HW3

1. (**Problem 4.36 in book**) A piston-cylinder device initially contains steam at 200 kPa, 200°C, and 0.4 m^3. At this state, a linear spring (F α x) is touching the piston but exerts no force on it. Heat is now slowly transferred to the steam, causing the pressure and the volume to rise to 250 kPa and 0.6 m^3, respectively. Show the process on a *P*-*v* diagram with respect to saturation lines and determine (a) the final temperature, (b) the work done by the steam, and (c) the total heat transferred. (Answers: (a) 606°C, (b) 45 kJ, (c) 288 kJ).

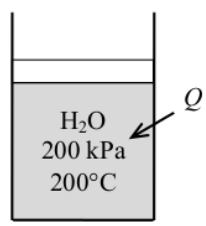
Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** The thermal energy stored in the cylinder itself is negligible. **3** The compression or expansion process is quasi-equilibrium. **4** The spring is a linear spring.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. Noting that the spring is not part of the system (it is external), the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_2 - u_1) \quad \text{(since KE = PE = 0)}$$

$$Q_{in} = m(u_2 - u_1) + W_{b,out}$$



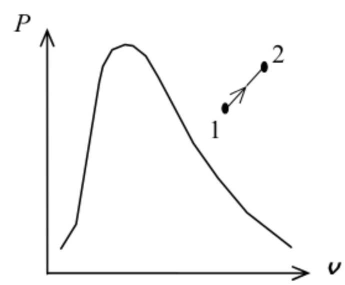
The properties of steam are (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 200 \text{ kPa} \\ T_1 = 200°\text{C} \end{array}\right\} \begin{array}{l} v_1 = 1.08049 \text{ m}^3/\text{kg} \\ u_1 = 2654.6 \text{ kJ/kg} \end{array}$$

$$m = \frac{V_1}{v_1} = \frac{0.4 \text{ m}^3}{1.08049 \text{ m}^3/\text{kg}} = 0.3702 \text{ kg}$$

$$v_2 = \frac{V_2}{m} = \frac{0.6 \text{ m}^3}{0.3702 \text{ kg}} = 1.6207 \text{ m}^3/\text{kg}$$

$$\left.\begin{array}{l} P_2 = 250 \text{ kPa} \\ v_2 = 1.6207 \text{ m}^3/\text{kg} \end{array}\right\} \begin{array}{l} T_2 = \mathbf{606°C} \\ u_2 = 3312.0 \text{ kJ/kg} \end{array}$$



(*b*) The pressure of the gas changes linearly with volume, and thus the process curve on a P-V diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

$$W_b = Area = \frac{P_1 + P_2}{2}(V_2 - V_1) = \frac{(200 + 250)\text{kPa}}{2}(0.6 - 0.4)\text{m}^3 \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) = \mathbf{45 \text{ kJ}}$$

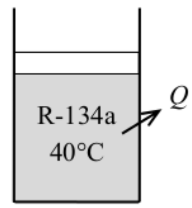
(*c*) From the energy balance we have

$$Q_{in} = (0.3702 \text{ kg})(3312.0 - 2654.6)\text{kJ/kg} + 45 \text{ kJ} = \mathbf{288 \text{ kJ}}$$

2. (**Problem 4.41 in book**) Saturated R-134a vapor at 40°C is condensed at constant pressure to a saturated liquid in a closed piston-cylinder system. Calculate the heat transfer and work done during this process, in kJ/kg.

Assumptions **1** The cylinder is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions involved other than the boundary work. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Analysis We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as



$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

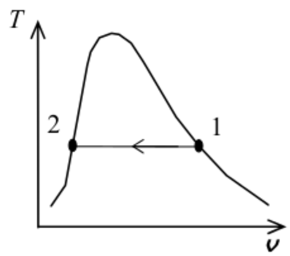
$$W_{b,\text{in}} - Q_{\text{out}} = \Delta U = m(u_2 - u_1) \quad \text{(since KE = PE = 0)}$$

$$Q_{\text{out}} = W_{b,\text{in}} - m(u_2 - u_1)$$

The properties at the initial and final states are (Table A-11)

$$\left.\begin{aligned} T_1 &= 40°\text{C} \\ x_1 &= 1 \end{aligned}\right\} \begin{aligned} \boldsymbol{v}_1 &= \boldsymbol{v}_g = 0.019952\ \text{m}^3/\text{kg} \\ u_1 &= u_g = 250.97\ \text{kJ/kg} \end{aligned}$$

$$\left.\begin{aligned} T_2 &= 40°\text{C} \\ x_2 &= 0 \end{aligned}\right\} \begin{aligned} \boldsymbol{v}_2 &= \boldsymbol{v}_f = 0.0008720\ \text{m}^3/\text{kg} \\ u_2 &= u_f = 107.38\ \text{kJ/kg} \end{aligned}$$



Also from Table A-11,

$$P_1 = P_2 = 1017.1\ \text{kPa}$$
$$u_{fg} = 143.60\ \text{kJ/kg}$$
$$h_{fg} = 163.00\ \text{kJ/kg}$$

The work done during this process is

$$w_{b,\text{out}} = \int_1^2 P d\boldsymbol{v} = P(\boldsymbol{v}_2 - \boldsymbol{v}_1) = (1017.1\text{kPa})(0.0008720 - 0.019952)\text{m}^3/\text{kg}\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = -19.41\ \text{kJ/kg}$$

That is,

$$w_{b,\text{in}} = \mathbf{19.41 kJ/kg}$$

Substituting into energy balance equation gives

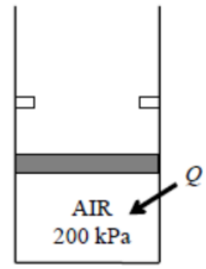
$$q_{\text{out}} = w_{b,\text{in}} - (u_2 - u_1) = w_{b,\text{in}} + u_{fg} = 19.41 + 143.60 = \mathbf{163.0\ kJ/kg}$$

Discussion The heat transfer may also be determined from

$$-q_{\text{out}} = h_2 - h_1$$
$$q_{\text{out}} = h_{fg} = 163.0\ \text{kJ/kg}$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process.

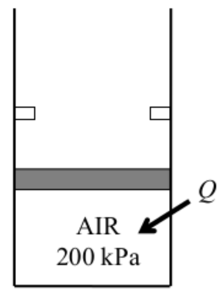
3. (**Problem 4.75 in book**) A piston-cylinder device, with a set of stops on the top, initially contains 3 kg of air at 200 kPa and 27°C. Heat is now transferred to the air, and the piston rises until it hits the stops, at which point the volume is twice the initial volume. More heat is transferred until the pressure inside the cylinder doubles. Determine the work done and the amount of heat transfer for this process. Also, show the process on a *P*-*v* diagram.



Assumptions **1** Air is an ideal gas with variable specific heats. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** The thermal energy stored in the cylinder itself is negligible.

Properties The gas constant of air is $R = 0.287$ kPa.m³/kg.K (Table A-1).

Analysis We take the air in the cylinder to be the system. This is a closed system since no mass crosses the boundary of the system. The energy balance for this closed system can be expressed as

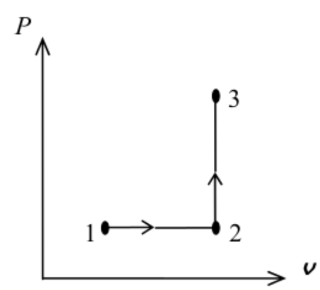


$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_3 - u_1)$$

$$Q_{in} = m(u_3 - u_1) + W_{b,out}$$

The initial and the final volumes and the final temperature of air are determined from



$$V_1 = \frac{mRT_1}{P_1} = \frac{(3\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})}{200\text{ kPa}} = 1.29\text{ m}^3$$

$$V_3 = 2V_1 = 2\times 1.29 = 2.58\text{ m}^3$$

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \longrightarrow T_3 = \frac{P_3}{P_1}\frac{V_3}{V_1}T_1 = \frac{400\text{ kPa}}{200\text{ kPa}}\times 2\times(300\text{ K}) = 1200\text{ K}$$

No work is done during process 2-3 since $V_2 = V_3$. The pressure remains constant during process 1-2 and the work done during this process is

$$W_b = \int_1^2 P\,dV = P_2(V_2 - V_1)$$

$$= (200\text{ kPa})(2.58 - 1.29)\text{ m}^3\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = \mathbf{258\text{ kJ}}$$

The initial and final internal energies of air are (Table A-17)

$$u_1 = u_{@300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_3 = u_{@1200\text{ K}} = 933.33\text{ kJ/kg}$$

Substituting,

$$Q_{in} = (3\text{ kg})(933.33 - 214.07)\text{kJ/kg} + 258\text{ kJ} = \mathbf{2416\text{ kJ}}$$

Alternative solution The specific heat of air at the average temperature of $T_{avg} = (300 + 1200)/2 = 750$ K is, from Table A-2b, $c_{v,avg} = 0.800$ kJ/kg.K. Substituting

$$Q_{in} = m(u_3 - u_1) + W_{b,out} \cong mc_v(T_3 - T_1) + W_{b,out}$$

$$= (3\text{ kg})(0.800\text{ kJ/kg.K})(1200 - 300)\text{ K} + 258\text{ kJ} = \mathbf{2418\text{ kJ}}$$

4. (**Problem 4.123 in book**) A piston-cylinder device contains helium gas initially at 150 kPa, 20oC, and 0.5 m3. The helium is now compressed in a polytropic process ($PV^n = const$) to 400 kPa and 140oC. Determine the heat loss or gain during this process. (Answers: 11.2 kJ loss).

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

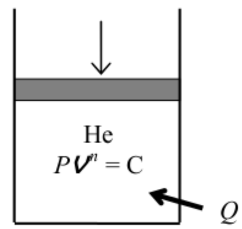
Properties The gas constant of helium is R = 2.0769 kPa.m^3/kg.K (Table A-1). Also, c_v = 3.1156 kJ/kg.K (Table A-2).

Analysis The mass of helium and the exponent n are determined to be

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kPa})(0.5 \text{ m}^3)}{(2.0769 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293 \text{ K})} = 0.123 \text{ kg}$$

$$\frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2} \longrightarrow V_2 = \frac{T_2 P_1}{T_1 P_2} V_1 = \frac{413 \text{ K}}{293 \text{ K}} \times \frac{150 \text{ kPa}}{400 \text{ kPa}} \times 0.5 \text{ m}^3 = 0.264 \text{ m}^3$$

$$P_2 V_2^n = P_1 V_1^n \longrightarrow \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n \longrightarrow \frac{400}{150} = \left(\frac{0.5}{0.264}\right)^n \longrightarrow n = 1.536$$



Then the boundary work for this polytropic process can be determined from

$$W_{\text{b,in}} = -\int_1^2 P\,dV = -\frac{P_2 V_2 - P_1 V_1}{1-n} = -\frac{mR(T_2 - T_1)}{1-n}$$

$$= -\frac{(0.123 \text{ kg})(2.0769 \text{ kJ/kg}\cdot\text{K})(413-293)\text{K}}{1-1.536} = 57.2 \text{ kJ}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. Taking the direction of heat transfer to be to the cylinder, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} + W_{\text{b,in}} = \Delta U = m(u_2 - u_1)$$

$$Q_{\text{in}} = m(u_2 - u_1) - W_{\text{b,in}}$$

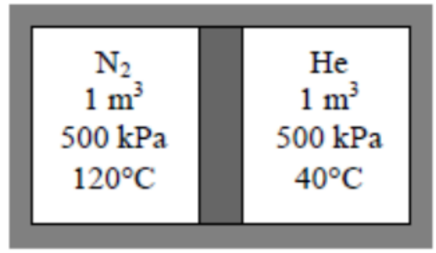
$$= mc_v(T_2 - T_1) - W_{\text{b,in}}$$

Substituting,

$$Q_{\text{in}} = (0.123 \text{ kg})(3.1156 \text{ kJ/kg}\cdot\text{K})(413 - 293)\text{K} - (57.2 \text{ kJ}) = \mathbf{-11.2 \text{ kJ}}$$

The negative sign indicates that heat is lost from the system.

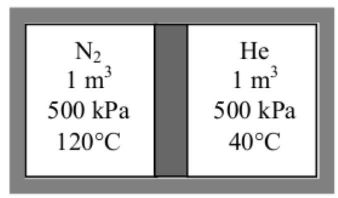
5. (**Problem 4.136 in book**) Consider a well-insulated horizontal rigid cylinder that is divided into two compartments by a piston that is free to move but does not allow either gas to leak into the other side. Initially, one side of the piston contains 1 m^3 of N_2 gas at 500 kPa and 120°C while the other side contains 1 m^3 of He gas at 500 kPa and 40°C. Now thermal equilibrium is established in the cylinder as a result of heat transfer through the piston. Using constant specific heats at room temperature, determine the final equilibrium temperature in the cylinder. What would be your answer if the piston were not free to move?



Assumptions **1** Both N_2 and He are ideal gases with constant specific heats. **2** The energy stored in the container itself is negligible. **3** The cylinder is well-insulated and thus heat transfer is negligible.

Properties The gas constants and the constant volume specific heats are R = 0.2968 kPa.m^3/kg.K is c_v = 0.743 kJ/kg·°C for N_2, and R = 2.0769 kPa.m^3/kg.K is c_v = 3.1156 kJ/kg·°C for He (Tables A-1 and A-2)

Analysis The mass of each gas in the cylinder is



$$m_{N_2} = \left(\frac{P_1 V_1}{RT_1}\right)_{N_2} = \frac{(500\text{ kPa})(1\text{ m}^3)}{(0.2968\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(393\text{ K})} = 4.287\text{ kg}$$

$$m_{He} = \left(\frac{P_1 V_1}{RT_1}\right)_{He} = \frac{(500\text{ kPa})(1\text{ m}^3)}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(313\text{ K})} = 0.7691\text{ kg}$$

Taking the entire contents of the cylinder as our system, the 1st law relation can be written as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$0 = \Delta U = (\Delta U)_{N_2} + (\Delta U)_{He}$$

$$0 = [mc_v(T_2 - T_1)]_{N_2} + [mc_v(T_2 - T_1)]_{He}$$

Substituting,

$$(4.287\text{ kg})(0.743\text{ kJ/kg}\cdot°\text{C})(T_f - 120)°\text{C} + (0.7691\text{ kg})(3.1156\text{ kJ/kg}\cdot°\text{C})(T_f - 40)°\text{C} = 0$$

It gives

T_f = **85.7°C**

where T_f is the final equilibrium temperature in the cylinder.

The answer would be the **same** if the piston were not free to move since it would effect only pressure, and not the specific heats.

Discussion Using the relation $PV = NR_uT$, it can be shown that the total number of moles in the cylinder is 0.153 + 0.192 = 0.345 kmol, and the final pressure is 515 kPa.