1. (Problem 1.23 in the Book) A transmission case measures $W = 0.3$ m on a side and receives a power input $P_i = 150$ hp from the engine. If the transmission efficiency is $\eta = 0.93$ and airflow over the case corresponds to $T_\infty = 30$C and $h = 200 \frac{W}{m^2 K}$, what is the surface temperature of the transmission.

**KNOWN:** Width, input power and efficiency of a transmission. Temperature and convection coefficient associated with air flow over the casing.

**FIND:** Surface temperature of casing.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Negligible radiation.

- There is no temperature difference given so there is no conduction.
- There is no radiation coefficient or surrounding temperature given so there is no radiation.

**ANALYSIS:** From Newton’s law of cooling,

$$q = hA_s (T_s - T_\infty) = 6hW^2 (T_s - T_\infty)$$

where the output power is $P_o = \eta P_i$ and the heat rate is

$$q = P_i - P_o = P_i (1-\eta) = 150 \text{hp} \times 746 \text{ W/hp} \times 0.07 = 7833 \text{ W}$$

Hence,

$$T_s = T_\infty + \frac{q}{6hW^2} = 30\text{C} + \frac{7833 \text{ W}}{6 \times 200 \text{ W/m}^2 \cdot \text{K} \times (0.3 \text{m})^2} = 102.5\text{C}$$

**COMMENTS:** There will, in fact, be considerable variability of the local convection coefficient over the transmission case and the prescribed value represents an average over the surface.
2. (Problem 1.28 in the Book) An overhead 30m long, uninsulated industrial steam pipe of 120mm diameter is routed through a building whose walls and air at at 30°C. Pressurized steam maintains a pipe surface temperature of 150°C, and the coefficient associated with natural convection is $h=10 \text{W/m}^2\cdot\text{K}$. The surface emissivity $\varepsilon=0.8$.

a) What is the rate of heat loss from the steam line?

b) If the steam is generated in a gas-fired boiler operating at an efficiency of $\eta_f =0.90$ and natural gas is priced at $C_g =$0.02 per MJ, what is the annual cost of heat loss from the line?

**ASSUMPTIONS:** (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

**ANALYSIS:** (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{\text{conv}} + q_{\text{rad}} = A \left[ h(T_s - T_\infty) + \varepsilon \sigma (T_s^4 - T_{\text{sur}}^4) \right]$$

Where $A = \pi DL = 11.3097 \text{m}^2$

Hence,

$$q = 11.3097 \text{m}^2 \left[ 10 \frac{W}{\text{m}^2\cdot\text{K}} (150 - 30) \text{K} + 0.8 \times 5.67 \times 10^{-8} \frac{W}{\text{m}^2\cdot\text{K}^4} (423^4 - 303^4) \right]$$

$$q = 11.3097 \text{m}^2 \left[ 1200 + 1069.9 \frac{W}{\text{m}^2\cdot\text{K}} \right] = 39243 \text{W}$$

(b) The annual energy loss is

$$E = qt = 18.405 \text{W} \times 3600 \text{ s/h} \times 24 \text{ h/d} \times 365 \text{ d/y} = 5.80 \times 10^{11} \text{ J}$$

$E = 39243 \text{W} \times 3600 \frac{\text{J}}{\text{h}} \times 24 \frac{\text{h}}{\text{d}} \times 365 \frac{\text{d}}{\text{y}} = 1.2376 \times 10^{12} \text{J}$

With a furnace energy consumption of $E_f = 1.3751 \times 10^{12} \text{ J}$, the annual cost of the loss is

$$C = C_g \frac{E_f}{E} = \frac{0.02}{\text{MJ}} \times 1.3751 \times 10^{12} \text{ J} = 27502 \text{ $}$$

**COMMENTS:** The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.
3. (Problem 1.67 in the Book) A photovoltaic panel of dimension $2m \times 4m$ is installed on the roof of a home. The panel is irradiated with a solar flux of $G_S = 700 \text{ W/m}^2$, oriented normal to the top panel surface. The absorptivity of the panel to the solar irradiation is $s = 0.83$, and the efficiency of conversion of the absorbed flux to electrical power is $\frac{P}{IsGSA} = 0.553 - 0.001T_p$, where $T_p$ is the panel temperature expressed in Kelvins and $A$ is the solar panel area. Determine the electric power generated for
(a) A still summer day, in which $T_{sur} = T_\infty = 35 \text{C}$, $h = 100 \text{ W/m}^2 \cdot \text{K}$, and
(b) A breezy winter day, for which $T_{sur} = T_\infty = -15 \text{oC}$, $h = 30 \text{ W/m}^2 \cdot \text{K}$. The panel Emissivity is 0.90.

**Schematic**

![Schematic Diagram](image_url)

**Assumptions**
1. Steady state
2. Uniform convection coefficient and panel temperature
3. No conduction

$A$ is the surface of the panel
$A = 2m \times 4m = 8m^2$

And the heat absorbed by the panel is:

$q_s = \alpha_s G_s A = 700 \times 0.83 \times 8 = 4648 \text{ W}$

Supposing that the panel temperature is constant, the amount of heat going out of the panel must be equal to the amount of heat that it receives, so:

$4648 = A[h(T_p - T_\infty) + \varepsilon \sigma (T_p^4 - T_{sur}^4)] = 8[10(T_p - 308) + 0.9 \times 5.67 \times 10^{-8} (T_p^4 - 308^4)]$

As we can see, this equation can not be solved analytically easily, so we use trial error method to solve this equation:

we define the equation again as

$\text{eq} = 8[10(T_p - 308) + 0.9 \times 5.67 \times 10^{-8} (T_p^4 - 308^4)] - 4648$

first guess

$T_p = 330 \text{K} \Rightarrow \text{eq} = -1400$

second guess

$T_p = 350 \text{K} \Rightarrow \text{eq} = 1500$

third guess

$T_p = 340 \text{K} \Rightarrow \text{eq} = 74$

forth try

$T_p = 339 \text{K} \Rightarrow \text{eq} = -69$
so, $T_p$ must be a number between 340 and 339 say 339.5
\[
\frac{P}{\alpha_s c_s A} = 0.553 - 0.001 T_p \quad \Rightarrow \quad P = 0.83 \times 700 \times 8 (0.553 - 0.001 \times 339.5) = 992.348 \text{W}
\]

b) In this part, solution is just similar to part a, so
\[
eq 8[30(T_p - 258) + 0.9 \times 5.67 \times 10^{-8} (T_p^4 - 258^4)] - 4648
\]
first guess
$T_p = 280 K \Rightarrow eq. = 1330$
second guess
$T_p = 270 K \Rightarrow eq. = -1400$
third guess
$T_p = 275 K \Rightarrow eq. = -42$
forth try
$T_p = 275.5 \Rightarrow eq. = 94.4$
So, $T_p$ must be a number between 275K and 275.5K say 275.25K
\[
\frac{P}{\alpha_s c_s A} = 0.553 - 0.001 T_p \quad \Rightarrow \quad P = 0.83 \times 700 \times 8 (0.553 - 0.001 \times 275.25) = 1291 \text{W}
\]
4. (Problem 2.32 in the Book) A plane wall of thickness \(2L = 60\, \text{mm}\) and thermal conductivity \(k = 5\, \text{W/m}\cdot\text{K}\) experiences uniform volumetric heat generation at a rate \(\dot{q}\) while convection heat transfer occurs at both of its surfaces \((x = -L, +L)\), each of which is exposed to a fluid of temperature \(T_\infty = 30\, \text{C}\). Under steady-state conditions, the temperature distribution is of the form \(T(x) = a + bx + cx^2\) where \(a = 86.0\, \text{C}\), \(b = -200\, \text{C/m}\), \(c = -2 \times 10^4\, \text{C/m}^2\), and \(x\) is in meters. The origin of the \(x\) coordinate is at the midplane of the wall.

**a)**

- \(T\) is distributed by the form 
  \[T(x) = 86 - 200x - 20000x^2\]
- \(T\) at \(-L\):
  \[T = 86 - 200(-0.03) - 20000(-0.03) = 74\, \text{C}\]
- \(T\) at \(+L\):
  \[T = 86 + 200(0.03) + 20000(0.03) = 62\, \text{C}\]
- And the maximum of temperature occurs in \(x = \frac{-(-200)}{2(-20000)} = 0.005\, \text{m} = 0.5\, \text{mm}\)
- And the coefficient of \(x^2\) is negative so the temperature distribution would be like the figure below.

**b)**

Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, the rate of volumetric heat generation can be determined.

\[
\frac{d}{dx} \left( \frac{dT}{dx} \right) + \frac{\dot{q}}{k} = 0 \quad \text{where} \quad T(x) = a + bx + cx^2
\]

\[
\frac{d}{dx} (0 + b + 2cx) + \frac{\dot{q}}{k} = (0 + 2c) + \frac{\dot{q}}{k} = 0
\]

\[
\dot{q} = -2ck = -2(-20000)5 = 2 \times 10^5\, \text{W/m}^3
\]

**c)**

The heat fluxes at the two boundaries can be determined using Fourier’s law and the temperature
distribution expression.

\[ q_x(x) = -k \frac{dT}{dx} \]

where \( T(x) = a + bx + cx^2 \)

\[ q_x(-L) = -k \left[ 0 + b + 2cx \right]_{x=-L} = -\left[ b - 2cL \right]k \]

\( q''_x(-L) = -5(-200 - 2 \times 20000 \times (-0.03)) = -5000 \text{ W/m}^2 \)

\( q''_x(L) = -5(-200 - 2 \times 20000 \times (0.03)) = 7000 \text{ W/m}^2 \)

From an overall energy balance on the wall as shown in the sketch below

\[ \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0, \]

\[ +q_x(-L) - q_x(+L) + 2\dot{q}L = 0 \]

\[ q''_x(L) = 1000 + 2 \times 10^5 \]

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location

\[ q''_x(x_{max}) = 0 \]
If the source of the heat generation is suddenly deactivated so that $\dot{q} = 0$, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) = \rho c_p \frac{\partial T}{\partial t}.$$

At the instant this occurs, the temperature distribution is still $T(x) = a + bx + cX^2$. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}_{st} = k \frac{\partial}{\partial x} [0 + b + 2cx] = k [0 + 2c] = 5 \text{ W/m$\cdot$K} \times 2 \left( -2 \times 10^4 \text{C/m$^2$} \right) = -2 \times 10^5 \text{ W/m}^3$$

With no heat generation, the wall will eventually ($t \to \infty$) come to equilibrium with the fluid, $T(x, \infty) = T_\infty = 30^\circ C$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis. The “initial” state is that corresponding to the steady-state temperature distribution, $T_i$, and the “final” state has $T_f = 30^\circ C$.

We’ve used $T_\infty$ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \quad \text{with} \quad E''_{in} = 0.$$ 

$$-E''_{out} = \rho c_p 2L \left( T_f - T_\infty \right) - \rho c_p \int_{-L}^{+L} \left( T_i - T_\infty \right) dx$$

$$E''_{out} = \rho c_p \int_{-L}^{+L} \left[ a + bx + cX^2 - T_\infty \right] dx = \rho c_p \left[ ax + bx^2 / 2 + cx^3 / 3 - T_\infty x \right]_{-L}^{+L}$$

$$E''_{out} = \rho c_p \left[ 2aL + 0 + 2cX^3 / 3 - 2T_\infty L \right]$$

$$E''_{out} = 2600 \times 800 [2 \times 86 \times 0.03 - 2 \times 20000 \times 0.03^3 - 2 \times 30 \times 0.03]$$

$$= 17.97 \times 10^6 \text{J/m}^2$$