1. **(Problem 5.5 in the Book except for part (e))**

For each of the following cases, determine an appropriate characteristic length $L_c$ and the corresponding Biot number $Bi$ that is associated with the transient thermal response of the solid object. State whether the lumped capacitance approximation is valid. If the temperature information is not provided, evaluate properties at $T = 300$ K. 

a. A toroidal shape of diameter $D = 65$ mm and cross-sectional area $A_c = 7\text{mm}^2$ is of thermal conductivity $k = 2.3 \text{ W/mK}$. The surface of the torus is exposed to a coolant corresponding to a convection coefficient of $h = 50 \text{ W/m}^2\text{K}$.

b. A long, hot AISI 302 stainless steel bar of rectangular cross-section has dimensions $w = 5$ mm, $W = 7$ mm, and $L = 150$ mm. The bar is subjected to a coolant that provides a heat transfer coefficient of $h = 10 \text{ W/m}^2\text{K}$ at all exposed surfaces.

c. A long extruded aluminum (Alloy 2024) tube of diameter of inner and outer dimensions $w = 25$ mm and $W = 30$ mm, respectively, is suddenly submerged in water, resulting in a convection coefficient of $h = 40 \text{ W/m}^2\text{K}$ at the four exterior tube surfaces. The tube is plugged at both ends, trapping stagnant air inside the tube.

d. An $L = 300$-mm-long solid stainless steel rod of diameter $D = 13$ mm and mass $M = 0.328$ kg is exposed to a convection coefficient of $h = 30 \text{ W/m}^2\text{K}$.

e. A long cylindrical rod of diameter $D = 20$ mm, density, specific heat $c_p = 1750 \text{ J/kgK}$, and thermal conductivity $k = 16 \text{ W/mK}$ is suddenly exposed to convective conditions with. The rod is initially at a uniform temperature of $T = 100^\circ\text{C}$ at $t = 225$ s.

f. Repeat part (f) but now consider a rod diameter of $D = 200$ mm.

**KNOWN:** Geometries of various objects. Material and/or properties. Cases (a) through (d): Convection heat transfer coefficient between object and surrounding fluid. Case (e): Emissivity of sphere, initial temperature, and temperature of surroundings. Cases (f) and (g): Initial temperature, spatially averaged temperature at a later time, and surrounding fluid temperature.

**FIND:** Characteristic length and Biot number. Validity of lumped capacitance approximation.
Case (a): \( D = 65 \text{ mm}, \ A_c = 7 \text{ mm}^2, \ k = 2.3 \text{ W/m} \cdot \text{K}, \ h = 50 \text{ W/m}^2 \cdot \text{K}. \)

Case (b): \( W = 7 \text{ mm}, \ w = 5 \text{ mm}, \ L = 150 \text{ mm}, \ h = 10 \text{ W/m}^2 \cdot \text{K}, \) AISI 302 stainless steel.

Case (c): \( w = 25 \text{ mm}, \ W = 30 \text{ mm}, \ h = 40 \text{ W/m}^2 \cdot \text{K} \) (\( L \) not specified), 2024 aluminum.

Case (d): \( L = 300 \text{ mm}, \ D = 13 \text{ mm}, \ M = 0.328 \text{ kg}, \ h = 30 \text{ W/m}^2 \cdot \text{K}. \) stainless steel.

Cases (f, g): \( D = 20 \text{ mm or 200 mm}, \ \rho = 2300 \text{ kg/m}^3, \ cp = 1750 \text{ J/kg} \cdot \text{K}, \ k = 16 \text{ W/m} \cdot \text{K}, \ T_\infty = 20^\circ \text{C}, \ Ti = 200^\circ \text{C}, \ T = 100^\circ \text{C} \) at \( t = 225 \text{ s}. \)

**ASSUMPTIONS:** (1) Constant properties

**PROPERTIES:** Table A.1, Stainless steel, AISI 302 \((T = 300 \text{ K}): k = 15.1 \text{ W/m} \cdot \text{K}. \) Aluminum 2024 \((T = 300 \text{ K}): k = 177 \text{ W/m} \cdot \text{K}. \)

**ANALYSIS:** Characteristic lengths can be calculated as \( Lc1 = V/As, \) or they can be taken conservatively as the dimension corresponding to the maximum spatial temperature difference, \( Lc2. \) The former definition is more convenient for complex geometries. The lumped capacitance approximation is valid for \( Bi = hLc/k < 0.1. \)

(a) The radius of the torus, \( ro, \) can be found from 2

\[ Ac = \pi r_o^2. \]

The characteristic lengths are

\[ L_{c1} = \frac{V}{A_i} = \frac{A_i \pi D}{2 \pi \sqrt{A_i / \pi}} = 1 \sqrt{A_i / \pi} = 1 \sqrt{7 \text{ mm}^2 / \pi} = 0.75 \text{ mm} \]

\[ L_{c2} = \text{Maximum center to surface distance} = \text{radius} = \sqrt{A_c / \pi} = \sqrt{7 \text{ mm}^2 / \pi} = 1.49 \text{ mm} \]
The corresponding Biot numbers are
\[ B_{i1} = \frac{hL_{c1}}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} \times 0.00075 \text{ m}}{2.3 \text{ W/m} \cdot \text{K}} = 0.016 \]
\[ B_{i2} = \frac{hL_{c2}}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} \times 0.00149 \text{ m}}{2.3 \text{ W/m} \cdot \text{K}} = 0.032 \]

The lumped capacitance approximation is valid according to either definition. 
(b) For this complex shape, we will calculate only \( L_{c1} \).
\[ I_{c1} = \frac{V}{A_s} = \frac{WwL}{2(W+w)l+2Ww} = \frac{7 \text{ mm} \times 5 \text{ mm} \times 150 \text{ mm}}{2(7 \text{ mm} + 5 \text{ mm}) \times 150 \text{ mm} + 14 \text{ mm} \times 5 \text{ mm}} = 1.43 \text{ mm} \]

Notice that the surface area of the ends has been included, and does have a small effect on the result – 1.43 mm versus 1.46 mm if the ends are neglected. The corresponding Biot number is
\[ B_{i1} = \frac{hL_{c1}}{k} = \frac{10 \text{ W/m}^2 \cdot \text{K} \times 0.00149 \text{ m}}{15.1 \text{ W/m} \cdot \text{K}} = 0.00095 \]

The lumped capacitance approximation is valid. Furthermore, since the Biot number is very small, the lumped capacitance approximation would certainly still be valid using a more conservative length estimate.

(c) Again, we will only calculate \( L_{c1} \). There will be very little heat transfer to the stagnant air inside the tube, therefore in determining the surface area for convection heat transfer, \( A_s \) only the outer surface area should be included. Thus,
\[ I_{c1} = \frac{V}{A_s} = \frac{(W^2-w^2)L}{4WL} = \frac{(30 \text{ mm})^2 - (25 \text{ mm})^2}{4 \times 30 \text{ mm}} = 2.29 \text{ mm} \]

The corresponding Biot number is
\[ B_{i1} = \frac{hL_{c1}}{k} = \frac{40 \text{ W/m}^2 \cdot \text{K} \times 0.00229 \text{ m}}{177 \text{ W/m} \cdot \text{K}} = 5.18 \times 10^{-4} \]

The lumped capacitance approximation is valid. Furthermore, since the Biot number is very small, the lumped capacitance approximation would certainly still be valid using a more conservative length estimate.

(d) We are not told which type of stainless steel this is, but we are told its mass, from which we can find its density:
\[ \rho = \frac{M}{V} = \frac{M}{\pi D^2 L / 4} = \frac{0.328 \text{ kg}}{\pi (0.013 \text{ m})^2 \times 0.3 \text{ m} / 4} = 8237 \text{ kg/m}^3 \]

This appears to be AISI 316 stainless steel, with a thermal conductivity of \( k = 13.4 \text{ W/m-K} \) at \( T = 300 \text{ K} \).

The characteristic lengths are
\[ L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi DL + 2\pi D^2 / 4} = \frac{DL / 4}{L + D / 2} = \frac{13 \text{ mm} \times 300 \text{ mm} / 4}{300 \text{ mm} + 13 \text{ mm} / 2} = 3.18 \text{ mm} \]
\[ L_{c2} = \text{Maximum center to surface distance} = \frac{D}{2} = 6.5 \text{ mm} \]
Notice that the surface area of the ends has been included in \( Lc1 \), and does have a small effect on the result. The corresponding Biot numbers are

\[
Bi_1 = \frac{hL_{c1}}{k} = \frac{15 \text{ W/m}^2 \cdot \text{K} \times 0.00318 \text{ m}}{13.4 \text{ W/m} \cdot \text{K}} = 0.0036
\]

\[
Bi_2 = \frac{hL_{c2}}{k} = \frac{15 \text{ W/m}^2 \cdot \text{K} \times 0.0065 \text{ m}}{13.4 \text{ W/m} \cdot \text{K}} = 0.0073
\]

The lumped capacitance approximation is valid according to either definition.

(f) The characteristic lengths are

\[
L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi D L} = \frac{D}{4} = 5 \text{ mm}
\]

\( L_{c2} = \text{Maximum center to surface distance} = D/2 = 10 \text{ mm} \)

We are not told the convection heat transfer coefficient, but we do know the fluid temperature and the temperature of the rod initially and at \( t = 225 \text{ s} \). If we assume that the lumped capacitance approximation is valid, we can determine the heat transfer coefficient from Equation 5.5:

\[
h = \frac{\rho C_L \ln \left( \frac{\theta_i}{\theta} \right)}{4t} = \frac{\rho C_L \ln \left( \frac{T_i - T_\infty}{T - T_\infty} \right)}{4t}
\]

\[
= \frac{2300 \text{ kg/m}^3 \times 0.020 \text{ m} \times 1750 \text{ J/kg} \cdot \text{K} \ln \left( \frac{200^\circ \text{C} - 20^\circ \text{C}}{100^\circ \text{C} - 20^\circ \text{C}} \right)}{4 \times 225 \text{ s}} = 72.5 \text{ W/m}^2 \cdot \text{K}
\]

The resulting Biot numbers are:

\[
Bi_i = \frac{hL_{c1}}{k} = \frac{72.5 \text{ W/m}^2 \cdot \text{K} \times 0.005 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 0.023
\]

\[
Bi_2 = \frac{hL_{c2}}{k} = \frac{72.5 \text{ W/m}^2 \cdot \text{K} \times 0.011 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 0.045
\]

The lumped capacitance approximation is valid according to either definition. This also means that it was appropriate to use the lumped capacitance approximation to calculate \( h \).

(g) With the diameter increased by a factor of ten, so are the characteristic lengths:

\[
L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi D L} = \frac{D}{4} = 50 \text{ mm}
\]

\( L_{c2} = \text{Maximum center to surface distance} = \frac{D}{2} = 100 \text{ mm} \)

Once again, we assume that the lumped capacitance approximation is valid to calculate the heat transfer coefficient according to

\[
h = \frac{\rho C_L \ln \left( \frac{\theta_i}{\theta} \right)}{4t} = \frac{\rho C_L \ln \left( \frac{T_i - T_\infty}{T - T_\infty} \right)}{4t}
\]

\[
= \frac{2300 \text{ kg/m}^3 \times 0.20 \text{ m} \times 1750 \text{ J/kg} \cdot \text{K} \ln \left( \frac{200^\circ \text{C} - 20^\circ \text{C}}{100^\circ \text{C} - 20^\circ \text{C}} \right)}{4 \times 225 \text{ s}} = 725 \text{ W/m}^2 \cdot \text{K}
\]

The resulting Biot numbers are:
The lumped capacitance approximation is not valid according to either definition. This means that the calculated value of $h$ is incorrect; therefore the above values of the Biot number are incorrect. However, we can still conclude that the $Bi$ number is too large for lumped capacitance to be valid by the following reasoning. If the lumped capacitance approximation were valid, then the calculated $h$ would be correct, and its value would be small enough to result in $Bi < 0.1$. Since the calculated Biot number does not satisfy the criterion to use the lumped capacitance approximation, the initial assumption that the lumped capacitance method is valid must have been false.

\[
Bi_1 = \frac{h L_1}{k} = \frac{725 \text{ W/m}^2 \cdot \text{K} \times 0.05 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 2.3
\]

\[
Bi_2 = \frac{h L_2}{k} = \frac{725 \text{ W/m}^2 \cdot \text{K} \times 0.1 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 4.5
\]
2. (Problem 5.22 in the book)
A plane wall of a furnace is fabricated from plain carbon steel \((k = 60 \text{W/mK}, \rho=7850\text{kg/m}^3, c = 430 \text{J/kgK})\) and is of thickness \(L = 10 \text{mm}\). To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of \(R_{t,f}^* = 0.01 \text{m}^2\text{K/W}\). The opposite surface is well insulated from the surroundings.

At furnace start-up the wall is at initial temperature of \(T_i = 300 \text{K}\), and combustion gases at \(T_\infty = 1300\text{K}\) enter the furnace, providing a convection coefficient of \(h = 25 \text{W/m}^2\text{K}\) at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of \(T_{s,i} = 1200 \text{K}\)? What is the temperature \(T_{s,o}\) of the exposed surface of the ceramic film at this time?

**KNOWN:** Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

**FIND:** (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

**PROPERTIES:** Carbon steel (given): \(\rho = 7850 \text{kg/m}^3, c = 430 \text{J/kgK}, k = 60 \text{W/mK}\).

**ANALYSIS:** The overall coefficient for heat transfer from the surface of the steel to the gas is

\[
U = (R_{\text{tot}}^*)^{-1} = \left( \frac{1}{h} + R_f^* \right)^{-1} = \left[ \frac{1}{25 \text{W/m}^2\cdot\text{K}} + 10^{-2}\text{m}^2\cdot\text{K/W} \right]^{-1} = 20 \text{W/m}^2\cdot\text{K}.
\]

Hence,

\[
\text{Bi} = \frac{UL}{k} = \frac{20 \text{W/m}^2\cdot\text{K} \times 0.01 \text{ m}}{60 \text{ W/m} \cdot \text{K}} = 0.0033
\]

And the lumped capacitance method can be used.

(a) It follows that
\[
\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau_t}\right) = \exp\left(-\frac{t}{RC}\right) = \exp\left(-\frac{Ut}{\rho L c}\right)
\]

\[
t = -\frac{\rho L c}{U} \ln \frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850 \text{ kg/m}^3 \times (0.01 \text{ m}) \times 430 \text{ J/kg} \cdot \text{K}}{20 \text{ W/m}^2 \cdot \text{K}} \ln \frac{1200 - 1300}{300 - 1300}
\]

\[
t = 3886 \text{s} = 1.08 \text{h}
\]

(b) Performing an energy balance at the outer surface (s,o),

\[
h (T_\infty - T_{s,o}) = \left( T_{s,o} - T_{s,i} \right) / R_f^\prime
\]

\[
T_{s,o} = \frac{h T_\infty + T_{s,i} / R_f^\prime}{h + (1 / R_f^\prime)} = \frac{25 \text{ W/m}^2 \cdot \text{K} \times 1300 \text{ K} + 1200 \text{ K}/10^{-2} \text{m}^2 \cdot \text{K}/\text{W}}{(25 + 100) \text{ W/m}^2 \cdot \text{K}}
\]

\[
T_{s,o} = 1220 \text{ K}.
\]
3. (Problem 5.29 in the book)
A long wire of diameter \( D = 2 \) mm is submerged in an oil bath of temperature \( t_\infty = 25^\circ \text{C} \). The wire has a electrical resistance per unit length of \( R_\ell' \). If a current of \( I = 100 \) A flows through the wire and the convection coefficient \( h = 400 \) W/m\(^2\)K, what is the steady-state temperature of the wire? From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1\(^\circ\)C of the steady-state value? The properties of the wire are, \( c = 500 \) J/kgK, and \( k = 20 \) W/mK.

**KNOWN:** Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.

**FIND:** Steady-state temperature of the wire. Time for the wire temperature to come within 1\(^\circ\)C of it’s steady-state value.

**SCHEMATIC:**

![Schematic diagram of a wire submerged in oil with convection](image)

**ASSUMPTIONS:** (1) Constant properties, (2) Wire temperature is independent of \( x \).

**PROPERTIES:** Wire (given): \( \rho = 8000 \) kg/m\(^3\), \( c_p = 500 \) J/kg\cdot K, \( k = 20 \) W/m\cdot K, \( R_\ell' = 0.01 \) \( \Omega \)/m.

**ANALYSIS:** Since

The lumped capacitance method can be used. The problem has been analyzed in Example 1.4, and without radiation the steady-state temperature is given by

\[ \pi D h (T - T_\infty) = I^2 R_\ell'. \]

Hence

\[ T = T_\infty + \frac{I^2 R_\ell'}{\pi D h} = 25^\circ \text{C} + \frac{(100 \text{A})^2 0.01 \text{\Omega}/\text{m}}{\pi (0.002 \text{m}) 400 \text{W/m}^2\cdot \text{K}} = 64.8^\circ \text{C}. \]

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.4)

\[ \frac{dT}{dt} = \frac{I^2 R_\ell'}{\rho c_p \left( \pi D^2 / 4 \right)} - \frac{4h}{\rho c_p D} (T - T_\infty). \]

With \( T = T_i = 25^\circ \text{C} \) at \( t = 0 \), the solution is

\[ \frac{T - T_\infty - \left( I^2 R_\ell' / \pi D h \right)}{T_i - T_\infty - \left( I^2 R_\ell' / \pi D h \right)} = \exp \left( -\frac{4h}{\rho c_p D} t \right). \]

Substituting numerical values, find
\[
\frac{63.8 - 25 - 39.8}{25 - 25 - 39.8} = \exp\left(-\frac{4 \times 400 \text{ W/m}^2 \cdot \text{K}}{8000 \text{ kg/m}^3 \times 500 \text{ J/kg} \cdot \text{K} \times 0.002 \text{ m}} t\right)
\]

\[t = 18.4 \text{s}.
\]
4. (Problem 5.87 in the book) A tile-iron consists of a massive plate maintained at 150°C by an embedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50°C for at least 2 min, but its temperature shouldn’t exceed 120°C to avoid deterioration of the adhesive. Assume the tile and subfloor to have an initial temperature of 25°C and to have equivalent thermophysical Properties of \( k = 0.15 \text{ W/mK} \) and \( \rho c_p = 1.5 \times 10^6 \text{ J/m}^3 \cdot \text{K} \).

a. How long will it take a worker using the tile-iron to lift a tile? Will the adhesive temperature exceed 120°C?

b. If the tile-iron has a square surface area 254 mm to the side, how much energy has been removed from it during the time it has taken to lift the tile?

**KNOWN:** Tile-iron, 254 mm to a side, at 150°C is suddenly brought into contact with tile over a subflooring material initially at \( T_i = 25°C \) with prescribed thermophysical properties. Tile adhesive softens in 2 minutes at 50°C, but deteriorates above 120°C.

**FIND:** (a) Time required to lift a tile after being heated by the tile-iron and whether adhesive temperature exceeds 120°C, (2) How much energy has been removed from the tile-iron during the time it has taken to lift the tile.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Tile and subflooring have same thermophysical properties, (2) Thickness of adhesive is negligible compared to that of tile, (3) Tile-subflooring behaves as semi-infinite solid experiencing one-dimensional transient conduction.

**PROPERTIES:** Tile-subflooring (given): \( k = 0.15 \text{ W/mK} \), \( \rho c_p = 1.5 \times 10^6 \text{ J/m}^3 \cdot \text{K} \), \( \alpha = k/\rho c_p = 1.00 \times 10^{-7} \text{ m}^2/\text{s} \).

**ANALYSIS:** (a) The tile-subflooring can be approximated as a semi-infinite solid, initially at a uniform temperature \( T_i = 25°C \), experiencing a sudden change in surface temperature \( T_s = T(0,t) = 150°C \). This corresponds to Case 1, Figure 5.7. The time required to heat the adhesive \( (x_0 = 4 \text{ mm}) \) to 50°C follows from Eq. 5.60
Using error function values from Table B.2. Since the softening time, Δts, for the adhesive is 2 minutes, the time to lift the tile is

\[ t_s = 48.7s = 0.81 \text{ min} \]

Using error function values from Table B.2. Since the softening time, Δts, for the adhesive is 2 minutes, the time to lift the tile is

\[ t_ℓ = t_o + Δt_s = (0.81 + 2.0)\text{ min} = 2.81\text{ min} \]

To determine whether the adhesive temperature has exceeded 120°C, calculate its temperature at \( t_ℓ = 2.81 \text{ min} \); that is, find \( T(x_0, t_ℓ) \)

\[
\frac{T(x_0, t_ℓ) - T_s}{T_1 - T_s} = \text{erf}\left(\frac{x_0}{2(\alpha t_o)^{1/2}}\right)
\]

\[
\frac{50 - 150}{25 - 150} = \text{erf}\left(\frac{0.004\text{ m}}{2\left(1.00 \times 10^{-7} \text{ m}^2/\text{s} \times t_o\right)^{1/2}}\right)
\]

\[ 0.80 = \text{erf}\left(6.325t_o^{-1/2}\right) \]

\[ t_s = 48.7s = 0.81 \text{ min} \]

Using error function values from Table B.2. Since the softening time, Δts, for the adhesive is 2 minutes, the time to lift the tile is

\[ t_ℓ = t_o + Δt_s = (0.81 + 2.0)\text{ min} = 2.81\text{ min} \]

To determine whether the adhesive temperature has exceeded 120°C, calculate its temperature at \( t_ℓ = 2.81 \text{ min} \); that is, find \( T(x_0, t_ℓ) \)

\[
\frac{T(x_0, t_ℓ) - 150}{25 - 150} = \text{erf}\left(\frac{0.004\text{ m}}{2\left(1.00 \times 10^{-7} \text{ m}^2/\text{s} \times 2.81 \times 60\text{s}\right)^{1/2}}\right)
\]

\[ T(x_0, t_ℓ) - 150 = -125\text{erf}(0.4880) = -125 \times 0.5098 \]

\[ T(x_0, t_ℓ) = 86°C \]

Since \( T(x_0, t_ℓ) < 120°C \), the adhesive will not deteriorate.

(c) The energy required to heat a tile to the lift-off condition is

\[ Q = \int_0^{t_ℓ} q_x'^{(s)}(0, t) \cdot A_s \, dt \]

Using Eq. 5.61 for the surface heat flux \( q_x''(t) = q_x''(0, t) \), find

\[
Q = \int_0^{t_ℓ} \frac{k(T_s - T_1)}{(\pi \alpha)^{1/2} A_s} \cdot \frac{dt}{t^{1/2}} = \frac{2k(T_s - T_1)}{(\pi \alpha)^{1/2} A_s 1/2}
\]

\[
Q = \frac{2 \times 0.15 \text{ W/m} \cdot \text{K} (150 - 25) \text{ C}}{(\pi \times 1.00 \times 10^{-7} \text{ m}^2/\text{s})^{1/2}} \times (0.254 \text{ m})^2 \times (2.81 \times 60\text{s})^{1/2} = 56 \text{ kJ}
\]