1. **(Problem 7.56 in the Book)** Hot water at 50°C is routed from one building in which it is generated to an adjoining building in which it is used for space heating. Transfer between buildings occurs in a steel pipe (k=60 W/mK) of 100-mm outside diameter and 8-mm wall thickness. During winter, representative environmental conditions involve air at $T_\infty = -5^\circ C$ and $V = 3$ m/s in cross flow over the pipe.

a. If the cost of producing the hot water is $0.1$ per kWh, what is the representative daily cost of heat loss from an uninsulated pipe to the air per meter of pipe length? The convection resistance associated with water flow in the pipe may be neglected.

b. Determine the savings associated with application of a 10-mm-thick coating of urethane insulation (k=0.026 W/mK) to the outer surface of the pipe.

**Schematic**

**Assumptions**
(1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible contact resistance between insulation and pipe, (4) Negligible radiation.

**Properties**

*Table A-4, air (p = 1atm, $T_f \approx 300K$):* $k_a = 0.0263 W/m \cdot K$, $\nu = 15.89 \times 10^{-6} m^2/s$, $Pr = 0.707$.

**Analysis**

(a) With $Re_D = \frac{VD_o}{\nu} = 3 m/s \times 0.1 m / 15.89 \times 10^{-6} m^2/s = 18,880$, application of the Churchill-Bernstein correlation yields

$$Nu_D = 0.3 + \frac{0.62(18,800)^{1/2}(0.707)^{1/3}}{1 + (0.4/0.707)^{2/3}}^{1/4}\left[1 + \left(\frac{18,880}{282,000}\right)^{5/8}\right]^{4/5} = 76.6$$

$$\bar{h} = \frac{k_a}{D_o} Nu_D = \frac{0.0263}{0.1 m} \times 76.6 = 20.1 W/m^2 \cdot K$$

Without the insulation, the total thermal resistance and heat loss per length of pipe are then

$$R_t^{(wo)} = \frac{ln(D_o/D_i)}{2 \pi k_p} + \frac{1}{\pi D_o \bar{h}} = \frac{ln(100/84)}{2 \pi \times 60 W/m \cdot K} + \frac{1}{\pi (0.1 m) 20.1 W/m^2 \cdot K}$$

$$= \left(4.63 \times 10^{-4} + 0.158\right) m \cdot K/W = 0.159 m \cdot K/W$$
The corresponding daily energy loss is
\[ Q'_{wo} = 0.346 \text{ kW} / \text{m} \times 24 \text{ h} / \text{d} = 8.3 \text{ kW} \cdot \text{h} / \text{m} \cdot \text{d} \]
and the associated cost is
\[ C'_{wo} = (8.3 \text{ kW} \cdot \text{h} / \text{m} \cdot \text{d}) (0.05 / \text{kW} \cdot \text{h}) = 0.415 / \text{m} \cdot \text{d} \]

(b) The conduction resistance of the insulation is
\[ R'_{cond} = \frac{\ln (D_o / D_i)}{2 \pi k_i} = \frac{\ln (120 / 100)}{2 \pi (0.026 \text{ W} / \text{m} \cdot \text{K})} = 1.116 \text{ m} \cdot \text{K} / \text{W} \]

Using the Churchill-Bernstein correlation with an outside diameter of \( D_o = 0.12 \text{ m} \), \( \text{Re}_D = 22,660 \), \( \text{Nu}_D = 83.9 \) and \( h = 18.4 \text{ W} / \text{m}^2 \cdot \text{K} \). The convection resistance is then
\[ R'_{conv} = \frac{1}{\pi D_o h} = \frac{1}{\pi (0.12 \text{ m}) (18.4 \text{ W} / \text{m}^2 \cdot \text{K})} = 0.144 \text{ m} \cdot \text{K} / \text{W} \]
and the total resistance is
\[ R'_{tot(w)} = (4.63 \times 10^{-4} + 1.116 + 0.144) \text{ m} \cdot \text{K} / \text{W} = 1.261 \text{ m} \cdot \text{K} / \text{W} \]

The heat loss and cost are then
\[ q'_{w} = \frac{T - T_\infty}{R'_{tot(w)}} = \frac{55 \text{ } ^\circ \text{C}}{1.261 \text{ m} \cdot \text{K} / \text{W}} = 43.6 \text{ W} / \text{m} = 0.0436 \text{ kW} / \text{m} \]
\[ C'_{w} = 0.0436 \text{ kW} / \text{m} \times 24 \text{ h} / \text{d} \times 0.05 / \text{kW} \cdot \text{h} = 0.052 / \text{m} \cdot \text{d} \]

The daily savings is then
\[ S' = C'_{wo} - C'_{w} = 0.363 / \text{m} \cdot \text{d} \]

Comments

(1) The savings are significant, and the pipe should be insulated. (2) Assuming a negligible temperature drop across the pipe wall, a pipe emissivity of \( \varepsilon_p = 0.6 \) and surroundings at \( T_{sur} = 268 \text{K} \), the radiation coefficient associated with the uninsulated pipe is \( h_r = \varepsilon \sigma (T + T_{sur}) (T^2 + T_{sur}^2) = 0.6 \times 5.67 \times 10^8 \text{ W} / \text{m}^2 \text{K}^4 (591 \text{K}) (323^2 + 268^2) \text{K}^2 = 3.5 \text{W} / \text{m}^2 \cdot \text{K} \). Accordingly, radiation increases the heat loss estimate of Part (a) by approximately 17%.
2. (Problem 7.69 in the book) A thermocouple is inserted into a hot air duct to measure the air temperature. The thermocouple \(T_1\) is soldered to the tip of a steel thermocouple well of length \(L = 0.1\) m and inner and outer diameters of \(D_i = 5\) mm and \(D_o = 8\) mm. A second thermocouple \(T_2\) is used to measure the duct wall temperature. Consider conditions for which the air velocity in the duct is \(V = 5\) m/s and the two thermocouples register temperatures of \(T_1 = 450\) K and \(T_2 = 375\) K. Neglecting radiation, determine the air temperature \(T_\infty\). Assume that, for steel, \(k = 35\) W/m.K, and for air, \(\rho = 0.774\) kg/m\(^3\), \(\mu = 251 \times 10^{-7}\) N·s/m\(^2\), \(k = 0.0373\) W/mK.

**Schematic**

**Assumptions**

1. Steady-state conditions,
2. Constant properties,
3. One-dimensional conduction along well,
4. Uniform convection coefficient,
5. Negligible radiation.

**Properties**

Steel (given): \(k = 35\) W/m·K; Air (given): \(\rho = 0.774\) kg/m\(^3\), \(\mu = 251 \times 10^{-7}\) N·s/m\(^2\), \(k = 0.0373\) W/m·K, \(Pr = 0.686\).

**Analysis**

Applying Equation 3.75 at the well tip (\(x = L\), where \(T = T_1\)),

\[
\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[ \cosh \frac{mL}{h} + \left( \frac{h}{mk} \right) \sinh \frac{mL}{h} \right]^{-1}
\]

\(m = \left( \frac{hP}{kA_c} \right)^{1/2} = \pi D_o = \pi (0.008\) m\) = 0.0251 m

\(A_c = \left( \frac{\pi}{4} \right) \left( D_o^2 - D_i^2 \right) = \left( \frac{\pi}{4} \right) (0.008^2 - 0.005^2)\) m\(^2\) = 3.06 \times 10^{-5}\) m\(^2\).

\(Re_D = \frac{\rho V D}{\mu} = \frac{0.774\) kg/m\(^3\) (5 m/s) 0.008 m}{251 \times 10^{-7}\) N·s/m\(^2\)} = 1233

\(C = 0.26, m = 0.6, n = 0.37\) and the Zhukauskas correlation yields

\[
\bar{Nu}_D = 0.26 Re_D^{0.6} Pr^{0.37} (Pr/Pr_\infty)^{1/4} \approx 0.26 (1233)^{0.6} (0.686)^{0.37} \times 1 = 16.2
\]

\[
\bar{h} = Nu_D \frac{k}{D_o} = 16.2 \frac{0.0373 \text{ W/m·K}}{0.008 \text{ m}} = 75.5 \text{ W/m}^2 \cdot \text{K}
\]
Heat conduction along the wall to the base at 375 K is balanced by convection from the air.

Hence

\[
m = \left[ \frac{\left(75.5 \text{ W/m}^2 \cdot \text{K}\right) \times 0.0251 \text{ m}}{(35 \text{ W/m} \cdot \text{K}) \times 3.06 \times 10^{-5} \text{ m}^2} \right]^{1/2} = 42.1 \text{ m}^{-1} \quad \text{mL} = \left(42.1 \text{ m}^{-1}\right) \times 0.10 \text{ m} = 4.21.
\]

With

\[
\frac{h}{\text{mk}} = \frac{75.5 \text{ W/m}^2 \cdot \text{K}}{42.1 \text{ m}^{-1}} = 0.0513
\]

find

\[
\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[33.6 + (0.0513)33.5\right]^{-1} = 0.0283 \quad T_\infty = 452.2 \text{ K}.
\]

Comments

Heat conduction along the wall to the base at 375 K is balanced by convection from the air.
3. **(Problem 7.77 in the book)** A spherical, underwater instrument pod used to make soundings and to measure conditions in the water has a diameter of 85 mm and dissipates 300 W.

   a. Estimate the surface temperature of the pod when suspended in a bay where the current is 1 m/s and the water temperature is 15°C.

   b. Inadvertently, the pod is hauled out of the water and suspended in ambient air without deactivating the power. Estimate the surface temperature of the pod if the air temperature is 15°C and the wind speed is 3 m/s.

**Schematic**

**Assumptions**

1. Steady-state conditions,
2. Flow over a smooth sphere,
3. Uniform surface temperatures,
4. Negligible radiation heat transfer for air (a) condition, and
5. Constant properties.

**Properties**

*Table A-6*, Water ($T_\infty = 15^\circ \text{C} = 288 \text{ K}$): \( \mu = 0.001053 \text{ N} \cdot \text{s/m}^2 \), \( \nu = 1.139 \times 10^{-6} \text{ m}^2/\text{s} \), \( k = 0.5948 \text{ W/m} \cdot \text{K} \), \( Pr = 8.06 \); *Table A-4*, Air ($T_\infty = 288 \text{ K}, 1 \text{ atm}$): \( \mu = 1.788 \times 10^{-5} \text{ N} \cdot \text{s/m}^2 \), \( \nu = 1.482 \times 10^{-5} \text{ m}^2/\text{s} \), \( k = 0.02534 \text{ W/m} \cdot \text{K} \), \( Pr = 0.710 \); Air ($T_s = 945 \text{ K}$): \( \mu_s = 4.099 \times 10^{-5} \text{ N} \cdot \text{s/m}^2 \).

**Analysis**

The energy balance for the submersed-in-water (w) and suspended-in-air (a) conditions are represented in the schematics above and have the form

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + E_{\text{gen}} = -q_{cv} + P_e = 0
\]

\[-h_D A_s (T_s - T_\infty) + P_e = 0\]

where \( A_s = \pi D^2 \) and \( h_D \) is estimated using the Whitaker correlation, Eq. 7.56,

\[
\overline{Nu_D} = 2 + \left[ 0.4 \text{Re}_D^{1/2} + 0.06 \text{Re}_D^{2/3} \right] \text{Pr}^{0.4} \left( \mu / \mu_s \right)^{1/4}
\]

where all properties except \( \mu_s \) are evaluated at \( T_\infty \). The results are tabulated below.
Comments

(1) While submerged and dissipating 300 W, the pod is safely operating at a temperature slightly above that of the water. When hauled from the water and suspended in air, the pod temperature increases to a destruction temperature (672°C). The pod gets smoked!

(2) The assumption that $\mu / \mu_s \approx 1$ is appropriate for the water (w) condition. For the air (a) condition, $\mu / \mu_s = 0.436$ and the final term of the correlation is significant. Recognize that radiation exchange with the surroundings for the air condition should be considered for an improved estimate.

(3) Why such a difference in $T_s$ for the water (w) and air (a) conditions? From the results table note that the Re$_D$, Nu$_D$, and $\overline{h}_D$ are, respectively, 4x, 7x and 170x times larger for water compared to air. Water, because of its thermophysical properties which drive the magnitude of $\overline{h}_D$, is a much better coolant than air for similar flow conditions.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Re$_D$ (x 10$^4$)</th>
<th>Nu$_D$</th>
<th>$\overline{h}_D$ (W/m$^2$K)</th>
<th>$T_s$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w) water</td>
<td>7.465</td>
<td>499</td>
<td>3491</td>
<td>18.8</td>
</tr>
<tr>
<td>(a) air</td>
<td>1.72</td>
<td>67.5</td>
<td>20.1</td>
<td>672</td>
</tr>
</tbody>
</table>
4. **(Problem 7.88 in the book)** A silicon chip \((k = 150 \text{ W/mK}, \rho = 2300 \text{ kg/m}^3, c_p = 700 \text{ J/kgK})\), 10 mm on a side and 1 mm thick, is connected to a substrate by solder balls \((k = 40 \text{ W/mK}, \rho = 10,000 \text{ kg/m}^3, c_p = 150 \text{ J/kgK})\) of 1-mm diameter, and during an accelerated thermal stress test, the system is exposed to the flow of a dielectric liquid \((k = 0.064 \text{ W/mK}, \nu = 10^{-6} \text{ m}^2/\text{s}, Pr = 25)\). As first approximation, treat the top and bottom surfaces of the chip as flat plates in turbulent, parallel flow and assume the substrate and lower chip surfaces to have a negligible effect on flow over the solder balls. Also assume point contact between the chip and the solder, thereby neglecting heat transfer by conduction between the components.

a. The stress test begins with the components at ambient temperature \((T_i = 20^\circ \text{C})\) and proceeds with heating by the fluid at \(T_\infty = 80^\circ \text{C}\). If the fluid velocity is \(V = 0.2 \text{ m/s}\), estimate the ratio of the time constant of the chip to that of a solder ball. Which component responds more rapidly to the heating process?

b. The thermal stress acting on the solder joint is proportional to the chip-to-solder temperature difference. What is temperature difference 0.25 s after the start of heating?

**Schematic**

![Schematic diagram of the system](image)

**Assumptions**

1. Solder balls and chips are spatially isothermal,
2. Negligible heat transfer from sides of chip,
3. Top and bottom surfaces of chip act as flat plates in turbulent parallel flow,
4. Heat transfer from solder balls may be approximated as that from an isolated sphere,
5. Constant properties.

**Properties**

Given. Dielectric liquid: \(k = 0.064 \text{ W/m-K}, \nu = 10^{-6} \text{ m}^2/\text{s}, Pr = 25\); Silicon chip: \(3 ; k = 150 \text{ W/m-K}, \rho = 2300 \text{ kg/m}^3, c_p = 700 \text{ J/kg-K}\); Solder ball: \(k = 40 \text{ W/m-K}, \rho = 10,000 \text{ kg/m}^3, c_p = 150 \text{ J/kg-K}\)

**Analysis**

(a) From Eq. 5.7, the thermal time constant is \(\tau_t = (\rho V c / h \ A_s)\). Hence,
\[
\frac{\tau_{t,ch}}{\tau_{t,sld}} = \frac{(\rho c)_{ch}(L^2 t)}{2h_{ch}L^2} \frac{\overline{h}_{sld}(\pi D^2)}{(\rho c)_{sld}(\pi D^3/6)} = \frac{3}{D} \frac{(\rho c)_{ch}}{\overline{h}_{sld} \overline{h}_{ch}}
\]

The convection coefficient for the chip may be obtained from Eq. 7.38 with \( A = 0 \), with \( \text{Re}_L = \frac{VL}{\nu} = 0.2 \, \text{m/s} \times 0.01 \, \text{m} / 10^{-6} \, \text{m}^2 / \text{s} = 2000 \).

\[
\overline{h}_{ch} = \frac{0.064 \, \text{W} / \text{m} \cdot \text{K}}{0.01 \, \text{m}} (0.037)(2000)^{4/5} (25)^{1/3} = 302 \, \text{W} / \text{m}^2 \cdot \text{K}
\]

The convection coefficient for the solder may be obtained from Eq. 7.56, with \( \text{Re}_D = \frac{VD}{\nu} = 0.2 \, \text{m/s} \times 0.001 \, \text{m} / 10^{-6} \, \text{m}^2 / \text{s} = 200 \). Neglecting the effect of the viscosity ratio,

\[
\overline{h}_{sld} = \frac{0.064 \, \text{W} / \text{m} \cdot \text{K}}{0.01 \, \text{m}} \left[ 2 + \left[ 0.4(200)^{1/2} + 0.06(200)^{2/3} \right] (25)^{0.4} \right] = 1916 \, \text{W} / \text{m}^2 \cdot \text{K}
\]

Hence,

\[
\frac{\tau_{t,ch}}{\tau_{t,sld}} = \frac{3}{D} \frac{(\rho c)_{ch}}{\overline{h}_{sld} \overline{h}_{ch}} = 3 \frac{2500 \, \text{kg} / \text{m}^3 \times 700 \, \text{J} / \text{kg} \cdot \text{K}}{10,000 \, \text{kg} / \text{m}^3 \times 150 \, \text{J} / \text{kg} \cdot \text{K}} \frac{1916 \, \text{W} / \text{m}^2 \cdot \text{K}}{302 \, \text{W} / \text{m}^2 \cdot \text{K}} = 20.4
\]

Hence, the solder responds much more quickly to the convective heating.

(b) From Eq. 5.6, the chip-to-solder temperature difference may be expressed as

\[
T_{ch} - T_{sld} = (T_i - T_\infty) \left\{ \exp \left[ \frac{-2 \overline{h}}{(\rho c)_{ch} t} \right] - \exp \left[ \frac{-6 \overline{h}}{(\rho c)_{sld} D} \right] \right\}
\]

\[
T_{ch} - T_{sld} = 60^\circ \text{C} \left\{ \exp \left[ \frac{604 \, \text{W} / \text{m}^2 \cdot \text{K}}{1610 \, \text{J} / \text{m}^2 \cdot \text{K}} \times 0.25 \, \text{s} \right] - \exp \left[ \frac{-11,496 \, \text{W} / \text{m}^2 \cdot \text{K}}{1500 \, \text{J} / \text{m}^2 \cdot \text{K}} \times 0.25 \, \text{s} \right] \right\}
\]

\[
T_{ch} - T_{sld} = 60^\circ \text{C} \left\{ 0.910 - 0.147 \right\} = 45.8^\circ \text{C}
\]

Comments

(1) The foregoing process is used to subject soldered chip connections (a major reliability issue) to rapid and intense thermal stresses. (2) Some heat transfer by conduction will occur between the chip and solder balls, thereby reducing the temperature difference and thermal stress. (3) Constriction of flow between the chip and substrate will reduce \( \overline{h}_{sld} \), as well as \( \overline{h}_{ch} \) at the lower surface of the chip, relative to values predicted by the correlations. The corresponding time constants would be increased accordingly. (4) With \( \text{Bi}_{ch} = \frac{\overline{h}_{ch} \, (t/2)}{k_{chip}} = 0.001 << 1 \) and \( \text{Bi}_{sld} = \frac{\overline{h}_{sld} \, (D/6)}{k_{sld}} = 0.008 << 1 \), the lumped capacitance analysis is appropriate for both components.