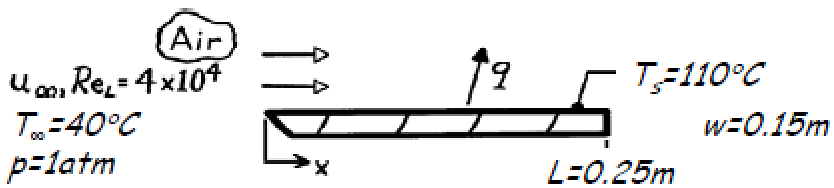
Mech 302 Heat Transfer HW6 SOLUTION

1. (Problem 7.17 in the Book) Air at a pressure of 1 atm and a temperature of 40°C is in parallel flow over the top surface of a flat plate that is heated to a uniform temperature of 110°C. The plate has a length of 0.25 m (in the flow direction) and a width of 0.15 m. The Reynolds number based on the plate length is 40,000. What is the rate of heat transfer from the plate to the air? If the free stream velocity of the air doubled and the pressure is increased to 10 atm, what is the rate of heat transfer?

KNOWN: Temperature, pressure and Reynolds number for air flow over a flat plate of uniform surface temperature.
FIND: (a) Rate of heat transfer from the plate, (b) Rate of heat transfer if air velocity is doubled and pressure is increased to 10 atm.

SCHEMATIC:




ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{x_c} = 5\times 10^5$

PROPERTIES: Table A-4, Air (Tf = 348K, 1 atm): k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}_L (w \times L)\ (T_s - T_\infty).$$

Since the flow is laminar over the entire plate for ReL = 4 ×10$^4$, it follows that

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(40{,}000)^{1/2}(0.70)^{1/3} = 117.9.$$

Hence $$\bar{h}_L = 117.9\frac{k}{L} = 117.9\frac{0.0299\ \text{W/m}\cdot\text{K}}{0.25\ \text{m}} = 14.1\ \text{W/m}^2\cdot\text{K}$$

and $$q = 14.1\frac{\text{W}}{\text{m}^2\cdot\text{K}}(0.15\text{m}\times 0.25\text{m})\ (110-40)^\circ\text{C} = 37.0\ \text{W}.$$

(b) With p2 = 10 p1, it follows that ρ2 = 10 ρ1 and ν2 = ν1/10. Hence

$$Re_{L,2} = \left(\frac{u_\infty L}{\nu}\right)_2 = 2\times 10\left(\frac{u_\infty L}{\nu}\right)_1 = 20\ Re_{L,1} = 8\times 10^5$$

And mixed boundary layer conditions exist on the plate. Hence

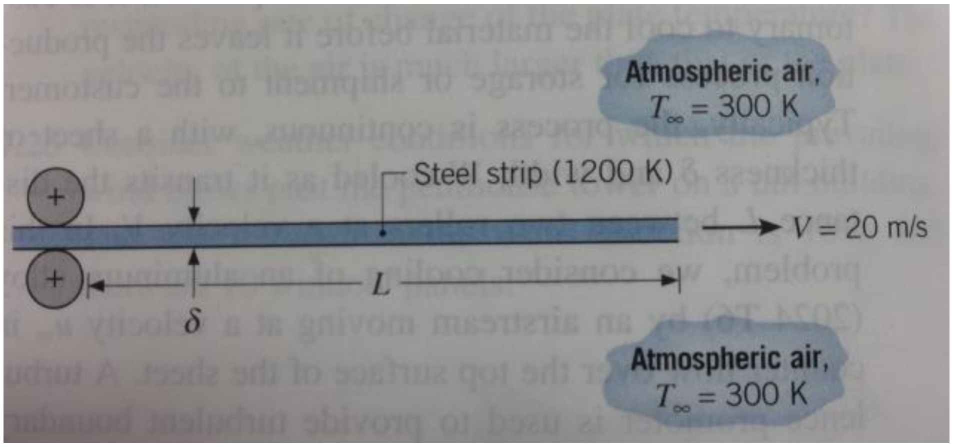
$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = \left(0.037\ Re_L^{4/5} - 871\right) Pr^{1/3} = \left[0.037 \times \left(8 \times 10^5\right)^{4/5} - 871\right] (0.70)^{1/3}$$

$$\overline{Nu}_L = 961.$$

Hence, $$\overline{h}_L = 961 \frac{k}{L} = 117.9 \frac{0.0299\ W/m \cdot K}{0.25\ m} = 114.9\ W/m^2 \cdot K$$

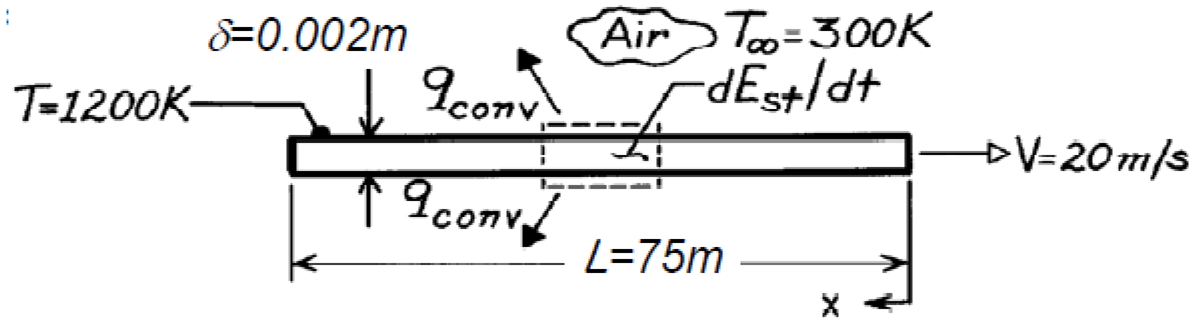
$$q = 114.9 \frac{W}{m^2 \cdot K} (0.15m \times 0.25m)\ (110 - 40)^\circ C = 301.6\ W.$$

2. 2. (Problem 7.32 in the book) A steel strip emerges from the hot roll section of a steel mill at a speed of 20 m/s and a temperature of 1200 K. Its length and thickness are L = 75 m and δ = 0.002m, respectively, and its density and specific heat are 7900 kg/$M^3$ and 640 J/kg K, respectively.




Accounting for heat transfer from the top and bottom surfaces and neglecting radiation and strip conduction, determine the time rate of change of the strip temperature at a distance of 0.1 m from the leading edge and at the trailing edge. Determine the distance from the leading edge at which the minimum cooling is achieved.

KNOWN: Length, thickness, speed and temperature of steel strip.
FIND: Rate of change of strip temperature 0.1 m from leading edge and at trailing edge. Location of minimum cooling rate.
SCHEMATIC:




ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Negligible longitudinal conduction in strip, (4) Critical Reynolds number is $5\times 10^5$
PROPERTIES: Steel (given): ρ = 7900 kg/m 3, cp = 640 J/kg·K. Table A-4, Air (T = 750K, 1 atm): ν = 76.4 × 10 -6 m 2 /s, k = 0.0549 W/m·K, Pr = 0.702.
ANALYSIS: Performing an energy balance for a control mass of unit surface area As riding with the strip,

$$-\dot{E}_{out} = dE_{st}/dt$$

$$-2h_x A_s (T - T_\infty) = \rho \delta A_s c_p (dT/dt)$$

$$dT/dt = \frac{-2h_x (T - T_\infty)}{\rho \delta c_p} = -\frac{2(900K)h_x}{7900\ kg/m^3 (0.002\ m) 640\ J/kg \cdot K} = -0.178 h_x\ (K/s).$$

At $x = 0.1$ m, $Re_x = \frac{Vx}{\nu} = \frac{20\ m/s(0.1m)}{76.4 \times 10^{-6}\ m^2/s} = 2.62 \times 10^4 < Re_{x,c}$. Hence,

$$h_x = (k/x) 0.332 Re_x^{1/2} Pr^{1/3} = \frac{0.0549\ W/m \cdot K}{0.1\ m}(0.332)\left(2.62 \times 10^4\right)^{1/2}(0.702)^{1/3} = 26.2 W/m^2 \cdot K$$

And at x = 0.1m, dT/dt = -4.66 K/s.

At the trailing edge, $Re_x = 1.96 \times 10^7 > Re_{x,c}$ Hence

$$h_x = (k/x) 0.0296 Re_x^{4/5} Pr^{1/3} = \frac{0.0549\ W/m \cdot K}{75\ m}(0.0296)\left(1.96 \times 10^7\right)^{4/5}(0.702)^{1/3} = 13.1 W/m^2 \cdot K$$

and at x = 75 m, dT/dt = -2.33 K/s.

The minimum cooling rate occurs just before transition; hence, for $Re_{x,c} = 5 \times 10^5$

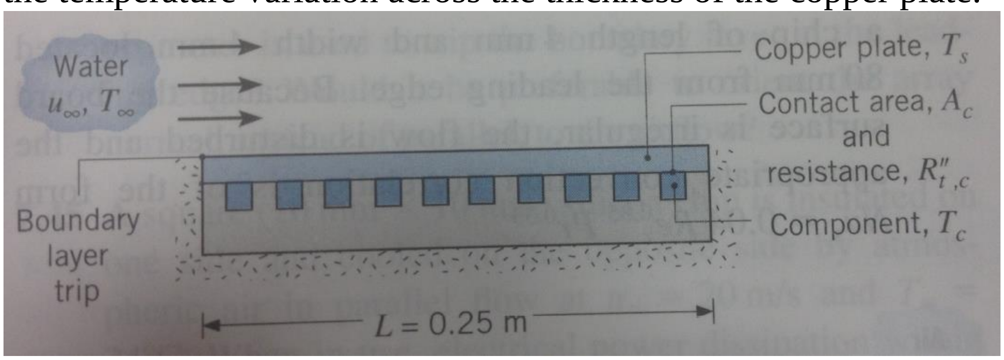
$$x_c = 5 \times 10^5 (\nu/V) = \frac{5 \times 10^5 \times 76.4 \times 10^{-6} m^2/s}{20\ m/s} = 1.91\ m$$

3. (Problem 7.35 in the book) One hundred electrical components, each dissipating 40 W, are attached to one surface of a square (0.25 m X 0.25 m) copper plate, and all the dissipated energy transferred to water in parallel flow over the opposite surface. A protuberance at the leading edge of the plate acts to trip the boundary layer (boundary layer becomes turbulent everywhere on the plate), and the plate itself may be assumed to be isothermal. The water velocity and temperature are $U_\infty$ = 2m/s and $T_\infty$ =17°C, and the water's thermophysical properties may be approximated as $v = 0.96\times 10^{-6} m^2/s$ , k = 0.620W/m.K , and Pr = 5.2.

a. What is the temperature of the copper plate?

b. If each component has a plate contact surface area of 1 cm2 and the corresponding contact resistance is $2\times 10^{-4} m^3K/W$, what is the component temperature?

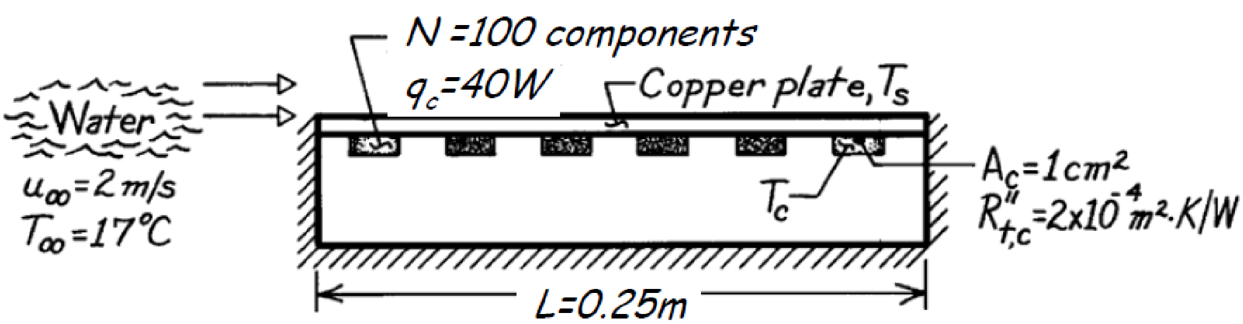
Neglect the temperature variation across the thickness of the copper plate.




KNOWN: Operating power of electrical components attached to one side of copper plate. Contact resistance. Velocity and temperature of water flow on opposite side.

FIND: (a) Plate temperature, (b) Component temperature.

SCHEMATIC:




ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss from sides and bottom, (4) Turbulent flow throughout.

PROPERTIES: Water (given): $\nu = 0.96 \times 10^{-6} m^2/s$, k = 0.620 W/m·K, Pr = 5.2.

ANALYSIS: (a) From the convection rate equation,

$$T_s = T_\infty + q/\bar{h}A$$

Where q = Nqc = 4000 W and A = m² = 0.0625m². The convection coefficient is given by the turbulent flow correlation

$$\bar{h} = \overline{Nu}_L\ (k/L) = 0.037 Re_L^{4/5} Pr^{1/3}\ (k/L)$$

Where

$$Re_L = (u_\infty L/\nu) = (2\ m/s \times 0.25m)/0.96 \times 10^{-6} m^2/s = 5.21 \times 10^5$$

And hence

$$\bar{h} = 0.037\left(5.21 \times 10^5\right)^{4/5} (5.2)^{1/3} (0.62\ W/m \cdot K/0.25\ m) = 5954\ W/m^2 \cdot K.$$

The plate temperature is then

$$T_s = 17^\circ C + 4000\ W/\left(5954\ W/m^2 \cdot K\right)(0.25\ m)^2 = 27.7^\circ C.$$

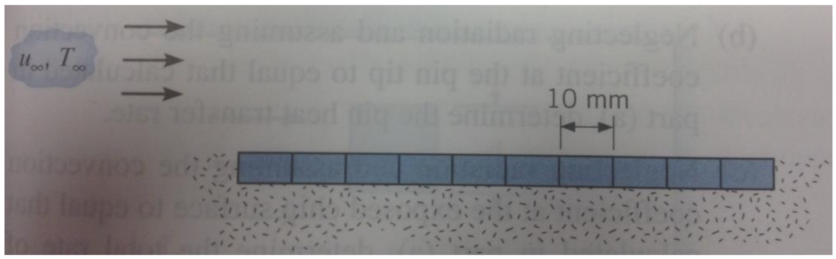
(b) For an individual component, a rate equation involving the component’s contact resistance can be used to find its temperature,

$$q_c = (T_c - T_s)/R_{t,c} = (T_c - T_s)/\left(R''_{t,c}/A_c\right)$$

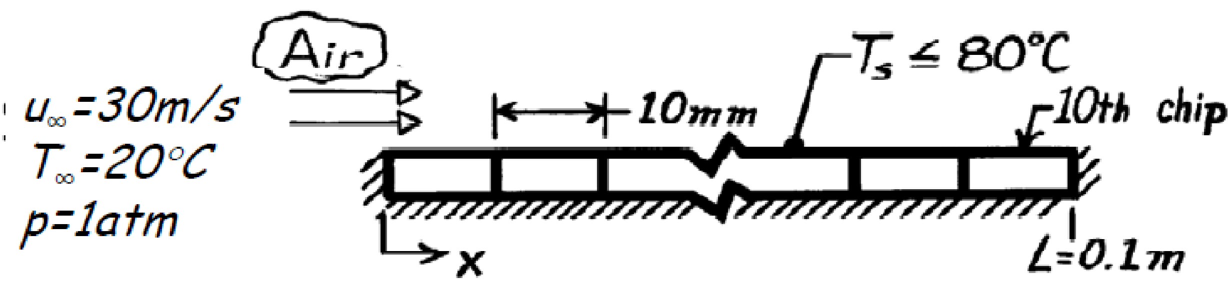
$$T_c = T_s + q_c R''_{t,c}/A_c = 27^\circ C + 40\ W\left(2 \times 10^{-4}\ m^2 \cdot K/W\right)/10^{-4}\ m^2$$

$$T_c = 108^\circ C.$$

4. (Problem 7.44 in the book) an array of 10 silicon chips, each of length L=10 mm on a side, is insulated on one surface and cooled on the opposite surface by atmospheric air in parallel flow with $T_\infty$ = 20°C and $U_\infty$= 30m/s. When in use, the same electrical power is dissipated in each chip, maintaining a uniform heat flux over the entire cooled surface. If the temperature of each chip may not exceed 80°C, what is the maximum allowable power per chip? What is the maximum allowable power if a turbulence promoter is used to trip the boundary layer at the leading edge?




KNOWN: Surface dimensions for an array of 10 silicon chips. Maximum allowable chip temperature. Air flow conditions.
FIND: Maximum allowable chip electrical power (a) without and (b) with a turbulence promoter at the leading edge.
SCHEMATIC:




ASSUMPTIONS: (1) Steady-state conditions, (2) Film temperature of 52°C, (3) Negligible radiation,
(4) Negligible heat loss through insulation, (5) Uniform heat flux at chip interface with air, (6) 5 $Re_{x,c}$= 5× $10^5$
PROPERTIES: Table A-4, Air (Tf = 325K, 1 atm): ν = 18.4 × 10 $10^{-6} m^2/s$, k = 0.0282 W/m·K,
Pr =0.703.
ANALYSIS: $Re_L = \frac{U_\infty L}{v} = \frac{40m}{s} \times 0.\frac{1m}{18}.4 \times \frac{10^{-6} m^2}{s} = 1.63 \times 10^5$ . Hence, flow is laminar over all chips without the promoter.

(a) For laminar flow, the minimum hx exists on the last chip. Approximating the average coefficient for Chip 10 as the local coefficient at x = 95 mm, $\bar{h}_{10}$= hx=0.095m.

$$\bar{h}_{10} = 0.453 \frac{k}{x} Re_x^{1/2} Pr^{1/3}$$

$$Re_x = \frac{u_\infty x}{\nu} = \frac{30 \text{ m/s} \times 0.095 \text{ m}}{18.4 \times 10^{-6} \text{ m}^2/\text{s}} = 1.549 \times 10^5$$

$$\bar{h}_{10} = 0.453 \frac{0.0282 \text{ W/m} \cdot \text{K}}{0.095} \left(1.549 \times 10^5\right)^{1/2} (0.703)^{1/3} = 47.1 \text{W/m}^2 \cdot \text{K}$$

$$q_{10} = \bar{h}_{10} A (T_s - T_\infty) = 47.1 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (0.01 \text{ m})^2 (80 - 20)^\circ \text{C} = 0.28 \text{ W}.$$

Hence, if all chips are to dissipate the same power and Ts is not to exceed 80°C.
qmax = 0.28 W.
(b) For turbulent flow,

$$\bar{h}_{10} = 0.0308 \frac{k}{x} Re_x^{4/5} Pr^{1/3} = 0.0308 \frac{0.0282 \text{W/m} \cdot \text{K}}{0.095 \text{ m}} \left(1.549 \times 10^5\right)^{4/5} (0.703)^{1/3} = 115 \text{W/m}^2 \cdot \text{K}$$

$$q_{10} = \bar{h}_{10} A (T_s - T_\infty) = 115 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (0.01 \text{ m})^2 (80 - 20)^\circ \text{C} = 0.69 \text{ W}.$$

Hence, qmax = 0.69 W.