ/kg} \\
T_5 &= 450^{\circ}\text{C} \quad \tau_5 = 7.2815 \text{ kJ/kg K} \\
P_6 &= ? \quad x_6 = 0.95 \quad (\text{Eq. 1}) \\
P_6 &= ? \quad h_{h_6} = (\text{Eq. 2}) \\
\eta_T &= \frac{h_5 - h_{h_6}}{h_5 - h_6} \rightarrow h_6 = h_3 - \eta_T (h_3 - h_{h_6}) = 3385.2 - (0.85)(3385.2 - h_{h_6}) \quad (\text{Eq. 3})
\end{align*}
\]

The pressure at state 6 may be determined by a trial-error approach from the steam tables or by using EES from the above three equations:

\[
P_6 = 9.73 \text{ kPa} \quad h_6 = 2463.3 \text{ kJ/kg}.
\]

(b) Then,

\[
\begin{align*}
h_3 &= h_{f,973 \text{ kPa}} = 189.57 \text{ kJ/kg} \\
\nu_1 &= \nu_{f,10 \text{ kPa}} = 0.001010 \text{ m}^3/\text{kg} \\
\omega_{p,\text{in}} &= \omega_{f}(P_2 - P_1)/\eta_p \\
&= (0.00101 \text{ m}^3/\text{kg})(12,500 - 9.73 \text{ kPa})\left(\frac{1 \text{ kJ}}{1 \text{ kPa m}^3}\right)(0.90) \\
&= 14.02 \text{ kJ/kg} \\
h_2 &= h_1 + \omega_{p,\text{in}} = 189.57 + 14.02 = 203.59 \text{ kJ/kg}
\end{align*}
\]

Cycle analysis:

\[
\begin{align*}
q_{\text{in}} &= (h_3 - h_2) + (h_5 - h_4) - 3476.5 - 203.59 + 3385.2 - 2463.3 - 3603.8 \text{ kJ/kg} \\
q_{\text{out}} &= h_6 - h_1 = 2463.3 - 189.57 = 2273.7 \text{ kJ/kg} \\
W_{\text{net}} &= \dot{m}(q_{\text{in}} - q_{\text{out}}) = (7.7 \text{ kg/s})(3603.8 - 2273.7) \text{kJ/kg} = 10,242 \text{ kW}
\end{align*}
\]

(c) The thermal efficiency is

\[
\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2273.7 \text{ kJ/kg}}{3603.8 \text{ kJ/kg}} = 0.369 = 36.9%.
\]