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George B. Dantzig: Operations Research Icon

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This article recalls some of George B. Dantzig's many contributions to operations research and the management sciences in his ninety-year lifetime.

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1. Introduction

The passing of George B. Dantzig in May 2005 has occasioned a stream of memorial articles reflecting on the life and major contributions of this remarkable man. These naturally began with editorial obituaries and progressed to feature stories in various news media. In most cases, the nature of the reading (or listening) audience has necessitated an emphasis on the considerable human interest side of the Dantzig story. Accordingly, many of the articles convey the heartfelt words of his students and colleagues, all of whom express their admiration for his warmth, generosity, and intelligence. While being no less heartfelt, the present piece will attempt to review George Dantzig's professional contributions for a more learned readership.

Trained as a mathematician, George Dantzig evolved into what can justly be called an operations research icon. He is, of course, widely known as the father of linear programming and the inventor of the simplex method. Indeed, he has done much to record for posterity his reminiscences of the origins of linear programming. There are at least five places to find this material: (Dantzig 1982, 1983, 1984, 1991; Dantzig and Thapa 1997). In addition, other writers—the present one included—have benefitted greatly from the inverview of George Dantzig conducted by Donald J. Albers (Albers and Reid 1986, Albers et al. 1990, Albers 2005). (See Cottle 2003, 2005; Dupacova and Morton 2005; and O'Connor and Robertson 2005 for a selection of articles that borrow heavily from the Albers-Dantzig interview.) Almost none of the memorial tributes fail to retell the now familiar legend of the writing on the blackboard that doctoral student George Dantzig thought was homework when, in fact, it was two unsolved problems in mathematical statistics. His determination in solving these difficult problems has been fashioned into a moral, if not an actual homily.

2. Early Professional Years

World events, primarily the Depression and World War II, shaped the careers of many scholars of that era. This was certainly so in Dantzig's case. After completing his master's degree at the University of Michigan, he joined the Bureau of Labor Statistics (BLS) as a junior statistician. There he worked on a project called "Urban Study of Consumer Purchases" (Albers and Reid 1986, Albers et al. 1990). At the BLS, Dantzig gained an introduction to practical applications of mathematics and statistics unlike anything he had seen before. There he became acquainted with Leontief's input-output model of the U.S. economy (Leontief 1951); this, he said, "changed the course of my career." And it was at the BLS that he was asked to review a paper written by the great mathematical statistician, Jerzy Neyman. This paper kindled Dantzig's desire to study for the Ph.D. under the supervision of Neyman who, in 1938, left the University of London to join the mathematics department at UC Berkeley. George Dantzig arrived there a year later and realized his ambition of studying under Neyman. His doctoral dissertation (Dantzig 1946) at Berkeley included the following acknowledgment:

> The two subjects found in this dissertation were inspired by the Lectures of Professor J. Neyman. They represented two "unsolved" problems in the Foundations of Mathematical Statistics. Appreciation is expressed to Professor George Polya for his valuable suggestions.

The two "unsolved" problems referred to in the acknowledgment are the ones he had mistaken for homework. As presented in the thesis, Part I was on the "Complete Form of the Neyman-Pearson Fundamental Lemma." Part II was "On the Non-Existence of Tests of 'Student's' Hypothesis Having Power Functions Independent of Sigma." These were published as two papers (Dantzig 1940) and (Dantzig and Wald 1951) separated by 11 years.[1]

After the summer of 1941, Dantzig took a leave of absence from his doctoral studies to join the war effort at the U.S.A.F. Headquarters Statistical Control in Washington. He became Chief of the Combat Analysis Branch and in 1944 was awarded the Exceptional Civilian Service Medal. In 1946, he returned to Berkeley to finish up the degree requirements for his Ph.D. which was awarded that year. Upon completion of his doctorate, George Dantzig declined the offer of a faculty position in mathematics at Berkeley in favor of returning as a Mathematical Advisor to the Comptroller of the U.S.A.F. It was in this capacity that he began the work for which he became so famous, and ultimately an operations research icon.

Dantzig's post-war professional responsibility at the Pentagon dealt with mechanization of the planning procedures to support time-staged deployment of training and supply activities. In response to this assignment, he conceived, in 1947, the linear programming problem and the simplex algorithm for its solution. This was to be the opportunity of a lifetime: one that made a deep impression on him, and one that led to his major influence on the world of applied science.

George Dantzig's Pentagon experience in the development of linear programming led him to become a strong advocate of real-world problems as the inspiration for interesting and challenging mathematical work. In speaking to Donald Albers (Albers et al. 1990) he said:

> Just because my mathematics has its origin in a real problem doesn't make it less interesting to me—just the other way around, I find it makes the puzzle I am working on all the more exciting. I get satisfaction out of knowing that I'm working on a relevant problem. I find that just as much mathematical ingenuity has to go into solving problems from a new developing area as from some old so-called pure math area.

This attitude is reflected in the powerful opening line in the preface to his classic monograph *Linear Programming and Extensions* (Dantzig 1963, p. vii): "The final test of a theory is its capacity to solve the problems which originated it." However much he may have immersed himself in abstract mathematics, George Dantzig was always able to relate his work to the solution of classes of real-world problems. This outlook contributed greatly to his affiliation with operations research and computer science, both of which were really in their infancy at the time. When it came to mathematics generally, he tended to blur distinctions between pure and applied mathematics as in the statement "I have never been able to tell the difference between the so-called pure and the non-pure and don't believe there is any" (Albers et al. 1990). Nonetheless, he knew what he didn't like: lack of interest in applications.

On September 9, 1948, Dantzig advanced his new discoveries in a session (on game theory) of the Econometric Society meeting taking place at the University of Wisconsin in Madison. Only the abstract of the talk, which he called "Programming in a Linear Structure," was published (Dantzig 1949). It was brief but revealed his awareness of the subject's potential to solve real-world problems and how it could be developed further. In the discussion period following his presentation, Dantzig was grilled by Harold Hotelling about the restrictiveness of the model's linearity assumptions. A concise and effective defense was offered by John von Neumann on the speaker's behalf. For some details on this episode, see (Cottle 2005).

The 1949 Conference on Activity Analysis of Production and Allocation held at the University of Chicago was a signal event in the history of our subject, and certainly in the life of George Dantzig. Tjalling Koopmans edited the proceedings volume (Koopmans 1951) of the conference and wrote its fascinating introduction.[2] After nearly four pages of scholarly exposition on earlier contributions, largely from economics, Koopmans says:

> The foregoing references to the main currents of thought that have inspired the present studies may already have helped to characterize the intent of this volume. The immediate occasion for it is to report a conference on "linear programming," held in Chicago at the Cowles Commission for Research in Economics on June 20–24, 1949.

It is interesting to note that in the summer of 1948 (a year before the conference), George Dantzig had agreed to adopt Koopmans's recommendation to shorten the name "Programming in a Linear Structure" to "Linear Programming." Although the new term appears in the passage just quoted, it is not used in the title of a single one of Dantzig's five papers in the proceedings (which had 25 papers in all). Instead, the titles he chose for his papers clearly conveyed their intent. There may have been other reasons for these choices, but I prefer to believe that Dantzig consciously avoided using an unfamiliar name that would not be understood.

From an examination of Dantzig's five papers in the proceedings volume, one can appreciate the breadth of his involvement. The first paper, written jointly with Marshall Wood, is a general discussion about the programming of interdependent activities and says in the very first sentence that it is "a generalization of the Leontief interindustry model." It lays out a bit of modeling philosophy using no mathematical symbols whatsoever. The next paper *is* mathematical and tends to be a bit axiomatic, beginning as it does with five postulates. The third paper belongs to a group of four papers concerned with game theory; it demonstrates the equivalence of the "programming problem" with the game problem. The fourth and fifth papers are part of a group of five articles on problems of computation. The ones by Dantzig are called "Maximization of a Linear Function of Variables Subject to Linear Inequalities," in which the simplex algorithm is presented, and "Application of the Simplex Method to a Transportation Problem." The first of these two papers acknowledges stimulating discussions with Leonid Hurwicz that culminated in the simplex approach. It also includes a nondegeneracy assumption and an informal statement on perturbation of the right-hand side as a way to handle degeneracy. In a paper on the transportation problem, Dantzig acknowledges the earlier work of Hitchcock (1941) and Koopmans (1947). At this stage in the subject's development, the work of Leonid Kantorovich (1939) was unknown in the West.

In his interview with Donald Albers, George Dantzig remarked that, in 1946, he turned down the job offer at Berkeley and took the job at the Pentagon instead. Then he added, "but I was just marking time[3] while looking for an academic position." This was a historically significant human decision; how very fortunate we are he made it. Although Dantzig was to mark time, as he put it, for 14 years, this period was one of great growth and productivity.

3. At the RAND Corporation

George Dantzig remained at the Pentagon until 1952, at which time he took a position in the Mathematics Department of the RAND Corporation in Santa Monica, California. In those days, the RAND Corporation was a "think tank" sponsored by the Air Force, so in a sense, Dantzig's move was from one part of the Air Force to another. But in this setting he was in a better position to build the foundations of linear programming, and that is what he did.

At RAND Dantzig worked with a stable of gifted mathematicians, economists, computer scientists, and an enlightened manager, John D. Williams. Dantzig's aims were broad: to develop the subject in a way that made it theoretically sound, computationally efficient, and indisputably useful. His published writings beginning around 1954 (and hence based on work initiated earlier) show the richness and diversity of his thinking and collaboration. One has only to look at George Dantzig's publication list to appreciate how much he accomplished while working at the RAND Corporation. With the benefit of hindsight, one can view the work of this period as a superb program of research based on the issues he identified in the abstract of his 1948 Madison talk (Dantzig 1949) and many other developments as well.

The topics treated in these accomplishments include the following:

(1) *Game theory*. Although this was not very high on his list of interests, game theory was nevertheless the theme of the session in which his Madison talk was given, and the abstract of the talk refers to the problem "of determining the Min-Max of a bilinear form." This was taken up in his paper (Dantzig 1956). See also §13-3 of (Dantzig 1963).

(2) *Linear programming theory and variants of the simplex method*. The papers of this group include a duality theorem based on the simplex method (Dantzig and Orden 1952), the well-known paper on the generalized simplex method (Dantzig et al. 1955) from which we get—among other things—the lexicographic technique of degeneracy resolution, the primal-dual algorithm for linear programs (Dantzig et al. 1956), and an inductive proof of the simplex method (Dantzig 1960b). In addition to these articles, there was an internal note on the dual simplex method.

(3) *Large-scale linear programming*. Dantzig (1949) proposed the use of computational techniques (algorithms) for solution of programming problems on large-scale digital computers, and it is reasonable to assume that he had in mind large-scale problems since he was interested in multistage planning models which are inherently large. Under this heading Dantzig produced (Dantzig 1955a) two classic papers (Dantzig and Wolfe 1960, 1961), another paper on the product form of the inverse (Dantzig and Orchard-Hays 1954), and a paper on solving dynamic Leontief models with substitution (Dantzig 1955b).

(4) *Linear programming under uncertainty*. Building on Ferguson and Dantzig (1956), Dantzig wrote a paper (Dantzig 1955c) that launched what was to become a major field of investigation. See also Dantzig and Madansky (1961).

(5) *Network optimization problems*. The transporation problem and network flow problems provided the inspiration for some of Dantzig's best work at RAND. With Ray Fulkerson, Dantzig wrote two papers, Fulkerson and Dantzig (1955) and Dantzig and Fulkerson (1956), on maximal flows through networks and on the max-flow min-cut theorem of networks, respectively. In this group of papers we also find a very important paper on the traveling salesman problem (Dantzig et al. 1954). A 49-city problem involving one city in each of the states of the "lower 48 states" plus Washington, D.C., this was the largest traveling salesman problem to be solved up to that time (1954). Another of Dantzig's publications that can be thought of in network terms is his paper on Dilworth's theorem in partially ordered sets (Dantzig and Hoffman 1956). They show how to use linear programming to find the smallest set of disjoint chains that cover the partially ordered set. In addition to these papers, Dantzig wrote an article (Dantzig 1960a) on the shortest route through a network.

(6) *Integer linear programming and discrete optimization*. George Dantzig recognized the great difficulty of solving linear programs in integers. His paper (Dantzig 1960c) makes the point by reducing many challenging combinatorial problems to such optimization problems. Other related papers on integer programming include Dantzig (1957, 1959).

(7) *Miscellaneous applications*. Eager to see linear programming put to use on practical problems, George Dantzig willingly joined modeling efforts. Publications resulting from these activities include his work on an approach to the chemical equilibrium problem (Dantzig et al. 1958, White et al. 1958), a paper on allocation of aircraft to routes with uncertain demand (Ferguson and Dantzig 1956), and a production smoothing problem (Dantzig and Johnson 1955).

(8) *Notes on linear programming*. During the early days of linear programming, there were very few books to be found on the subject. Thus, while working at RAND, George Dantzig produced a series of internal reports called *Notes on Linear Programming: Part I*, ... These were to form much of the content of his yet to be published book, *Linear Programming and Extensions* (Dantzig 1963).

George Dantzig's employment at RAND ended in 1960. Interviewing him 24 years later, Donald Albers asked, "What caused you to leave RAND and return to the academic world?" (Albers and Reid 1986). Dantzig's reply is interesting. He said:

> My leaving had to do with the way we teamed up to do our research. In the beginning I was part of a team with Ray Fulkerson and Selmer Johnson. For a time we did great things together. Then after a while, although we remained good friends, each of us got busy doing his own thing. ... There were no new people being hired to work with us as disciples.

It is quite possible that the times (and the working conditions) at RAND were a-changin' in a way that made academia look more appealing.

4. At UC Berkeley

After 14 years of marking time, George Dantzig found an academic position, possibly of just the sort he once sought. In 1960, he became a professor at the University of California at Berkeley in the Department of Industrial Engineering. The chair of the department was Ronald W. Shephard, a man Dantzig may have known from his days as a doctoral student with Jerzy Neyman (see Reid 1982, p. 211).

By the time of Dantzig's return to Berkeley as a faculty member, operations research and the management sciences had grown as fields of study and become organizations: ORSA founded in 1952 and TIMS founded in 1953. In early 1961, Dantzig established the Operations Research Center (ORC) as a focal point for teaching and research activities in OR. Despite its inconvenient and unattractive location at the Richmond Field Station in Richmond, miles from the Berkeley campus, the ORC managed to draw in other faculty (such as William S. Jewell, Robert M. Oliver, Ronald W. Shephard, and Ronald W. Wolff) and a group of eager doctoral students. Here were the missing disciples Dantzig wished for when still at RAND.

At UC Berkeley, George Dantzig launched his outstanding career as a teacher of linear programming and network flow theory and as the principal mentor of many doctoral students. Indeed, he was the thesis advisor of many student who became leaders in mathematical programming or some other branch of operations research.[4]

As mentioned earlier, Dantzig's *Notes on Linear Programming* grew into the monumental text/treatise *Linear Programming and Extensions*, published in 1963. Several graduate students had the responsibility to proofread—and, simultaneously, the opportunity to learn from—this great source of knowledge. *Linear Programming and Extensions* and Dantzig's fertile mind provided a wealth of material for doctoral dissertations. Among the thesis topics (and their student authors) were integer programming (Ellis Johnson), large-scale linear programming (Richard Van Slyke and Saul Gass), linear programming under uncertainty (Roger Wets), and nonlinear programming (Richard Cottle).

In addition to directing the ORC, finishing *Linear Programming and Extensions*, and supervising 11 doctoral students, Dantzig collaborated with a group working on problems in the life sciences, continued research on large-scale LP, and began some work on optimal control. In doing all these things, George Dantzig established himself as a leader in academic circles.

5. At Stanford University

In 1966, George Dantzig left UC Berkeley and took a position at Stanford University, joining the Computer Science Department and the Program in Operations Research. The latter became a department the next year. Although Dantzig held a joint appointment, his activity was centered within operations research.[5] Although George Dantzig was a planner, he was not much interested in organizational politics or managerial detail. Nonetheless, he began his tenure at Stanford by serving as president of The Institute of Management Sciences (TIMS), and, a few years later (1973–1974), he served as the first chairman of the new Mathematical Programming Society.

Under Dantzig's leadership the OR department's mathematical programming activities grew in size and international visibility. The addition of faculty and staff, as well as a steady stream of scholars visiting for various durations, made the department a bastion of mathematical programming research. Among the many (now well-known) figures who populated the resident research staff were Philip Gill, Walter Murray, Michael Saunders, John Tomlin, and Margaret Wright. They, along with numerous faculty members, some from other departments, formed the Systems Optimization Laboratory (SOL), which exists to this day, though with fewer participants. The SOL's purpose as articulated in a visionary article was to develop "computational methods and associated computer routines for numerical analysis and optimization of large-scale systems" (Dantzig et al. 1973).

Over his 30 years at Stanford, Dantzig spearheaded other large projects. One of these was the development of a Mathematical Programming Language (MPL) to be used in testing proposed optimization algorithms. Another was the Model Development Laboratory whose purpose was to create mathematical models and collect data for real-world problems. Within this setting, Dantzig's pet project—especially from 1975 to the late 1980s—was a large macroeconomic model called PILOT. To quote from one of many papers by George Dantzig, Patrick McAllister, and John Stone (1988), the aim of PILOT was "to assess the impact of old and proposed new technologies on the growth of the U.S. economy, and how the state of the economy and economic policy may affect the pace at which innovation and modernization proceeds." In the PILOT project Dantzig found a context for combining the three sides of research that greatly interested him: economic modeling, large-scale programming algorithms, and the computation of optimal solutions or the solution of economic equilibrium (complementarity) problems.

Dantzig acquired, of course, great international stature. He enjoyed cordial relationships with those who led the operations research communities around the world. In the mid 1970s, George Dantzig began his long-term association with IIASA, the International Institute for Applied Systems Analysis (in Laxenburg, Austria). At one point, he headed its Methodology Group. Dantzig also had close links to the Technical University of Linkøping (Sweden) and the Technion (Haifa, Israel).

While at Stanford, George Dantzig produced 41 doctoral students. The range of subjects of their theses attests to the breadth and vitality of his interests. The distribution goes about like this: large-scale linear programming (8), stochastic programming (6), combinatorial optimization (4), nonlinear programming (4), continuous linear programming (3), networks and graphs (3), complementarity and computation of economic equilibria (2), dynamic linear and nonlinear programming (2), probability (2), other (7). The skillful supervision that Dantzig brought to the dissertation work of these students and the energy they subsequently conveyed to the operations research community fortified his iconic reputation.

Recognition of George Dantzig's greatness came repeatedly in the form of honors and awards. In 1971, he was elected to the National Academy of Sciences. In 1975, he received the National Medal of Science (presented by President Gerald Ford) and the John von Neumann Theory Prize (presented by ORSA and TIMS). That same year he was elected to the American Academy of Arts and Sciences.[6] In all, he collected eight honorary doctorates and many other decorations. At various times beginning in 1970, he was listed on the editorial board of 22 different journals.

Though he became an emeritus professor in 1985, George Dantzig taught and did research for another 13 years. During that period, it seemed as though stochastic programming gained the lion's share of his attention. When Gerd Infanger came along in 1989, Dantzig encouraged him to participate in the research program on stochastic optimization. That was the beginning of a new partnership. During the 1990s, Dantzig and Infanger coauthored seven papers. Among these were Dantzig and Infanger (1992a, b, 1993). But this was not Dantzig's only partnership in those years. He realized that much had happened since 1963 and that something was needed to replace *Linear Programming and Extensions*. So, in collaboration with Mukund Thapa, he wrote a more contemporary two-volume text (Dantzig and Thapa 1997, 2003) on linear programming and extensions.[7]

6. At Rest

George Dantzig left us on May 13, 2005 after months of medical complications and painful decline. His remarkable professional life was faithfully devoted to his vision of enabling mankind to make decisions about complex systems and issues using mathematical programming. He went about this in a thoughtful and noble way. His legacy is not simply the concept of linear programming and the invention of the simplex method. He worked on a much larger canvas, one with room for the full ramifications and potential of the subject and many others related to it. His approach was ingenious and inventive rather than imitative. He knew how to get things done and often to make the doing serve more than one purpose. His manner was kindly and welcoming, not brusque. He was thoroughly dedicated to his work and loyal to his students, friends, and colleagues. It is not surprising that he was so loved and admired by operations researchers everywhere.

Endnotes

1. Dantzig's involvement in the war effort is the explanation for the gap in publication years (Dantzig 2002). He recalled that he was so busy with his work in Washington that he forgot about publishing Part I of his dissertation. Then, one day, he received a letter from Abraham Wald who explained that a paper of his had been accepted for publication and that he had subsequently learned that the same result was already reported in Dantzig's dissertation. Wald asked Dantzig what he thought should be done. Since the paper was still in galley proof, Dantzig suggested that his name be added as an author. This is how the Dantzig-Wald paper of 1951 came about.
2. See http://cowles.econ.yale.edu/P/cm/m13.htm for the introduction as well as the contents of this volume.
3. This language is to be found on p. 68 of the book *More Mathematical People* (Albers et al. 1990). Its chapter on George Dantzig is based on a nearly identical article in the *College Mathematical Journal* (Albers and Reid 1986), which does *not* include the phrase "marking time while."
4. For a more or less correct version of Dantzig's "academic family tree" see www.genealogy.ams.org/.
5. The Albers interview (Albers and Reid 1986) does not explore the reasons for Dantzig's moving from Berkeley to Bay Area rival, Stanford. Reluctant to discuss his motivation for the switch, Dantzig used to quip that OR program (later department) chairman, Gerald J. Lieberman, promised him a parking place adjacent to the office. Although this arrangement did not survive the relocation of the department office, Dantzig remained at Stanford!
6. Sadly, for Dantzig and his admirers in the mathematical programming community, the 1975 Nobel Prize in Economics was awarded to L. V. Kantorovich and T. C. Koopmans. Some saw a geopolitical motive in this selection.
7. Dantzig and Thapa did further work toward completing two more volumes of the series.

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