

The Class Cover Problem and its Applications to Pattern Recognition

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Outline

- History of the Class Cover Problem (CCP)

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 - CCP-based Classifiers

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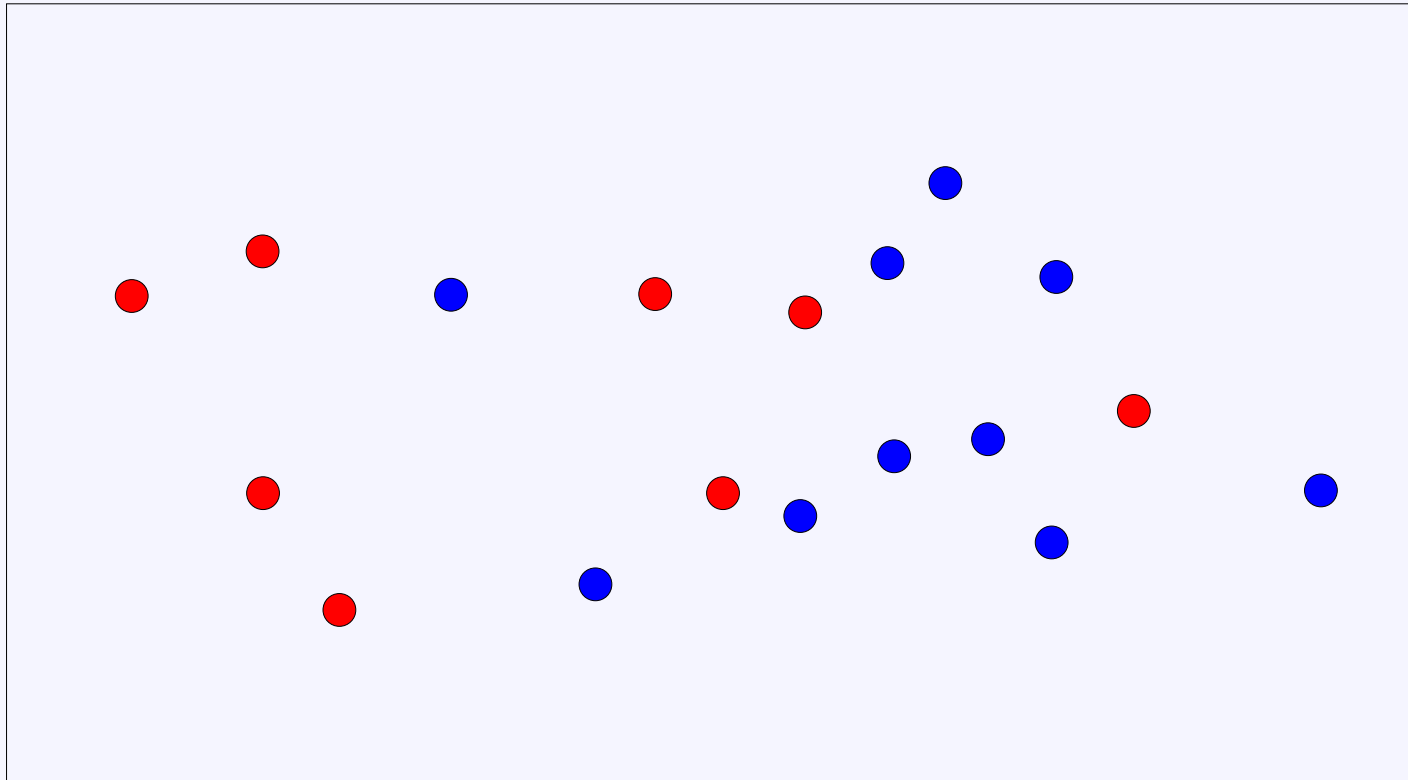
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 - CCP-based Classifiers
 - Simulation Results

History of the Class Cover Problem

- Approximate Distance Clustering
Cowen and Priebe
Advances in Applied Mathematics, 19 (1997)
- Class Cover Problem
Cannon and Cowen
Annals of Math and A.I. to appear
- Closely Related Topics
 - Sphere Covers, Hochbaum and Maass (J. of the A.C.M. 1985)
 - Sphere Digraphs, Maehara (J.G.T. 1984)
 - Sphere of Attraction Graphs,
McMorris and Wang (Con. Num. 2000)

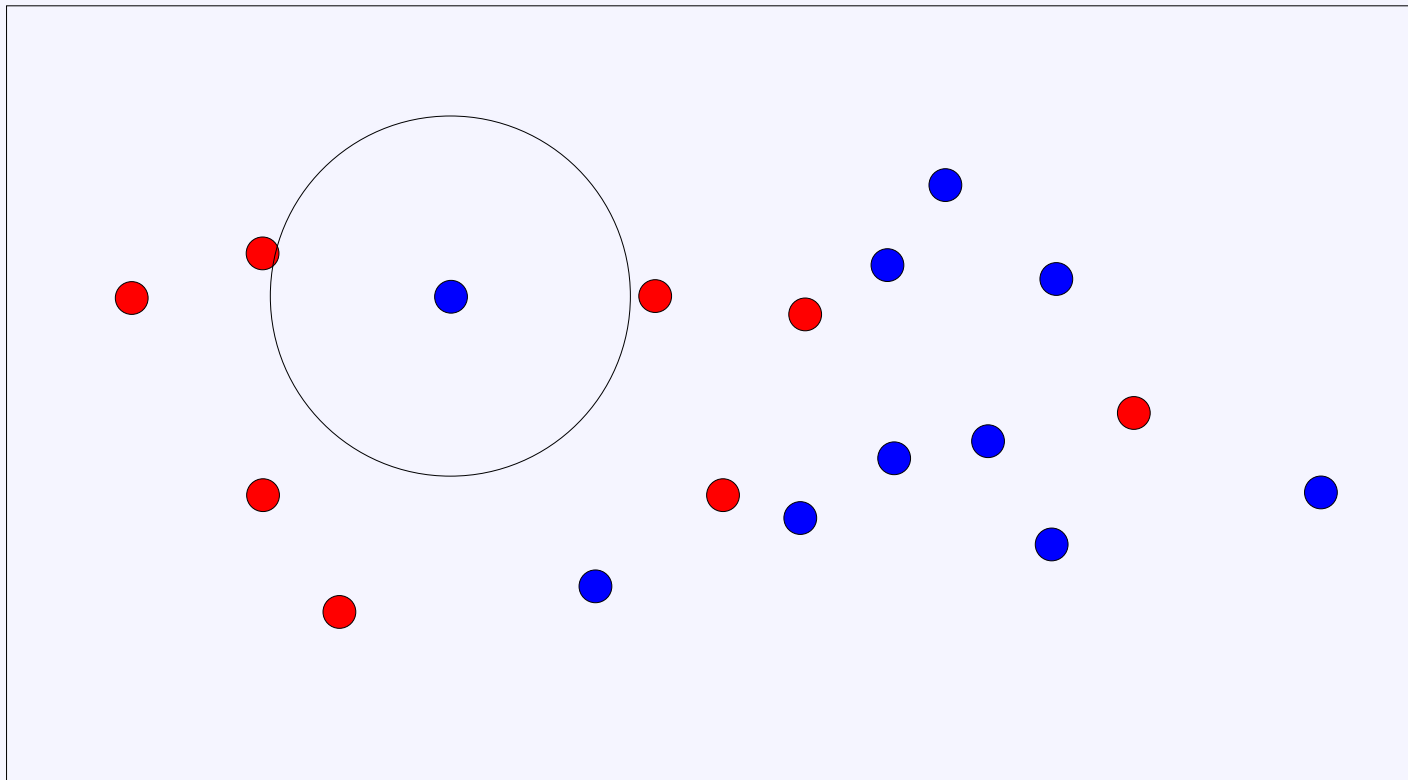
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Input: (Ω, d) (dissimilarity space) with $X, Y \subset \Omega$ and $X =$ target class.
Goal : Find the smallest set of “balls” (a cover), centered at points in X , such that every point in X is in at least one of the balls and no point in Y is in any ball.



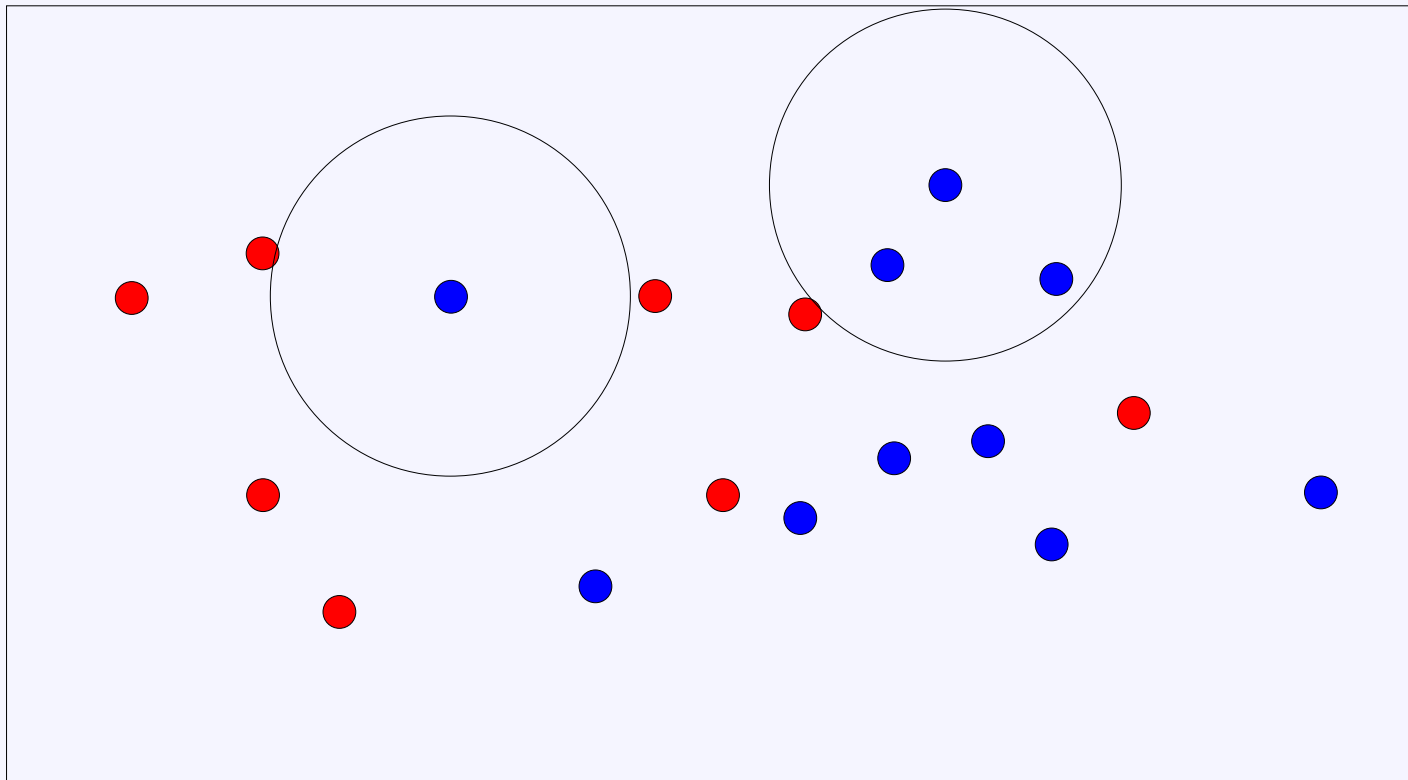
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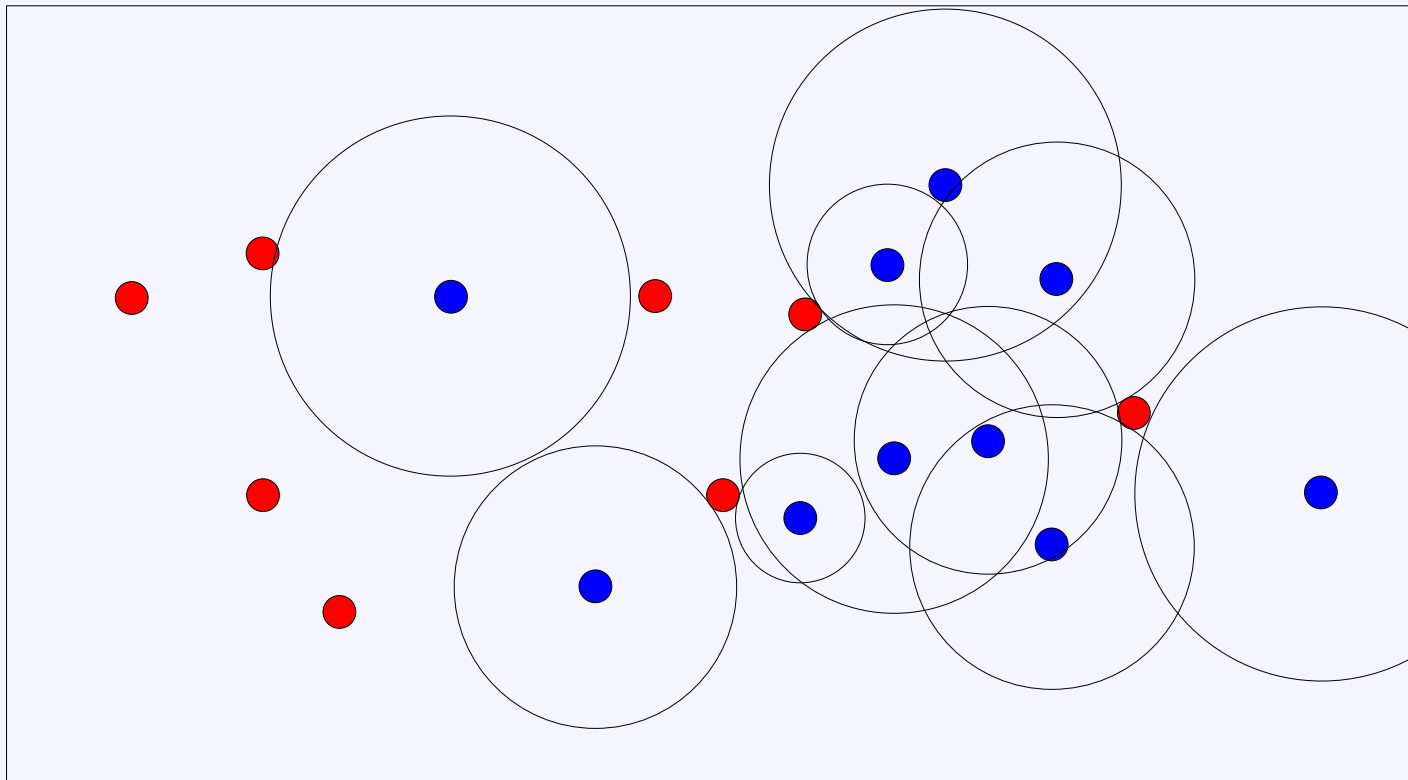
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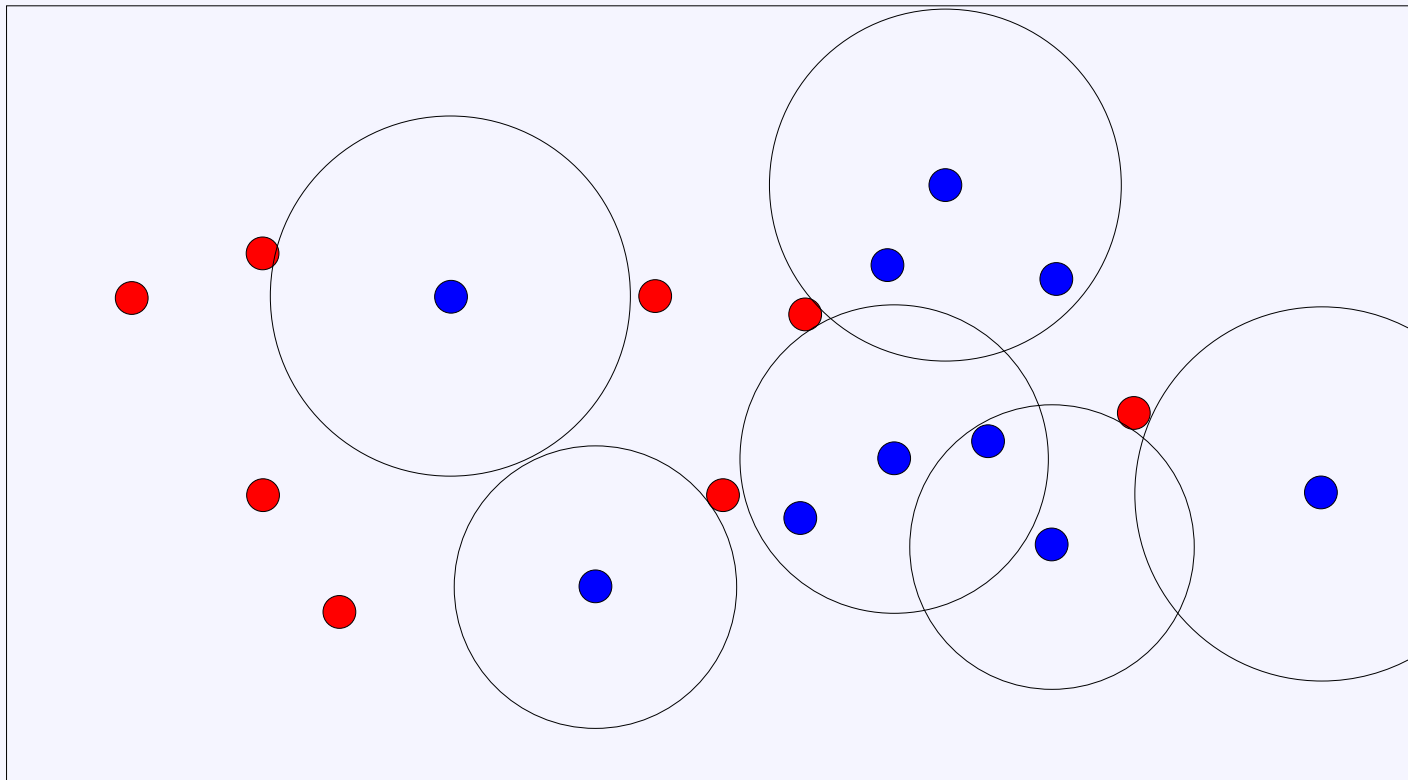
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Catch Digraphs

Given a collection of sets $\{S_1, S_2, \dots, S_n\}$ with associated “base” points $\{t_1, t_2, \dots, t_n\}$.

We form the *catch digraph* D with $V = \{v_1, v_2, \dots, v_n\}$ and a directed edge from v_i to v_j iff $t_j \in S_i$.

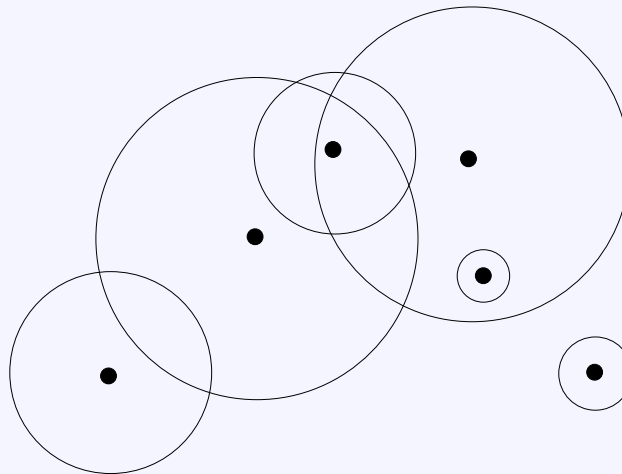
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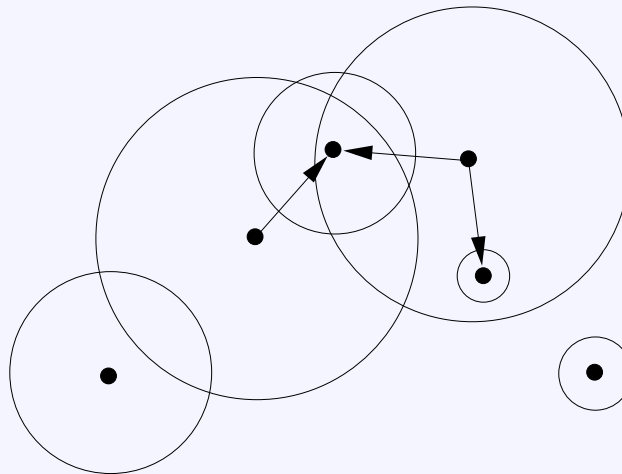


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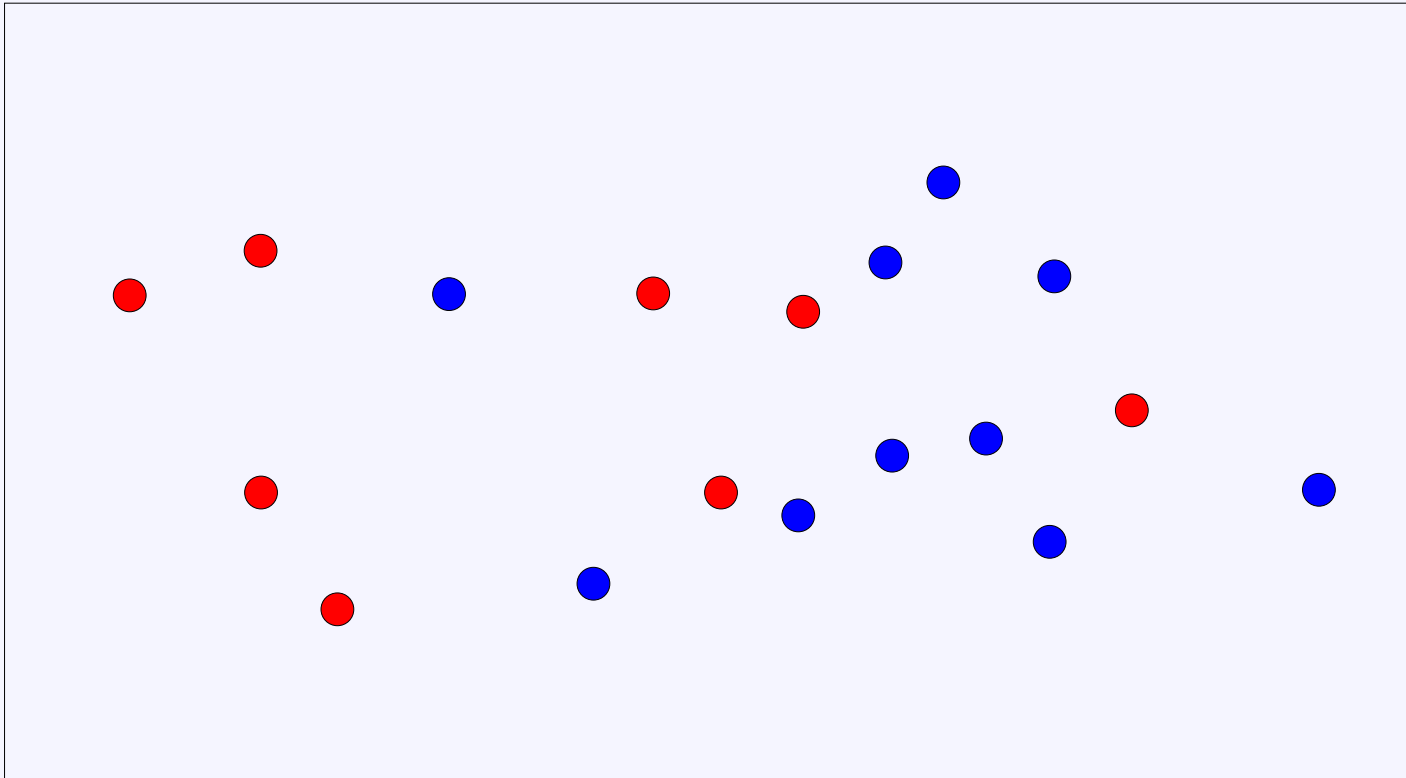


Class Cover Catch Digraphs (CCCD)

For any sets $X, Y \subset \Omega$ we can define the *class cover catch digraph* to be the catch digraph formed from the sets $B_i = \{z \in \Omega : d(z, x_i) < r_i\}$ and associated base points $x_i \in X$.

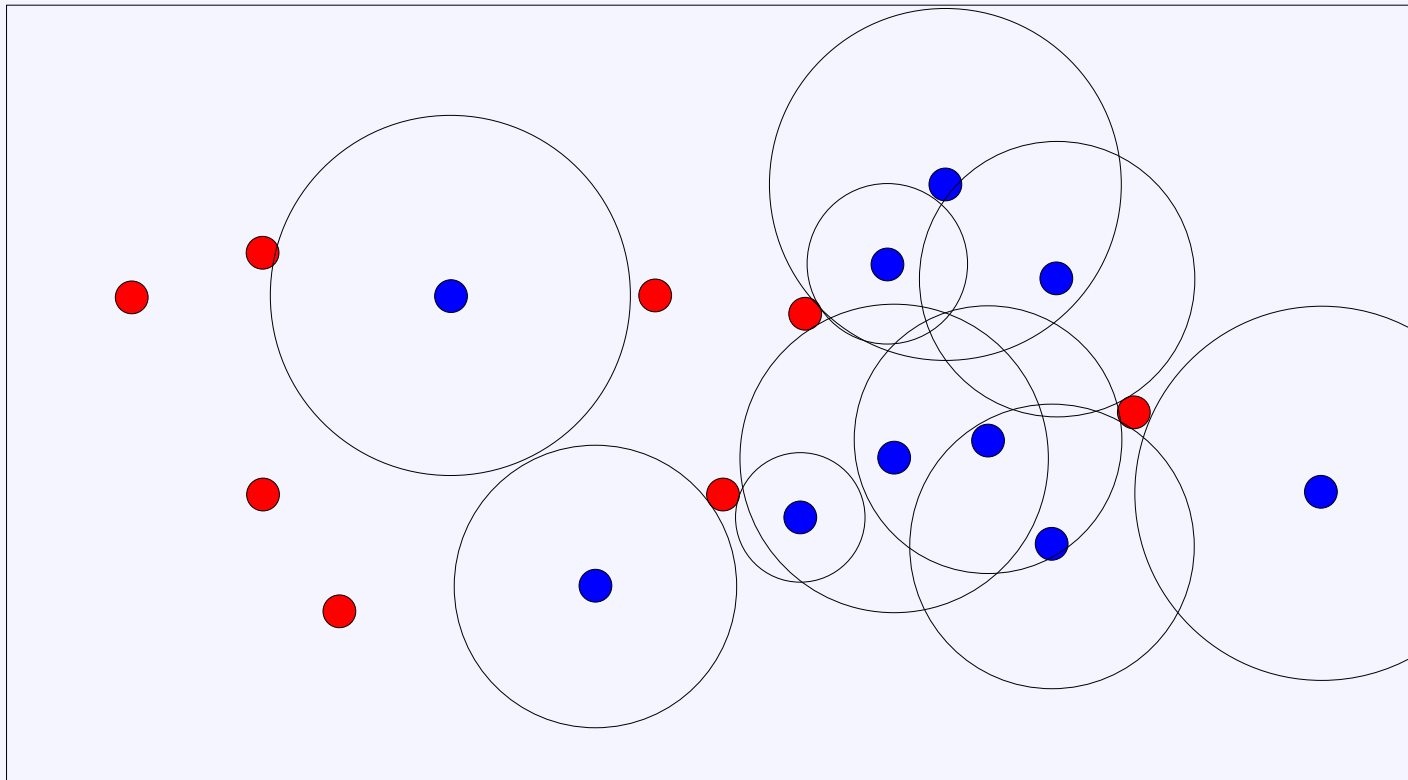
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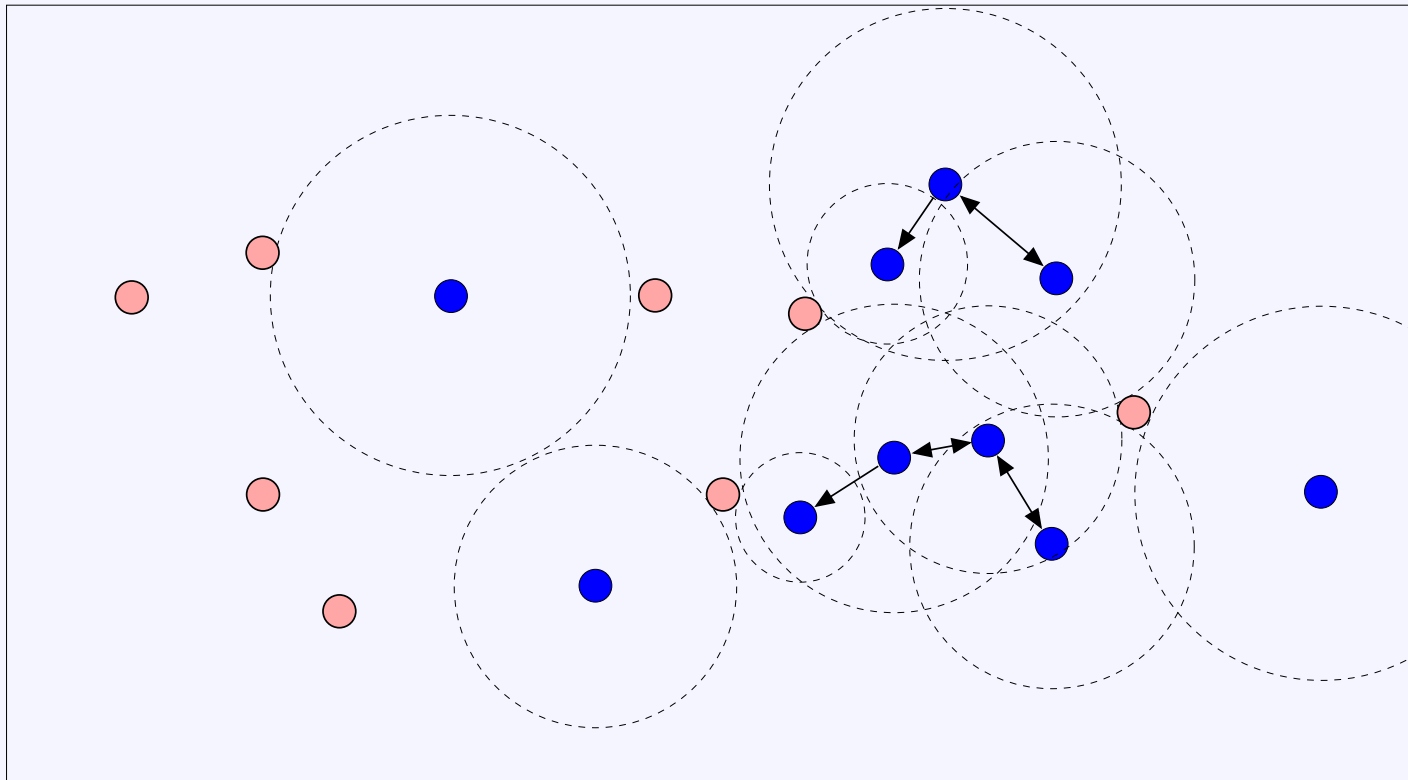
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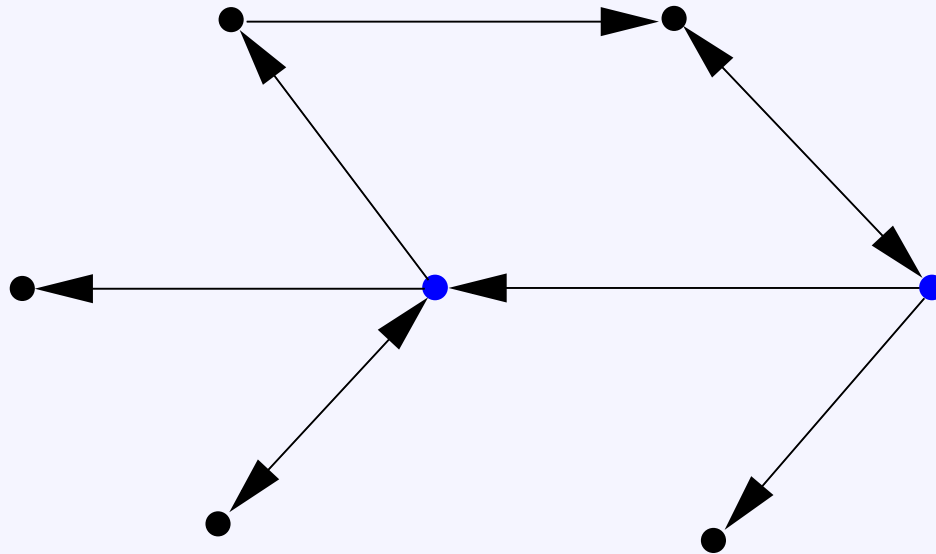
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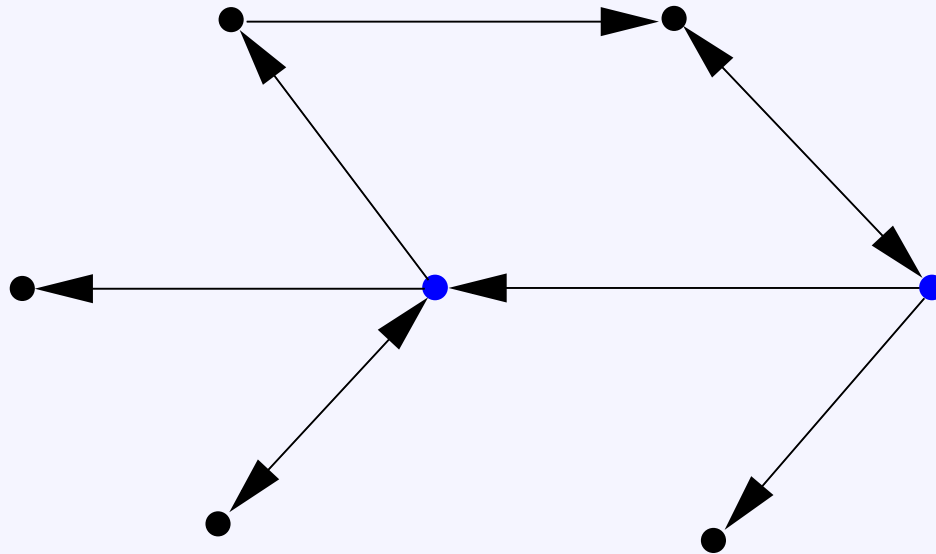
Dominating Sets

Define a *dominating set*, S , of a digraph $D = (V, A)$ as follows: $S \subset V$ such that $\forall v \in V$, $v \in S$ or $\exists w \in S$ such that $(w, v) \in A$.



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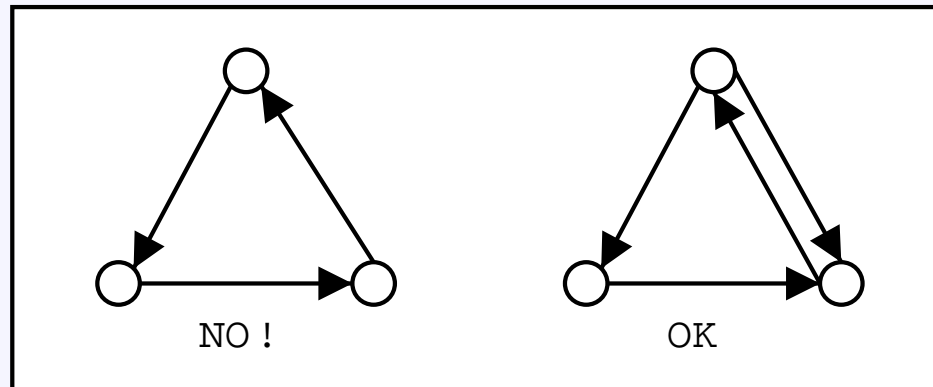


Solution to CCP \Leftrightarrow Minimum Size Dominating Set in CCCD

Euclidean CCCD's

If $X, Y \subset \mathbb{R}^q$ and d is the standard Euclidean metric, then we call the resulting digraph a Euclidean CCCD.

What digraphs are Euclidean CCCD's?



Definition: A simple cycle is a directed cycle with no bidirected edges.

Euclidean CCCD's

Theorem 1 *A digraph is a Euclidean CCCD if and only if it has no simple cycles.*

In other words. . .

Given $X, Y \in \mathbb{R}^q$, they induce a digraph with no simple cycles.

Given a digraph D with no simple cycles, we can find $X, Y \in \mathbb{R}^q$ which induce a digraph isomorphic to D (in fact we will need only one Y point, but we may need many dimensions).

Proof of characterization theorem

(\Leftarrow) Given D with a smallest simple cycle v_1, v_2, \dots, v_l .

- $(v_i, v_{i+1}) \in A$ and $(v_i, v_{i-1}) \notin A$ implies $d(x_i, x_{i-1}) > d(x_i, x_{i+1})$
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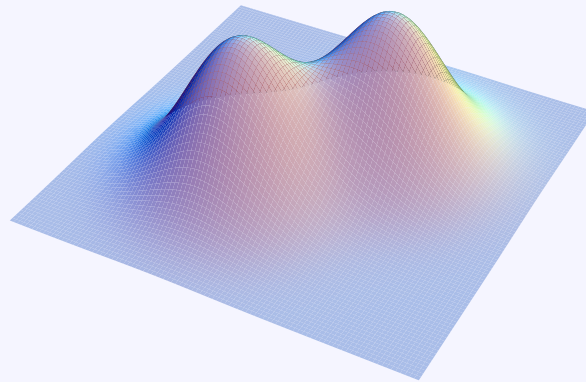
- Show Euclidean interpoint distances exist which preserve the linear order using MDS.

Random Model of CCP

n target class points are chosen from a distribution F_X

m non-target class points are chosen from a distribution F_Y

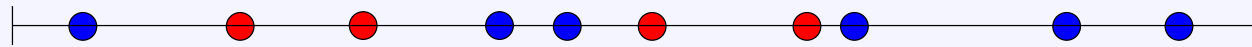
$\Gamma_{n,m} :=$ the domination number of the CCCD induced by the sample.



Question: What is the distribution of the random variable $\Gamma_{n,m}$?

Random Model in \mathcal{R}

Let F_X and F_Y be the uniform distribution on $[0, 1]$. In this case we use $\Gamma_{n,m}$ to represent the case with n target class points and m non-target class points.

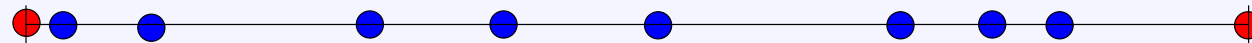


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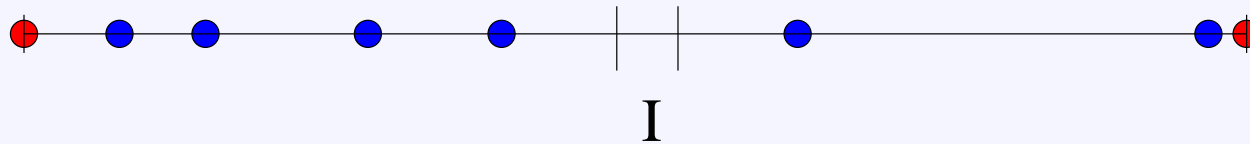


We need only consider the simplified case where $y_1 = 0, y_2 = 1$ and F_X is the uniform distribution on $[0, 1]$. We call such an induced CCCD C_n^* .



Random Model in \mathcal{R}

$$\Gamma(C_n^*) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } I \cap X \neq \emptyset \\ 2 & \text{otherwise} \end{cases}$$

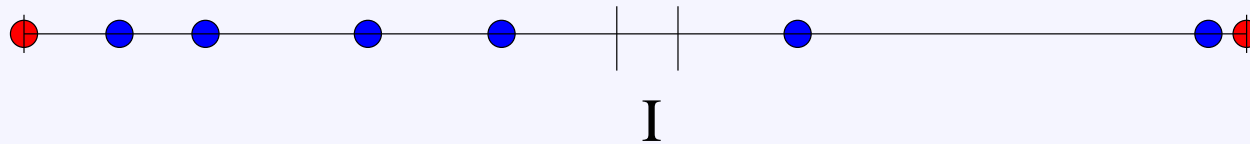


where $I = \left[\frac{x_n}{2}, \frac{1+x_1}{2} \right]$

$$\kappa(n) = P[\Gamma(C_n^*) = 1] = \frac{5}{9} + \frac{4}{9} \cdot 4^{-(n-1)}$$

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This result allows the calculation of...

Random Model in \mathcal{R}

... probability mass function for $\Gamma_{n,m}$

$$P[\Gamma_{n,m} = d] =$$

$$\frac{n!m!}{(n+m)!} \sum_{\vec{n} \in \Delta_{n,m+1}^{Z_{n+1}}} \sum_{\vec{d} \in \Delta_{d,m+1}^{Z_3}} \alpha(d_1, n_1) \cdot \alpha(d_{m+1}, n_{m+1}) \prod_{j=2}^m \beta(d_j, n_j)$$

and...

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and... expected value of $\Gamma_{n,m}$,

$$E[\Gamma_{n,m}] = \frac{2n}{n+m} + \frac{n!m(m-1)}{(n+m)!} \sum_{i=1}^n \frac{(n+m-i-1)!}{(n-i)!} \cdot (2 - \kappa(i))$$

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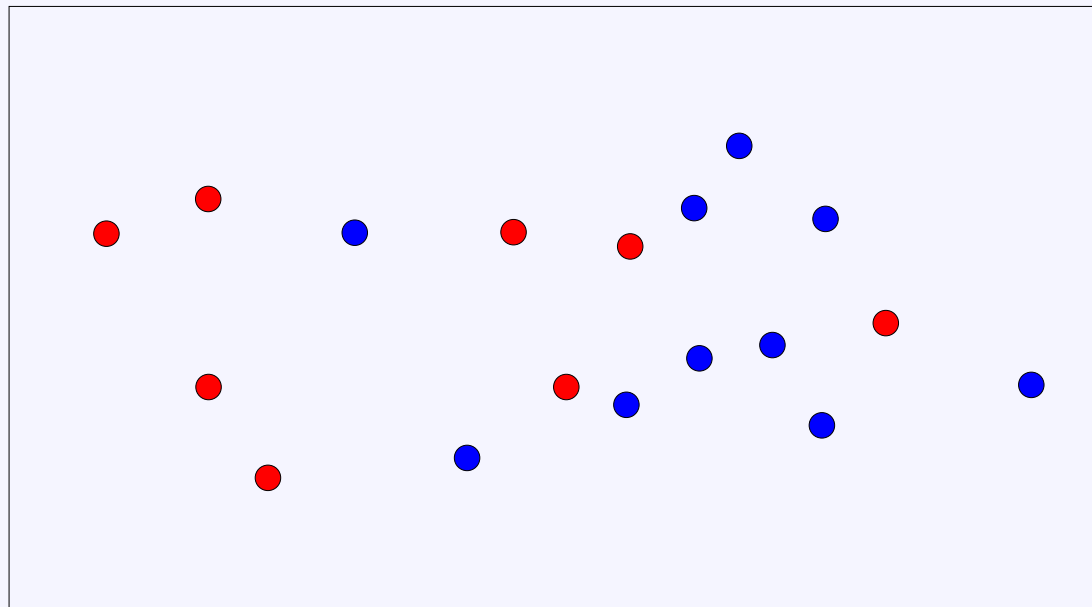
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and... almost sure limit of $\Gamma_{n,m}$,

$$\lim_{n \rightarrow \infty} \frac{\Gamma_{\lfloor an \rfloor, n}}{n} = \frac{a(13a+12)}{3(a+1)(3a+4)} \text{ a.s. } \quad a \in (0, \infty).$$

Application to Classification

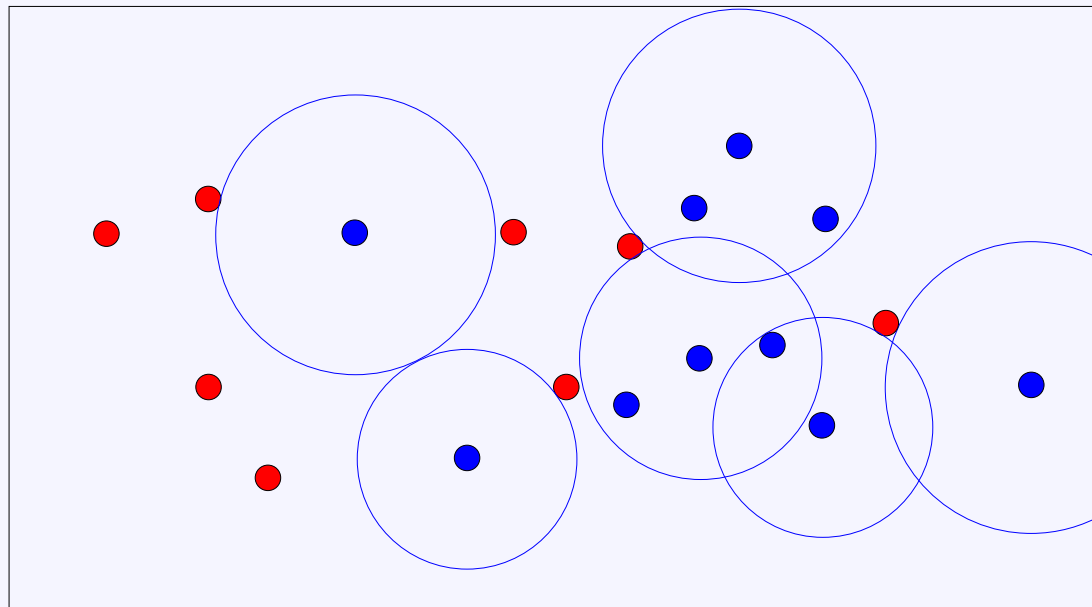
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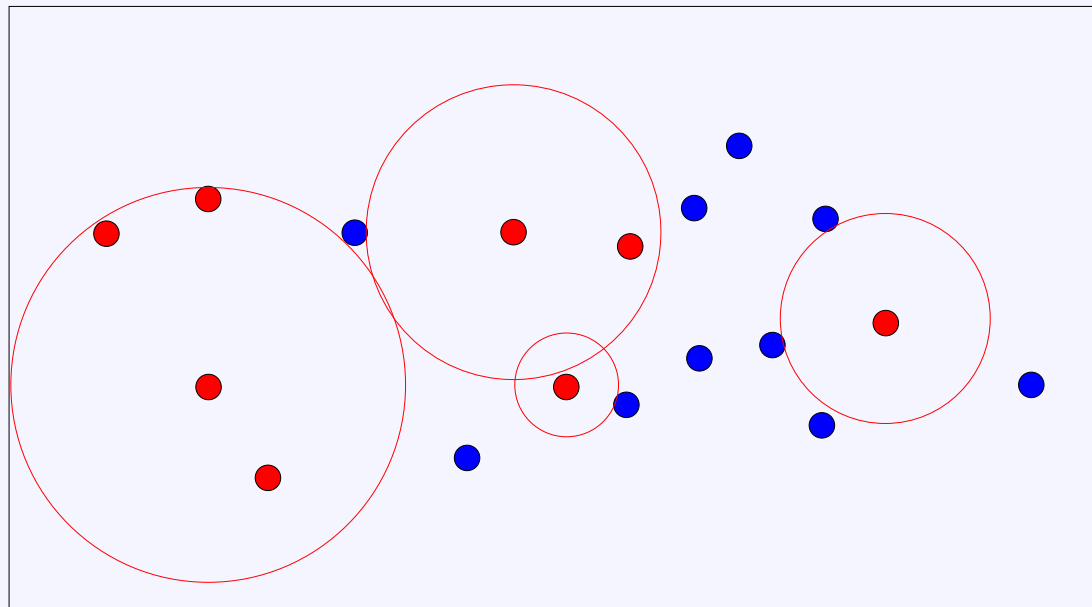
Find a cover C_X for X .



Application to Classification

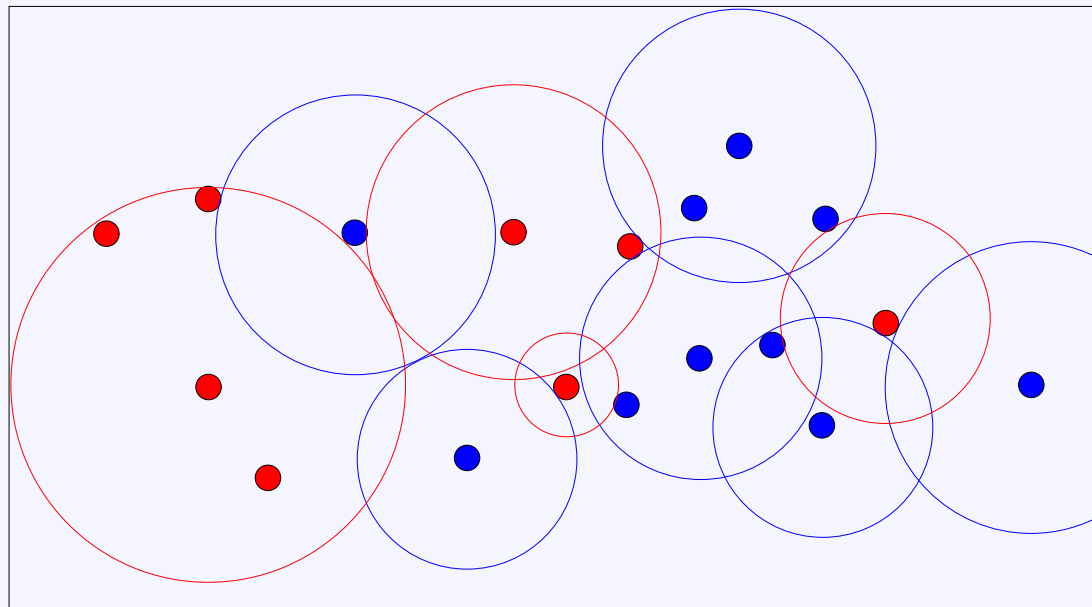
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Find a cover C_Y for Y .



Application to Classification

Our framework for using the CCP in classification is a reduced nearest neighbor with a dissimilarity dependent on the balls in the classifier.



Create a dissimilarity function $\rho(z, C)$ to measure the dissimilarity between a point $z \in \Omega$ and a cover C .

Naive or Pre-classifier

Given training data $X, Y \subset \Omega$ and covers C_X, C_Y . Define

$$\rho(z, C) := 1 - I\{z \in C\}$$

Using the reduced nearest neighbor framework,

$$g(z) := \begin{cases} 1 & : \rho(z, C_1) < \rho(z, C_2) \\ 2 & : \rho(z, C_1) > \rho(z, C_2) \\ 0 & : \text{otherwise} \end{cases}$$

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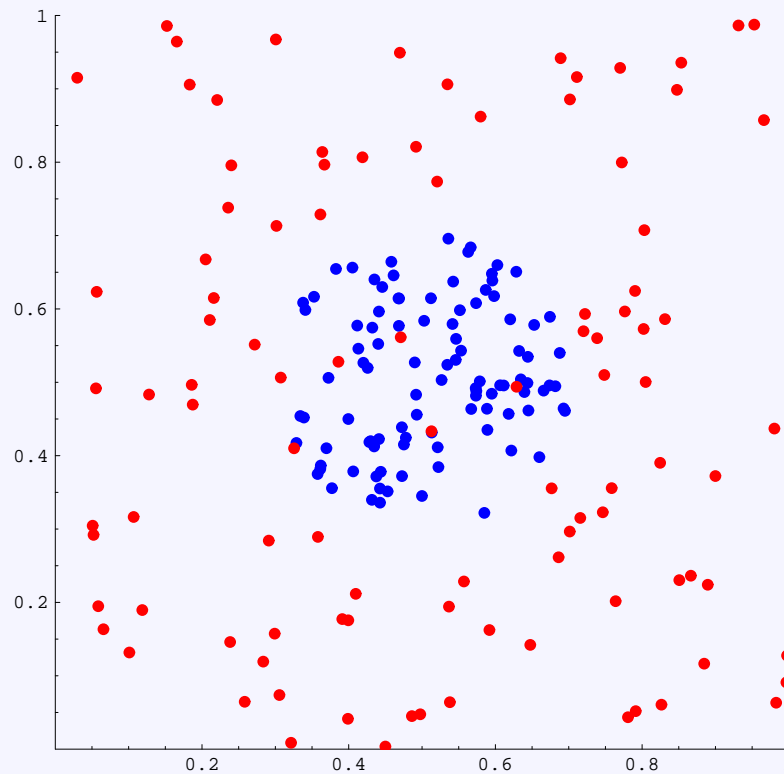
or more simply,

$$g(z) := \begin{cases} 1 & : z \in C_1 \cap C_2^c \\ 2 & : z \in C_2 \cap C_1^c \\ 0 & : \text{otherwise} \end{cases}$$

Simulation: Disk

100 Red Points $\sim U([0, 1] \times [0, 1])$.

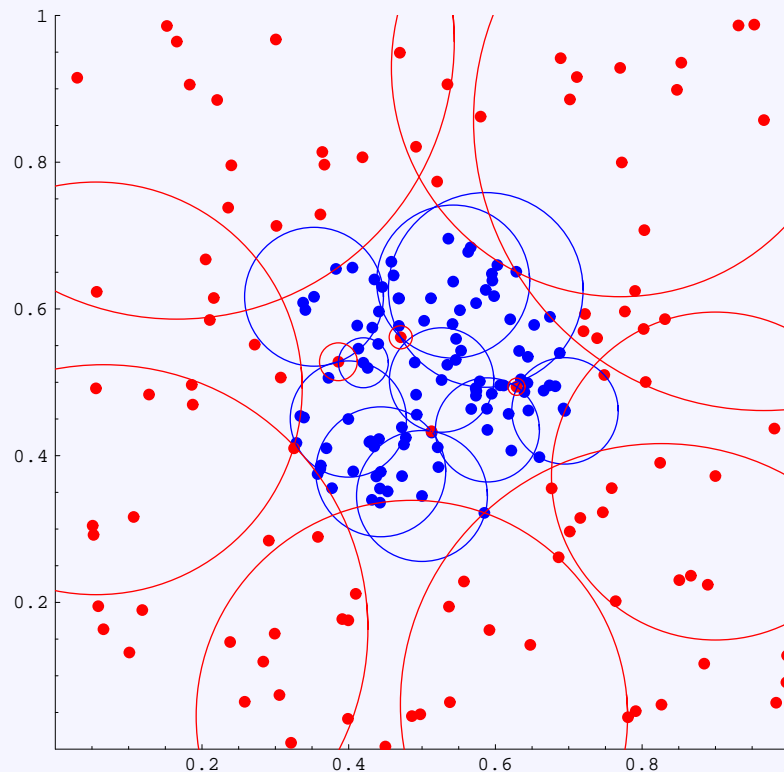
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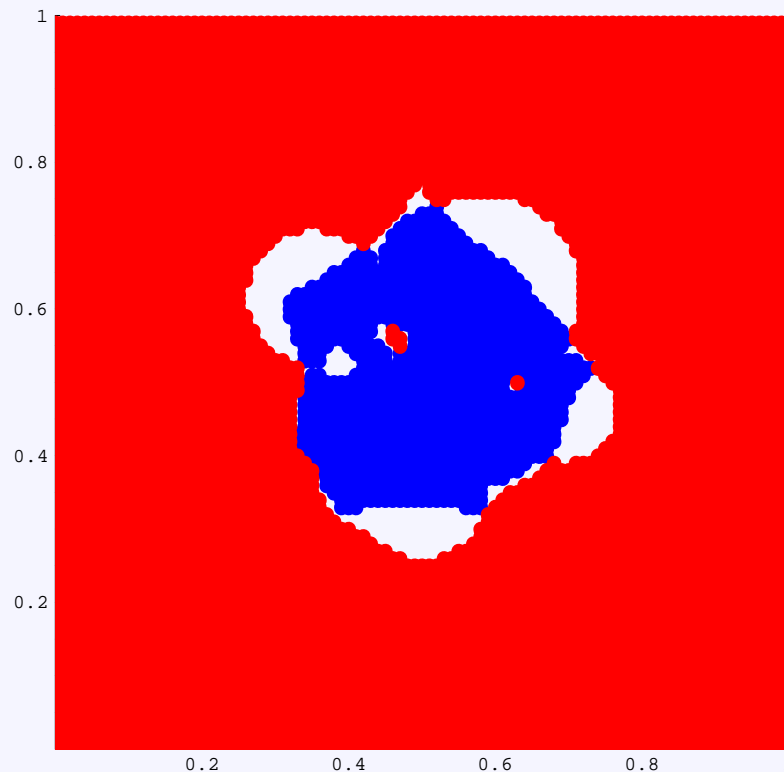


Drawbacks of Naive Classifier - overfitting

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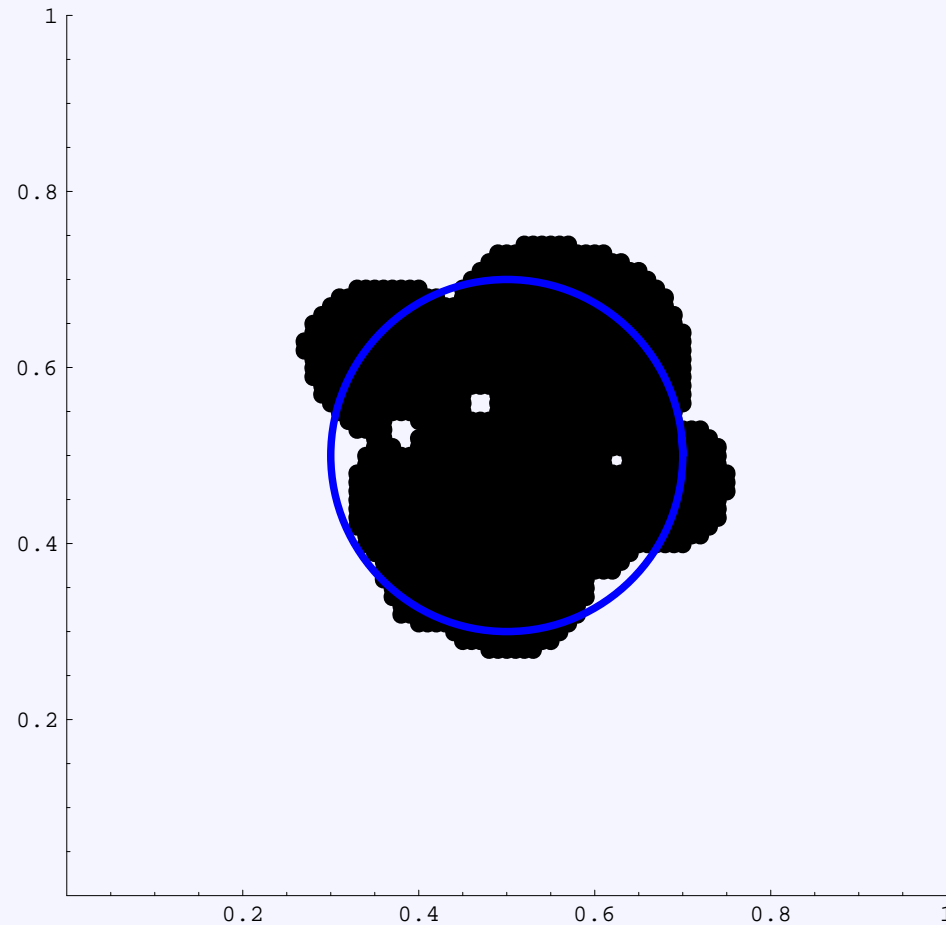
Drawbacks of Naive Classifier - overfitting and “no decision” regions.

Scaled Dissimilarity Function

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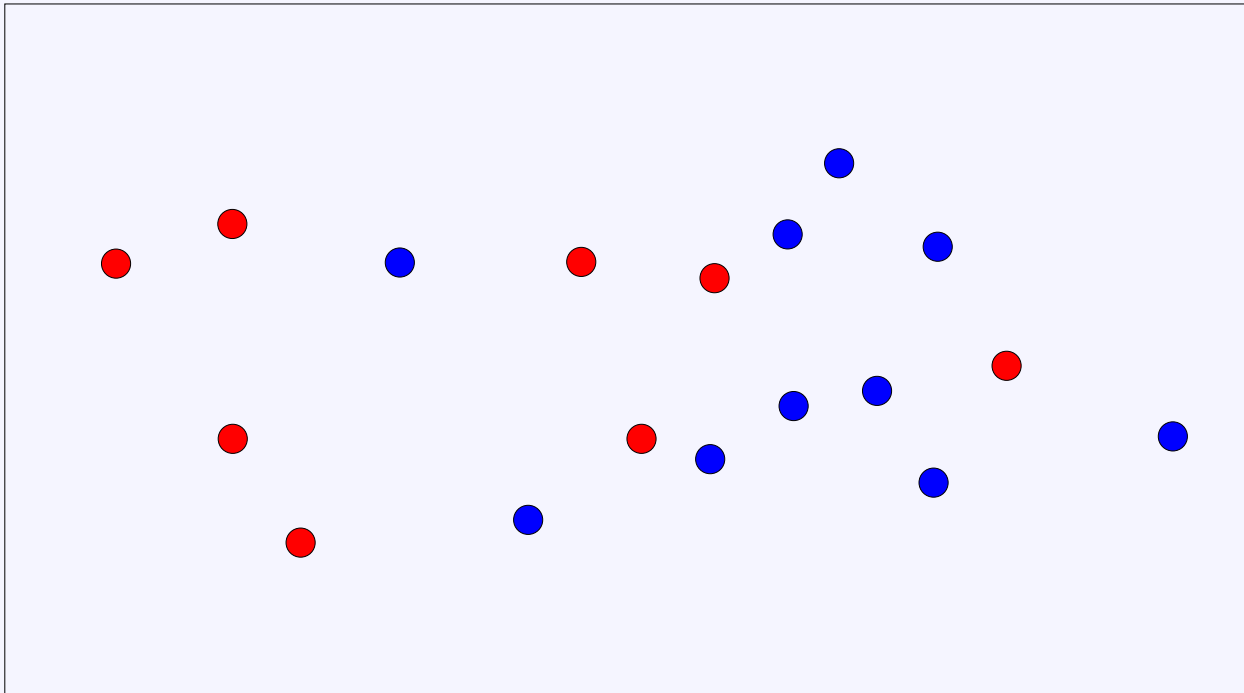
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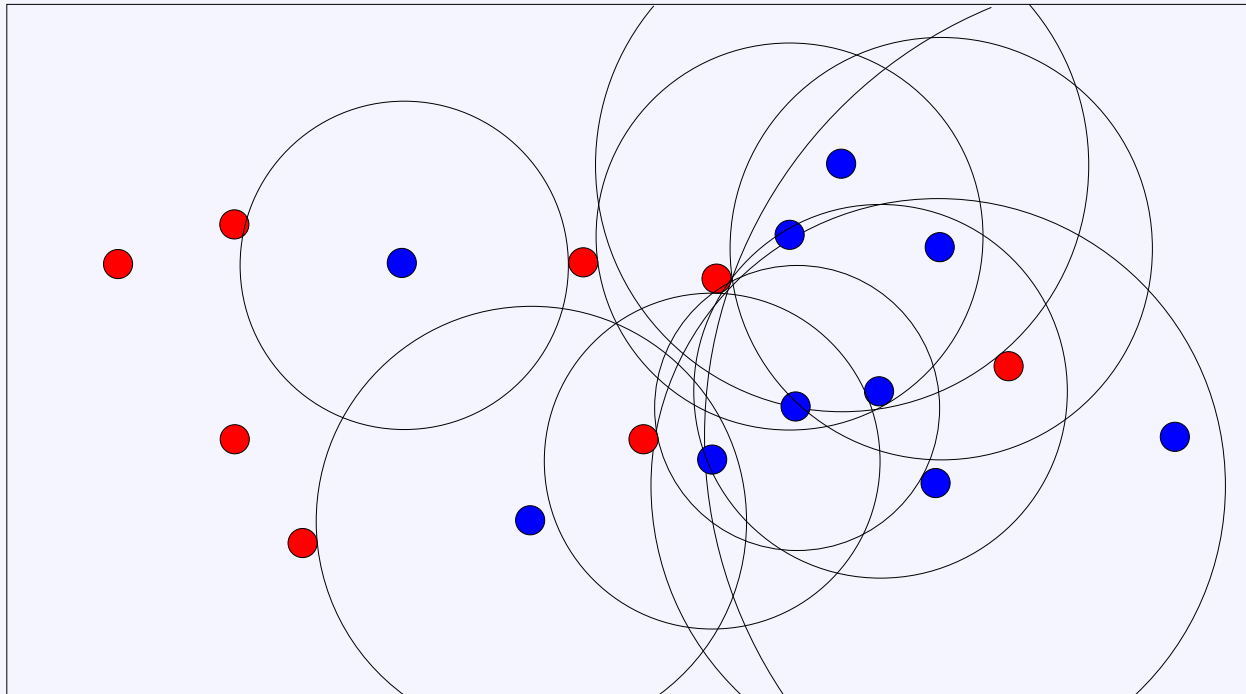
α, β CCP

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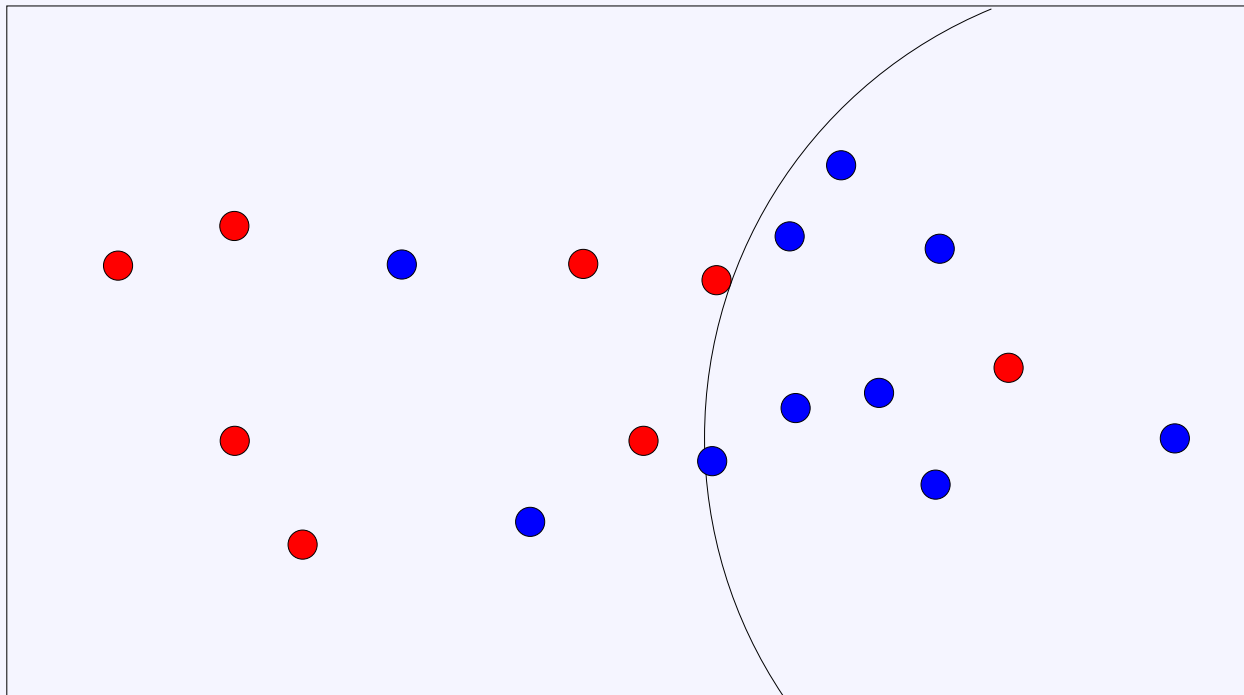
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$CCP(\alpha, \beta) :=$ the CCP where each ball may cover β of non-target class points and the cover may “miss” at most α of the target class points.

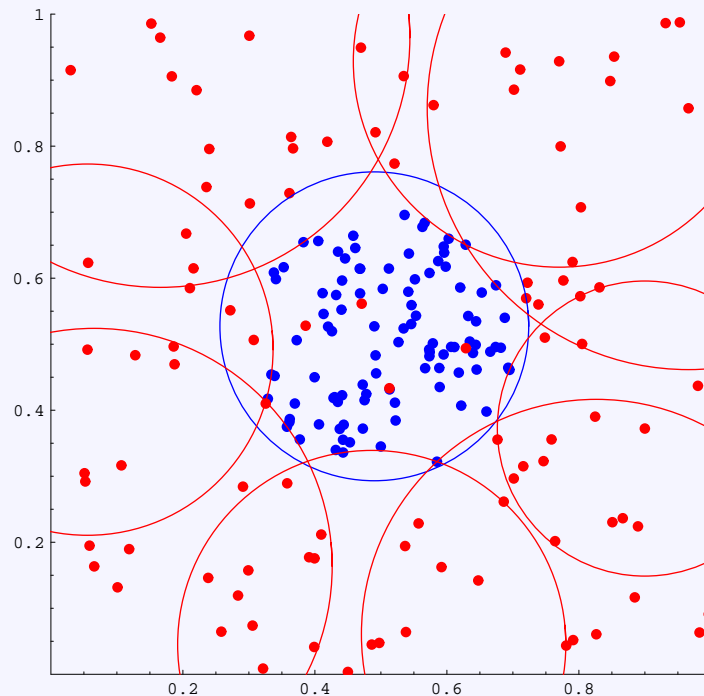


$$\beta = 1 \quad \alpha = 2$$

α, β CCP

Benefits of α, β CCP.

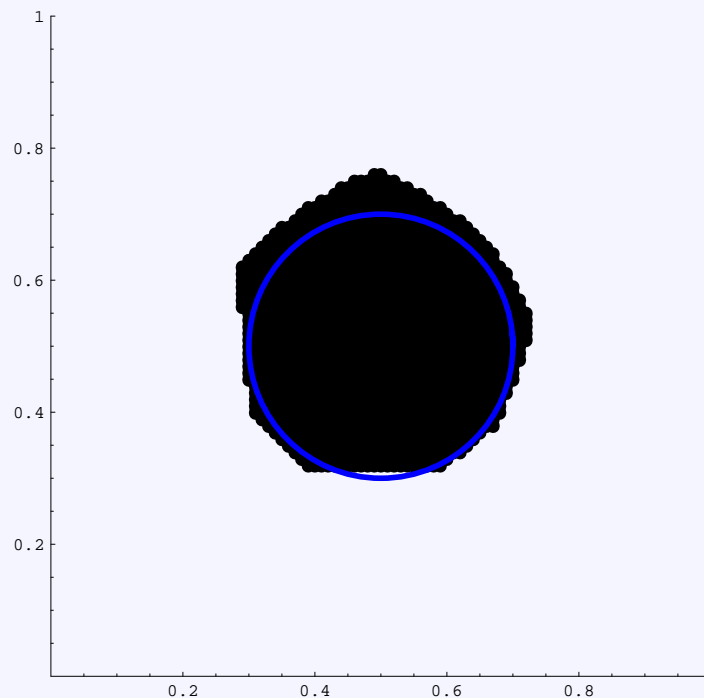
- α parameter allows us to “ignore” outlying target class points. Moving toward modeling the discriminant region.
- β parameter allows us to “ignore” outlying non-target class points.



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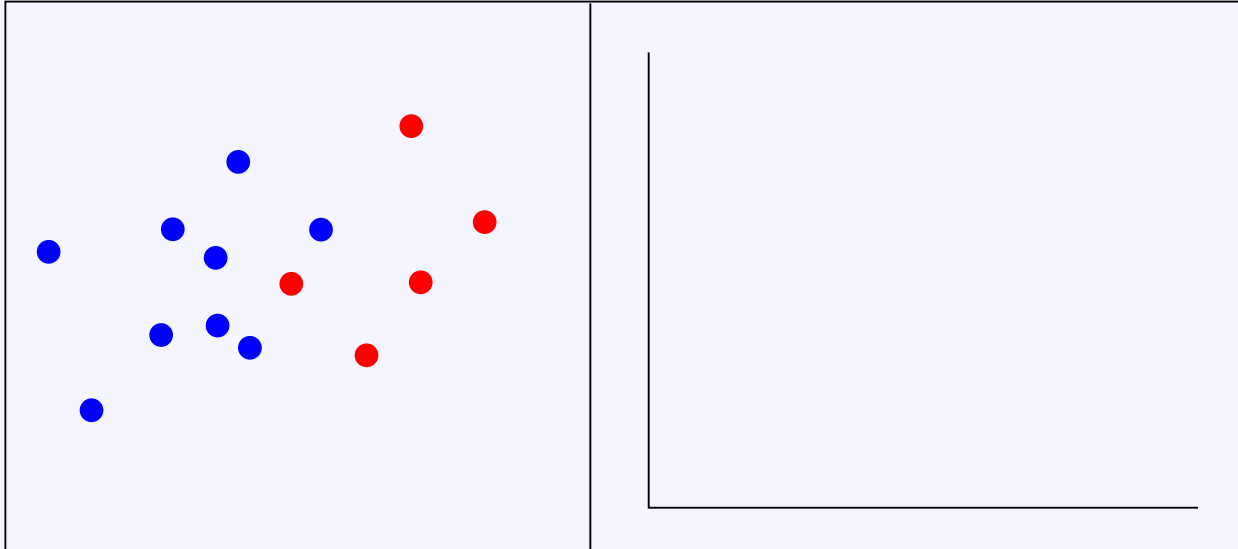


Disadvantages of α, β CCP.

- User must choose parameters α, β for each class.
- β parameters are global. We would only like balls to cover non-target class points in the discriminant region of the target class.

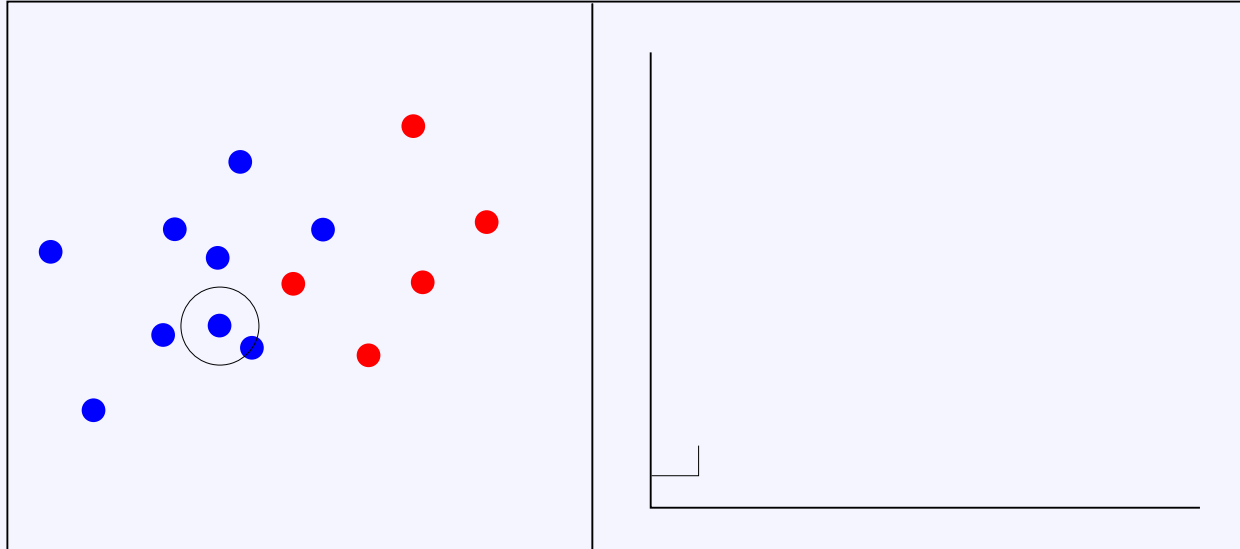
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



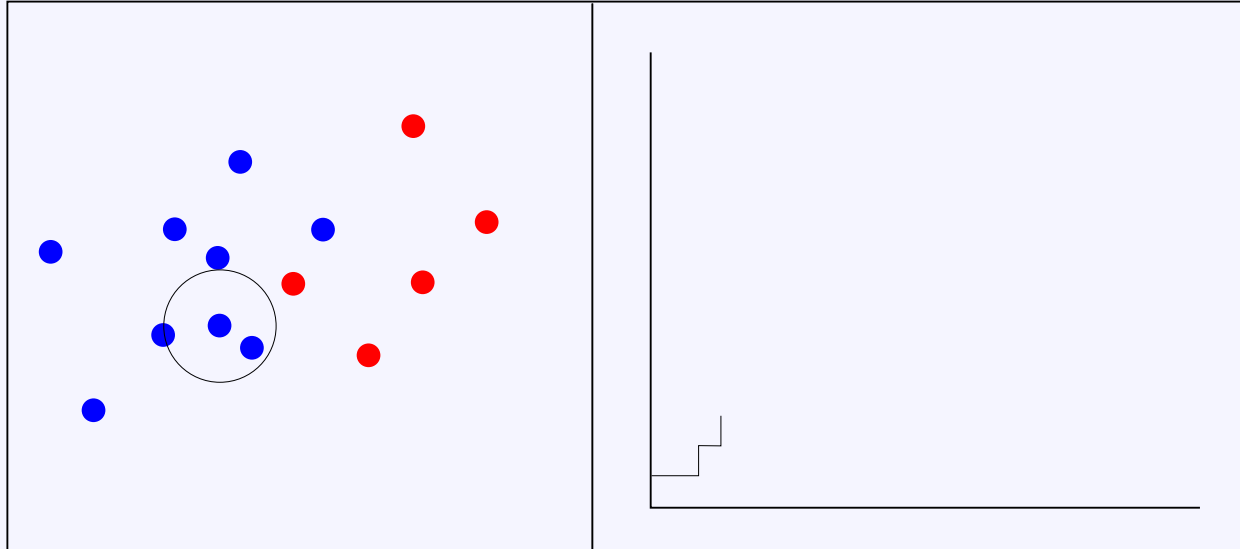
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



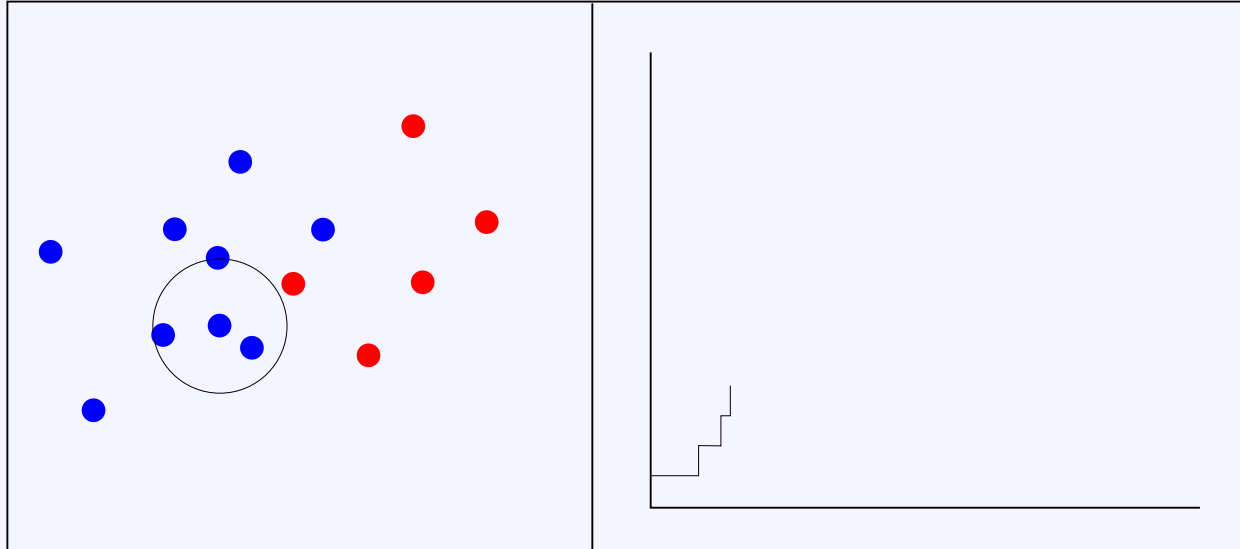
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



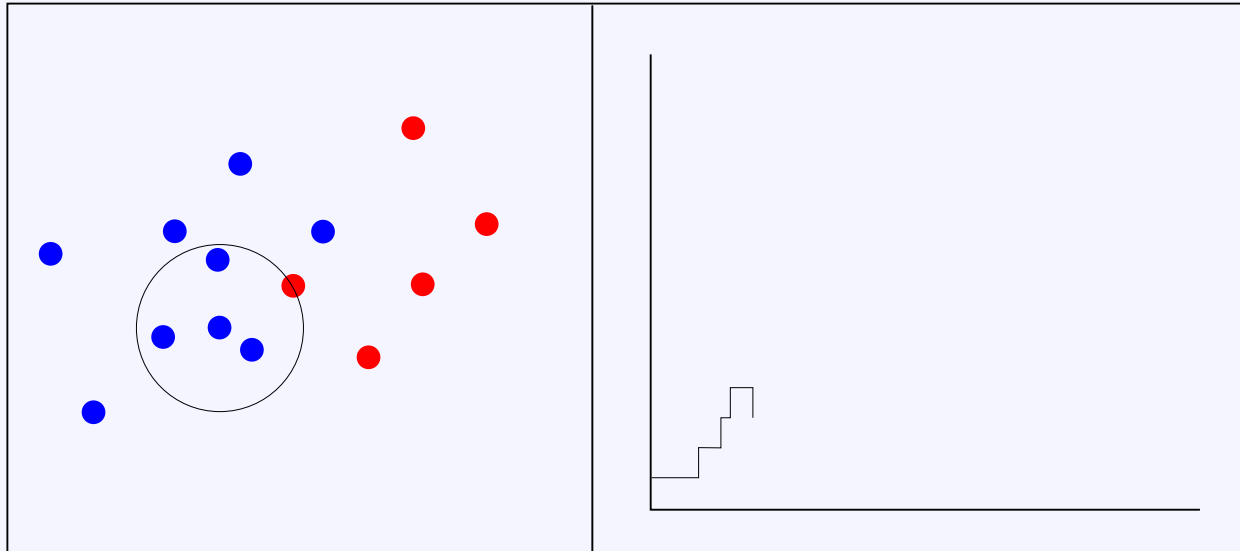
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



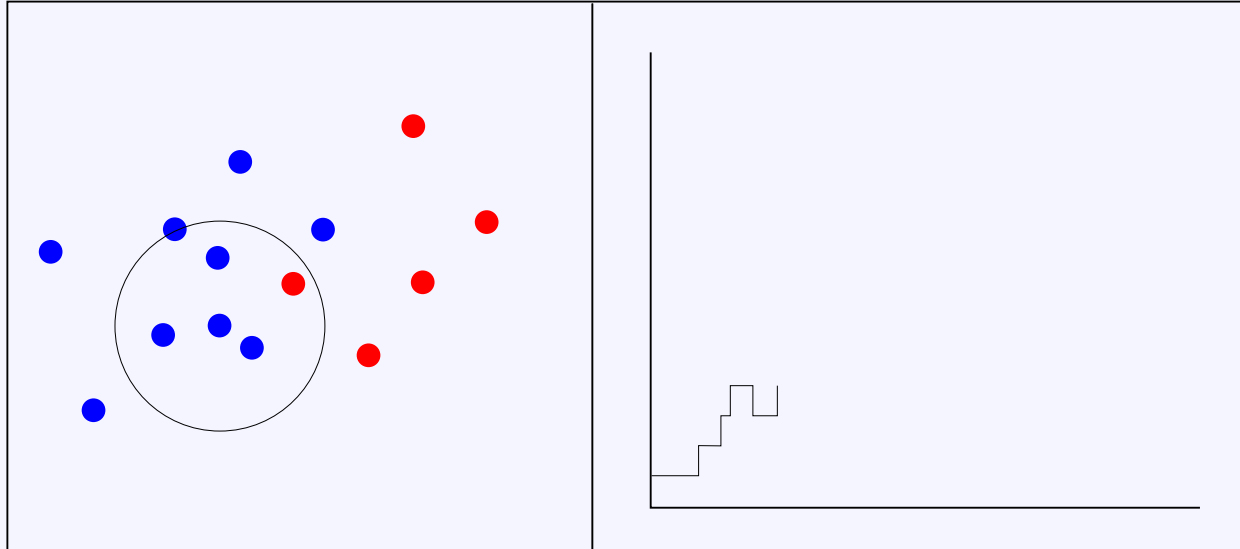
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



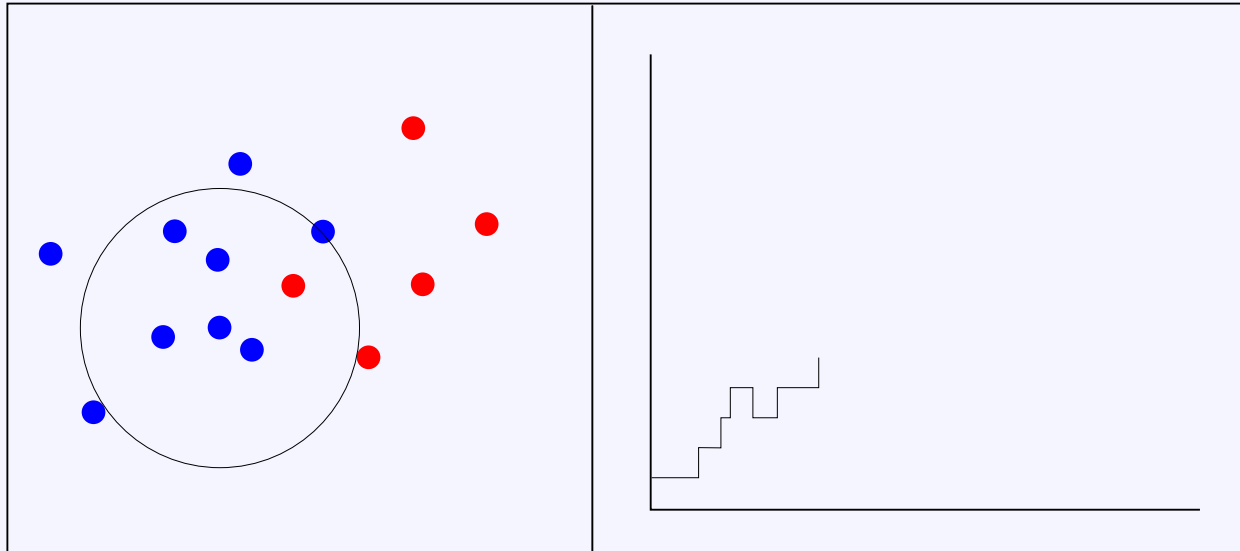
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



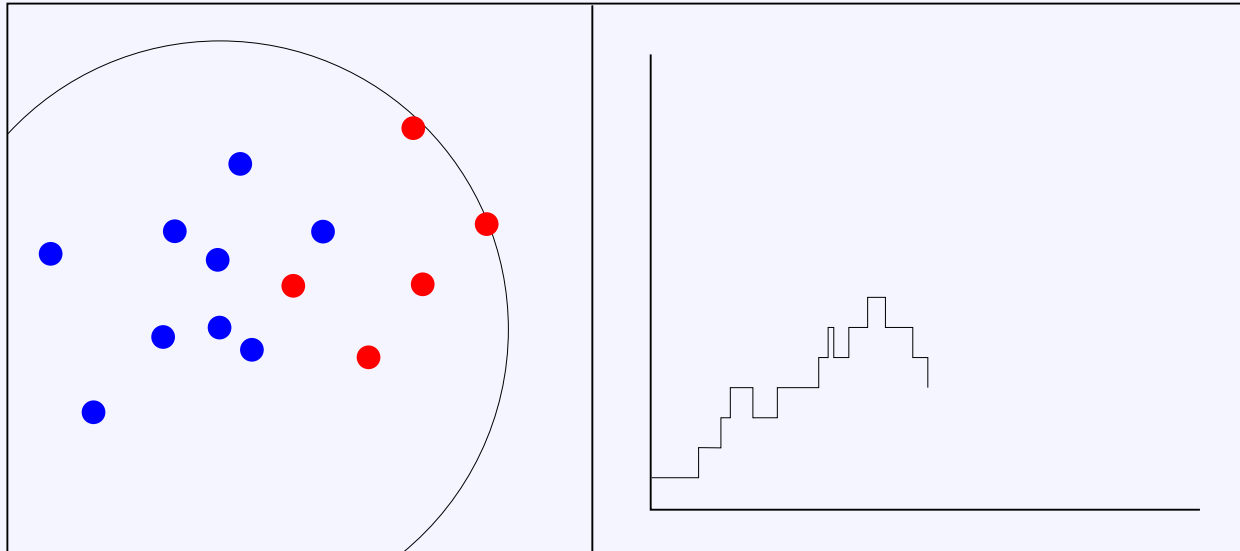
Random Walk CCP

Idea: Let each point choose the radius of its covering ball.



Random Walk CCP

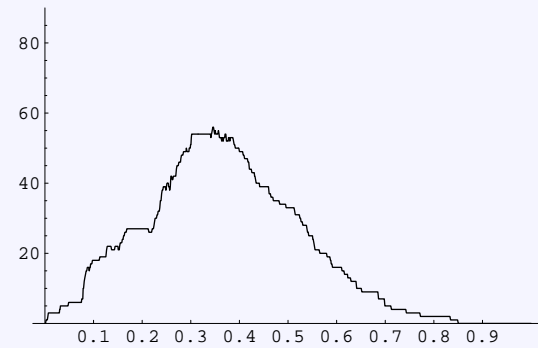
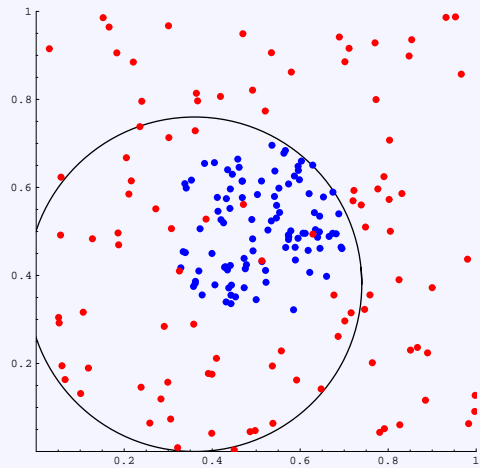
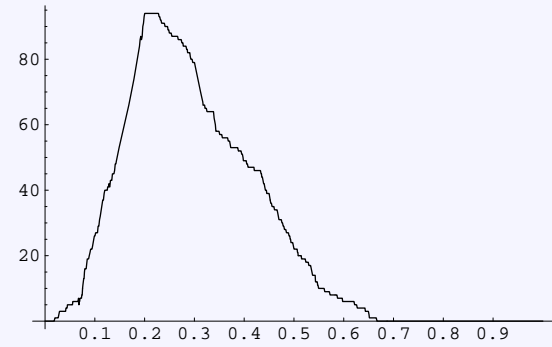
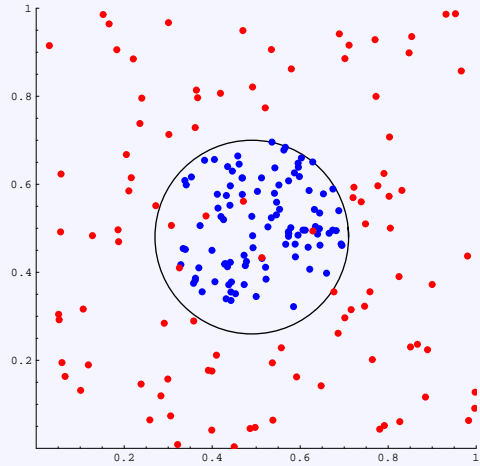
Idea: Let each point choose the radius of its covering ball.



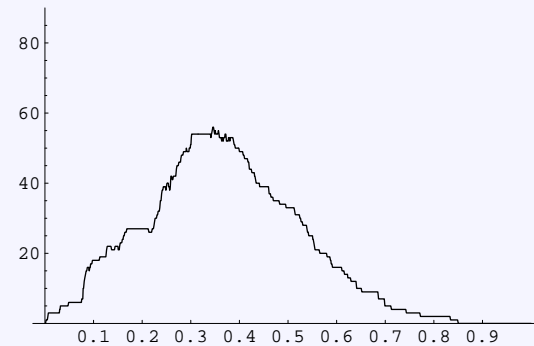
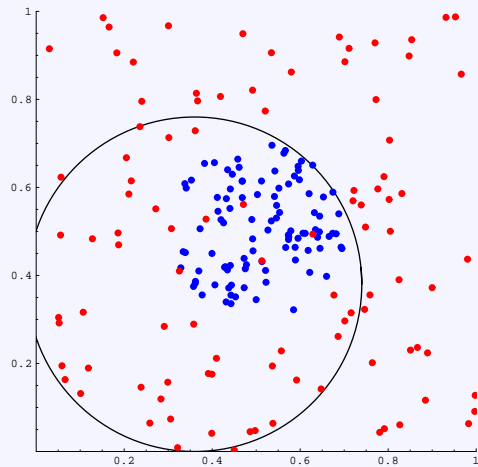
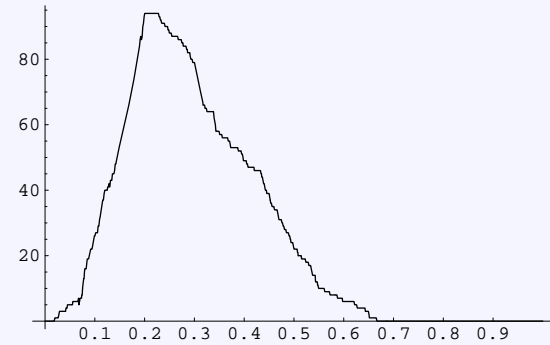
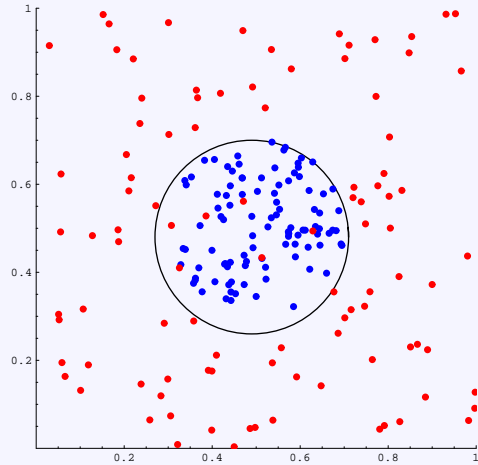
We choose the radius according to:

$$r^* = \operatorname{argmax}_{r \geq 0} \{RW(r) - f(r)\}$$

Random Walk CCP



Random Walk CCP



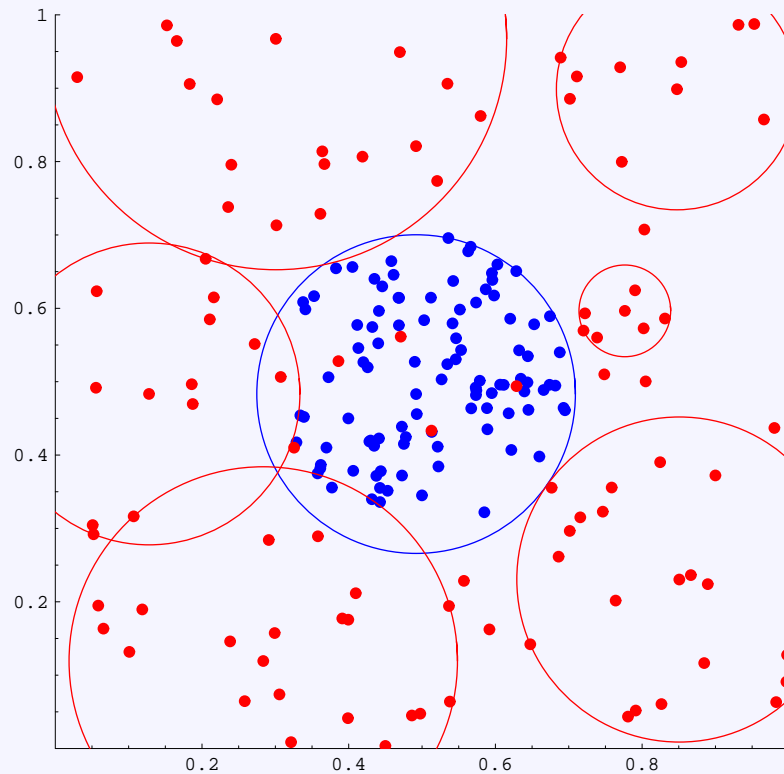
We will need the concept of a score for a covering ball.

Random Walk CCP

We want to choose the cover that is made up of the “best” or highest scoring covering balls instead of the fewest number of balls.

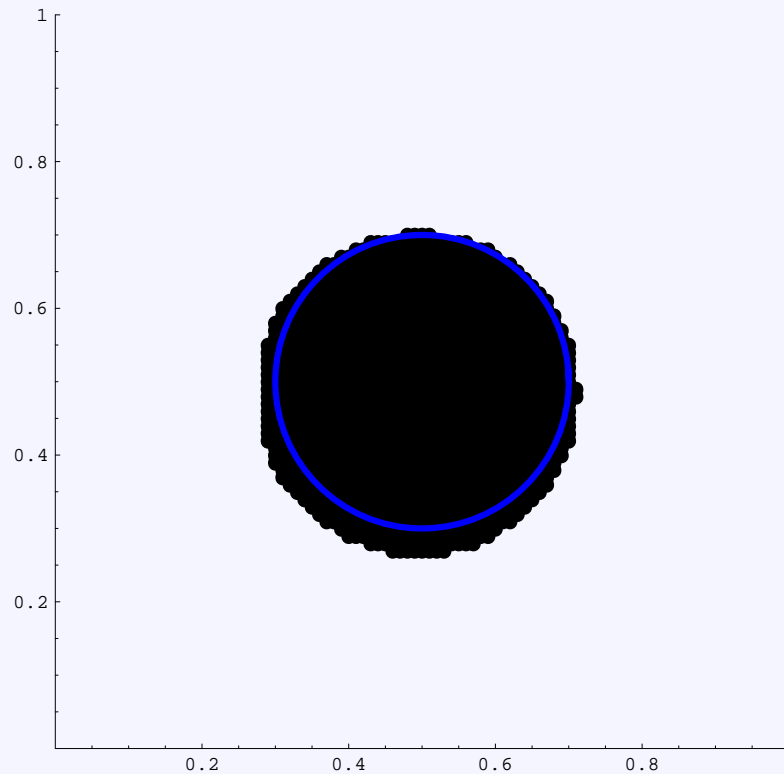
Random Walk CCP

We want to choose the cover that is made up of the “best” or highest scoring covering balls instead of the fewest number of balls.



Random Walk CCP

We want to choose the cover that is made up of the “best” or highest scoring covering balls instead of the fewest number of balls.



Random Walk CCP

Benefits of Random Walk.

- Same motivation as α, β version, but adaptively chooses the α, β parameters for each ball.
- Each covering ball for a class lies approximately in the discriminant region of that class.
- Covers are less complex than pre-classifier and α, β classifiers.

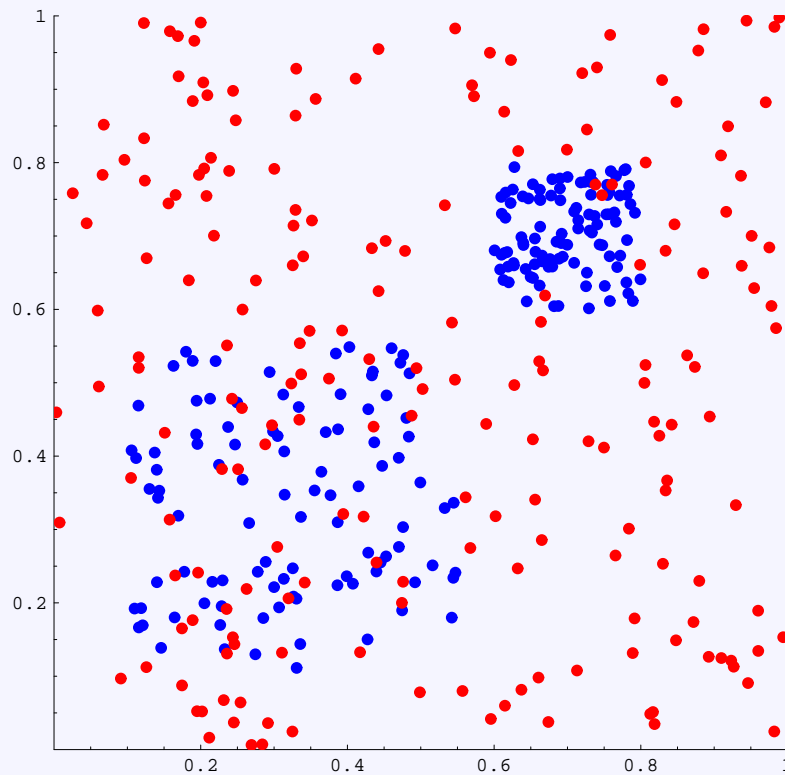
For Disk data,

- Pre-classifier - $\Gamma_X = 10, \Gamma_Y = 12$
- α, β classifier - $\Gamma_X = 1, \Gamma_Y = 8$
- Random Walk Classifier - $\Gamma_X = 1, \Gamma_Y = 6$

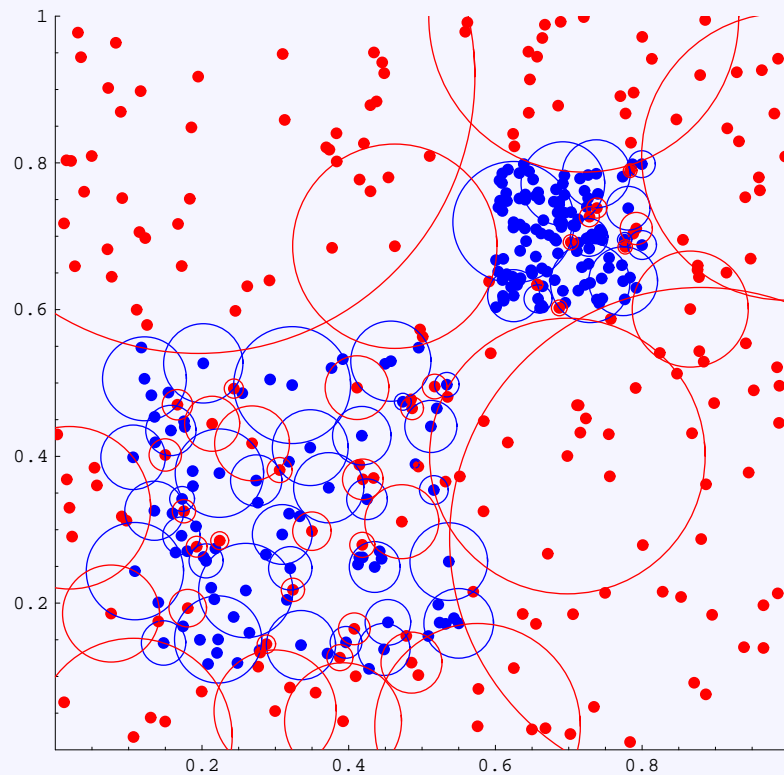
Simulation: Data

Red Points $\sim U([0, 1] \times [0, 1])$.

Blue Points $\sim \frac{1}{2}U([0.1, 0.55] \times [0.1, 0.55]) + \frac{1}{2}U([0.6, 0.8] \times [0.6, 0.8])$



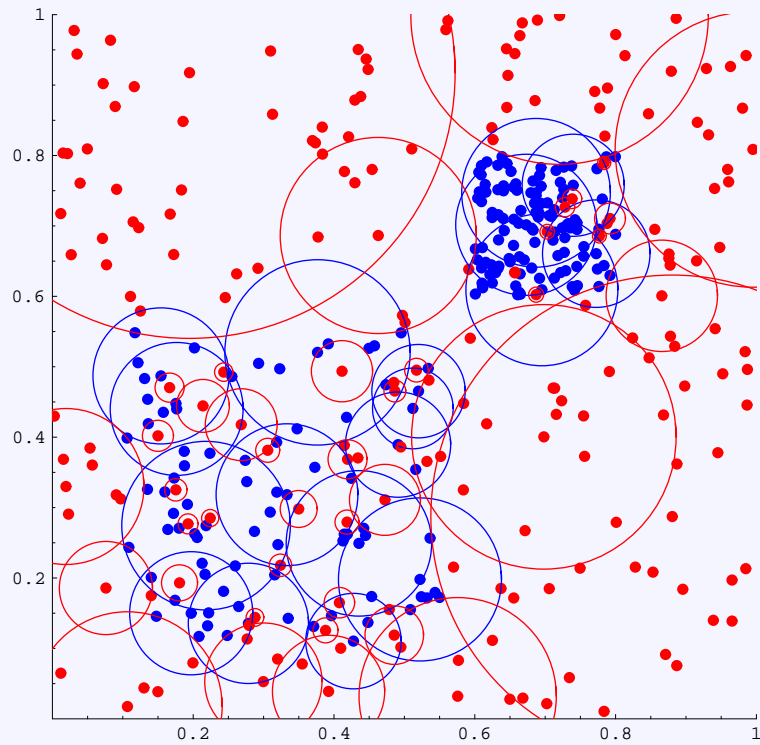
Simulation: Covers



Pure Proper Cover

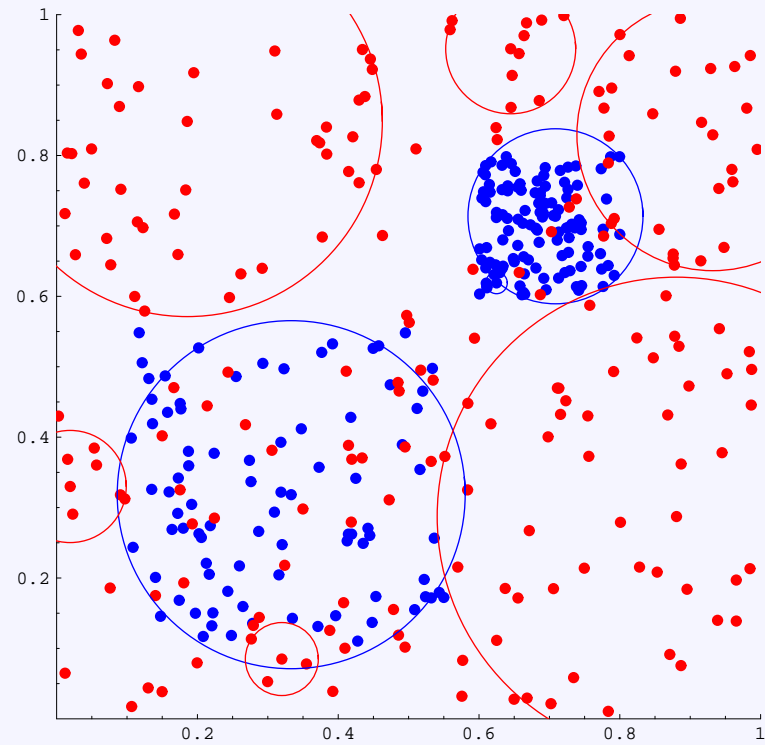
$$\Gamma_X = 46 \quad \Gamma_Y = 44$$

Simulation: Covers



α, β Cover

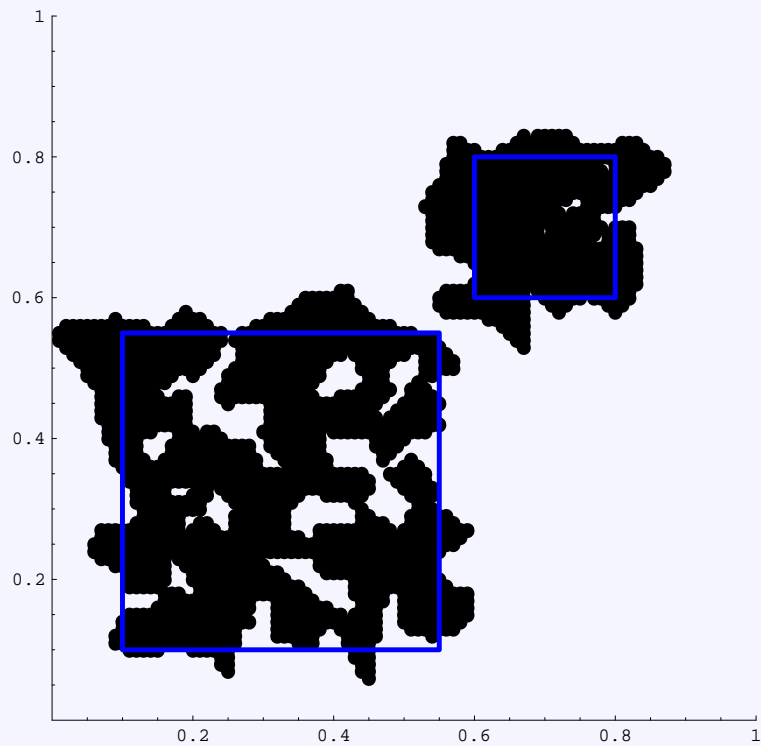
$$\Gamma_X = 14 \quad \Gamma_Y = 44$$



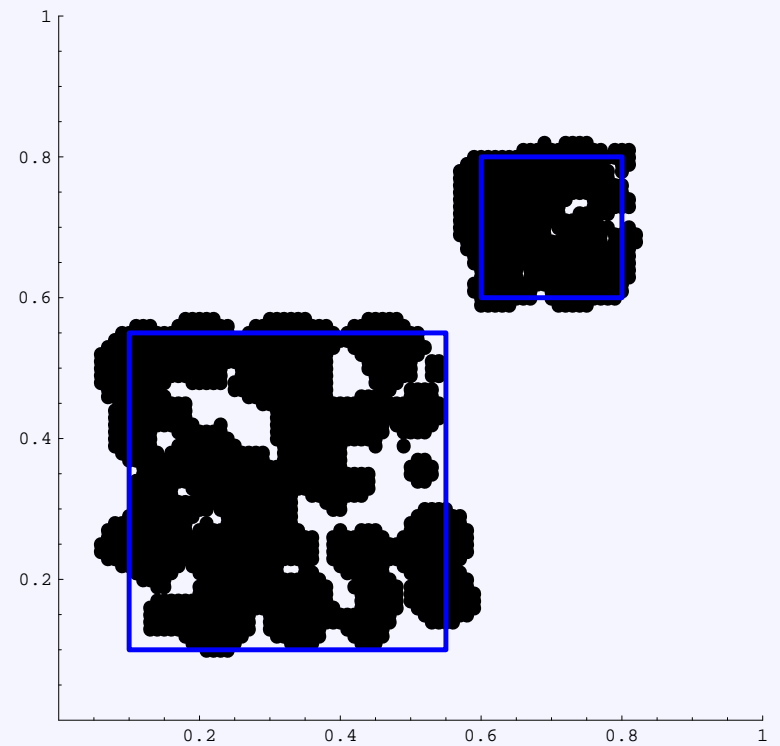
RW Cover

$$\Gamma_X = 3 \quad \Gamma_Y = 6$$

Simulation: Classifier Results

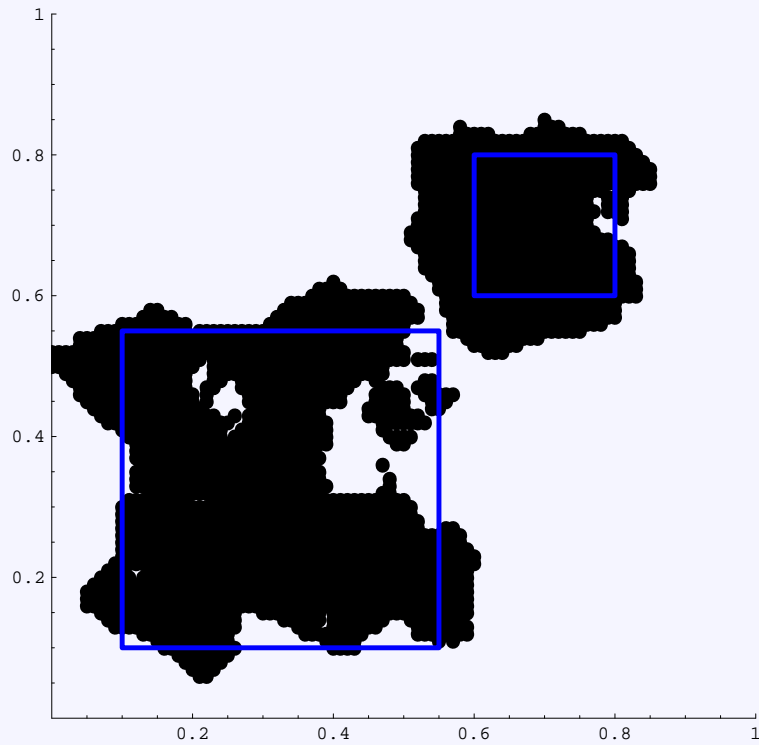


NN

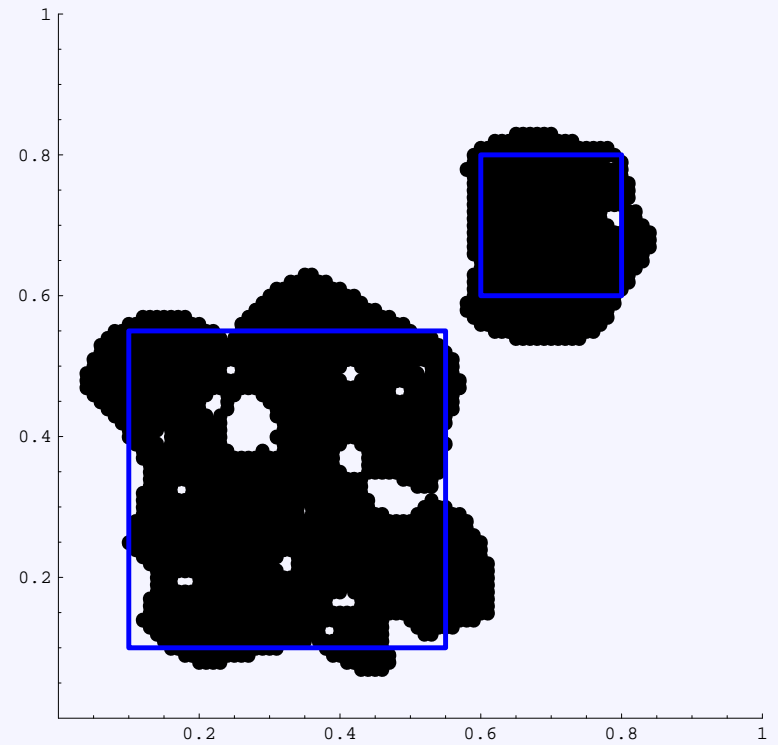


Naive CCP

Simulation: Classifier Results

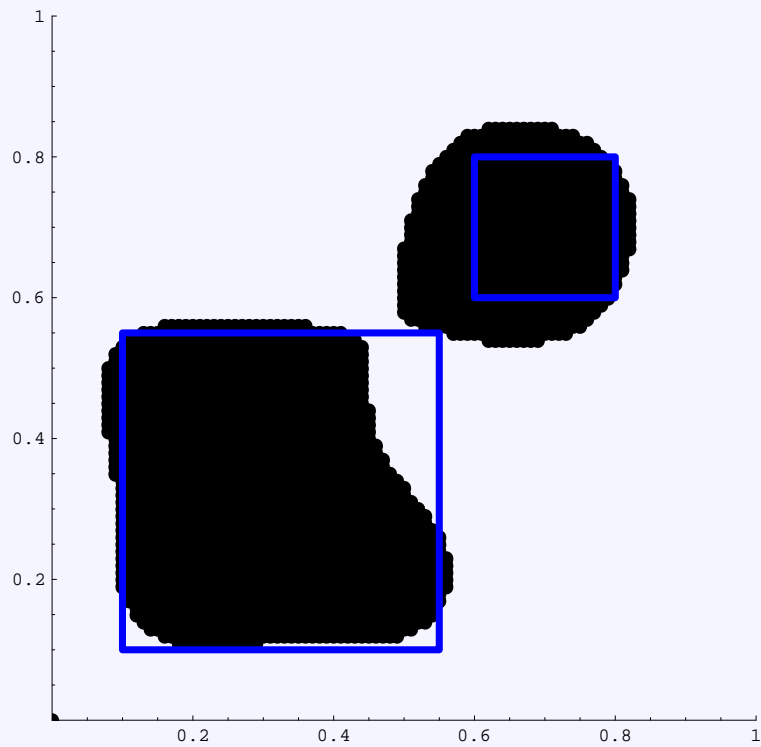


k-NN

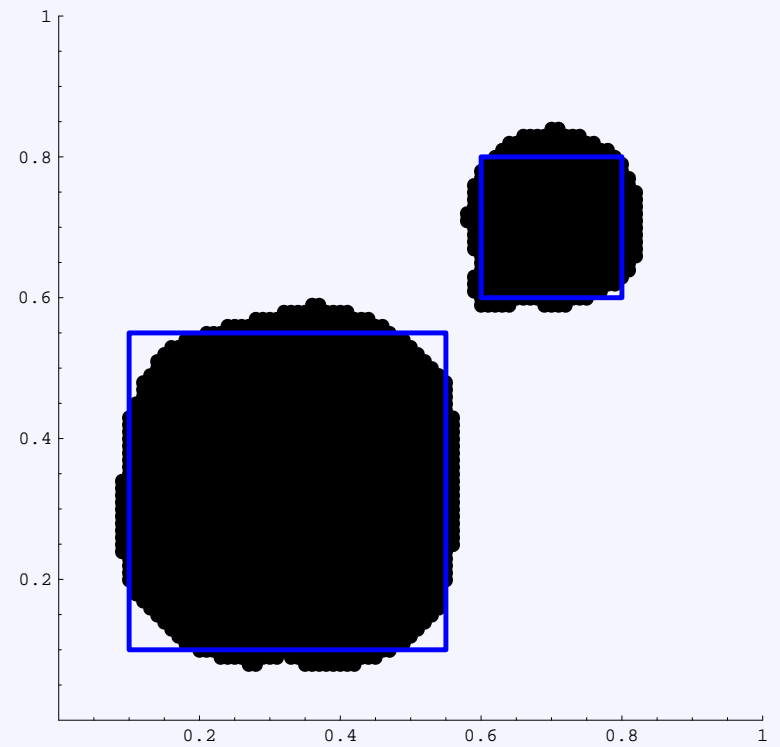


α, β CCP

Simulation: Classifier Results



SVM



RW-CCP

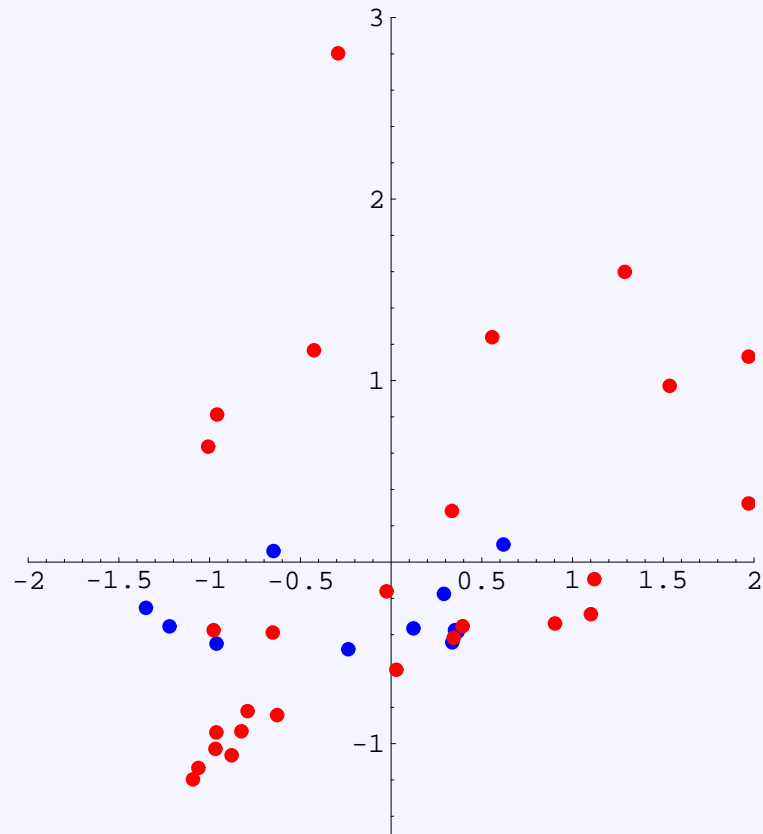
Simulation: Classifier Results

Here are the estimates for the misclassification rate \hat{L} as observed after 1000 trials.

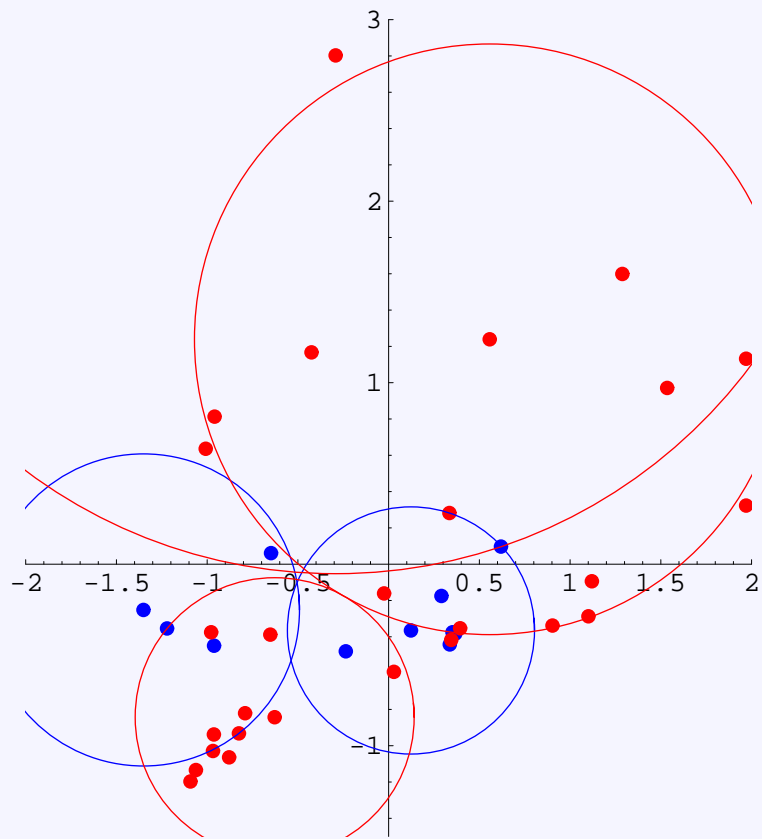
Tr. Size	NN	k-NN	SVM	Nv. CCP	α, β CCP	RW-CCP
50	0.242	.0.240	0.201	0.228	0.212	0.212
100	0.224	0.212	0.184	0.213	0.190	0.183
200	0.210	0.188	0.168	0.201	0.171	0.165
500	0.199	0.166	0.152	0.194	0.154	0.153

Experiment: Minefield Data

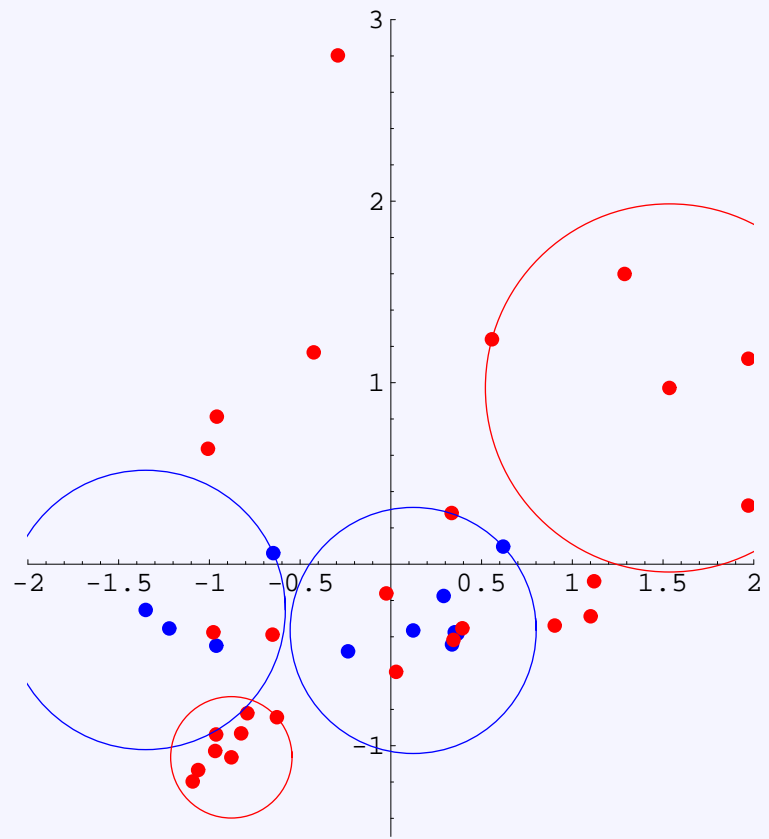
39 observations of multispectral images in a minefield.



Experiment: Covers

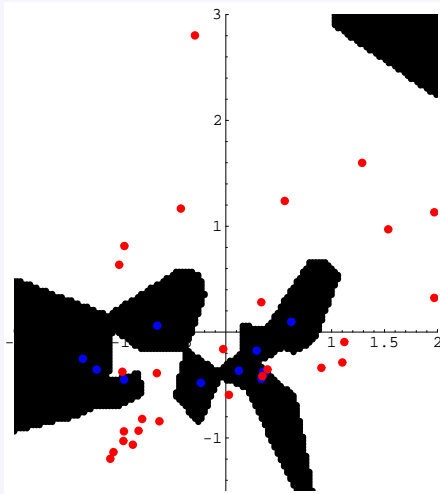


α, β -CCCD

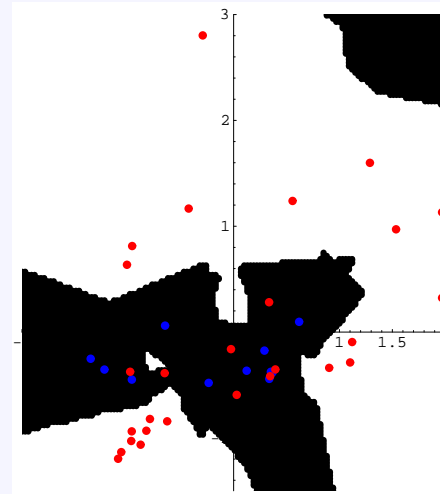


RW-CCCD

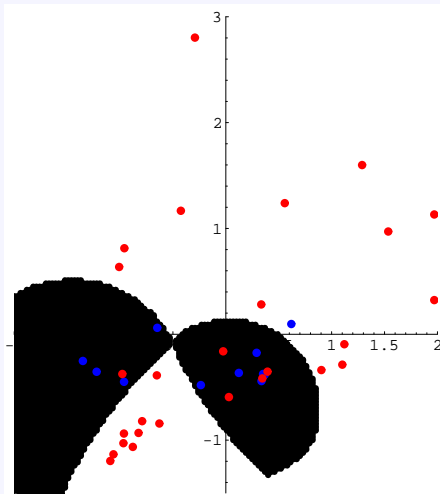
Experiment: Classifiers



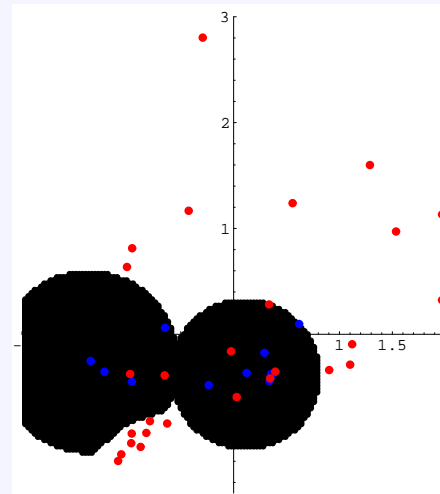
NN



k-NN



α, β CCCD



RW-CCCD

The End

CCCD home page : <http://www.mts.jhu.edu/~devinney/cccd>

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SVM results courtesy of SVM_light by T. Joachims
(http://www-ai.cs.uni-dortmund.de/svm_light)

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