
Disaster Management: Strengthening the Links of a Network for Disaster Response

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June 2006

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Outline

- Disaster mitigation
 - Disasters in Turkey and their impact
 - OR applications in disaster mitigation
 - Investment in disaster resilient public infrastructure
 - Problem definition: Strengthening the links of a transportation network
 - Two-stage stochastic programming model
 - Results and solution method
 - Conclusions and future work
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Disaster Mitigation

- Any one, or a combination, of the following can cause serious disruption and damage to communities, infrastructure, the environment and the economic activities:
 - Earthquake, bushfire
 - Flood, storm, landslide
 - Tsunami, tornado
 - Meteorite strike, volcanic eruption
 - Terrorist attacks



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Disaster Mitigation

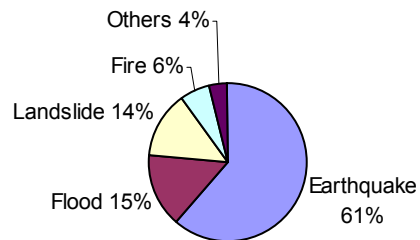
- ***‘Disaster mitigation’*** involves
 - taking measures in advance of, or after, a natural or man-made disaster
 - with the goal of eliminating or decreasing its impact on the society and the environment.



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Disasters in Turkey

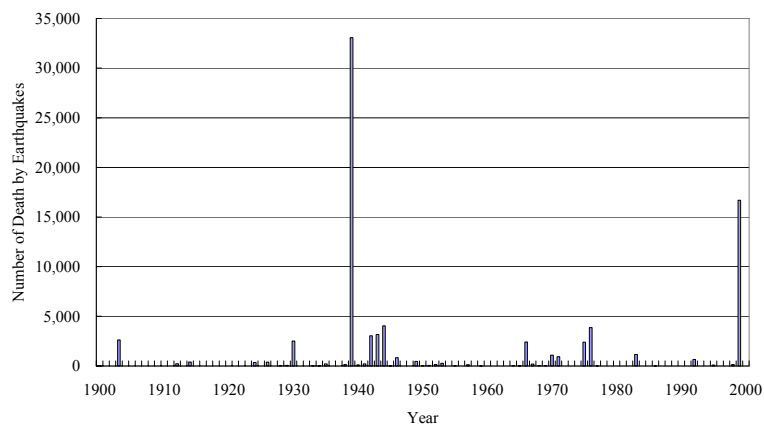
Types of natural disasters that occur in Turkey



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Impact of Earthquakes in Turkey

■ Number of deaths by earthquake



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Impact of Earthquakes in Turkey

- **Pelin G. Bakir and Hasan M. Boduroglu, 2001:**
 - **The last two earthquakes in Marmara have been catastrophic disasters**
 - **August 17, 1999 Kocaeli earthquake**
 - One million people trapped under debris,
 - Eighteen thousand people had lost their lives
 - Hundreds of thousands had been evacuated from their homes.
 - Roads had buckled and bridges had fallen down
 - **On 12 November 1999 another earthquake hit Düzce that caused 894 deaths and 4,948 injuries**
 - **Both of the disasters had serious social, administrative, legal, economical as well as environmental consequences**

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What can be done?

- **Investment in disaster resilient public infrastructure**
- **Geographic Information Systems (GIS) based decision support systems for disaster management**
 - **Information gathering**
 - **Coordination**
 - **Resource allocation**

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What can be done?

- **Develop plans and tools for post-disaster logistics**
 - **Supply the required resources and commodities to the affected areas as soon as possible to enable emergency response, rescue and relief operations**

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Investment in disaster resilient public infrastructure

- **The functionality of **infrastructure networks** is critical for effective disaster mitigation and response**
- **These networks are vulnerable to failure under natural or man-made disasters**
- **Their **vulnerability** can be **reduced** by engineering activities that enhance their components structurally leading to **higher survival probabilities**.**

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Investment in disaster resilient public infrastructure

- **These activities require large expenditure**
- **Limited resources must be allocated to obtain the maximum benefit**
- **Measures of benefit**
 - **Post-disaster functionality**
 - **Enabling efficient emergency response**

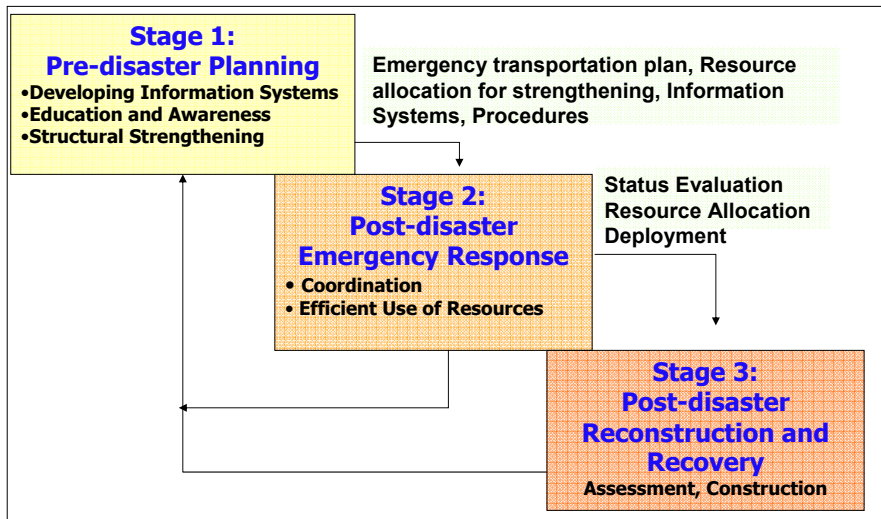
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Disaster Response

- **Post-disaster logistics**
 - **Transportation of search and rescue teams to affected areas**
 - **Transportation of injured people to hospitals**
 - **Transportation of people to shelters**
 - **Transportation of equipment, medical aid, food to shelters**
 - **In minimum time possible**

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Disaster Management



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Impact of Disasters on Transportation Networks

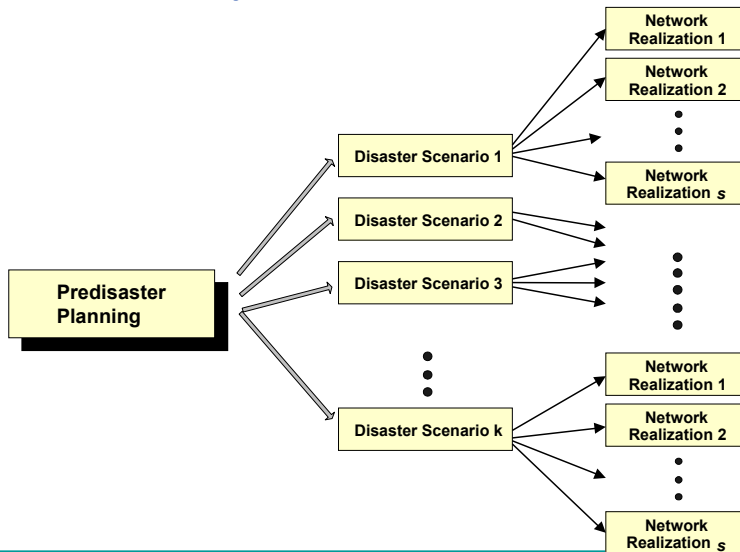
- Highway structures may be damaged or collapse after a disaster
- In a transportation network, links/nodes may fail after a disaster
- Various scenarios arise due to operational or non-operational state of the links



Collapsed section of I-10 (Santa Monica freeway), West Los Angeles. The freeway had been built across relatively soft soils (drained wetlands), the probable reason for the structural failure. This section of freeway was repaired and made serviceable in three months time. Photograph by Kerry Sieh.

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Uncertainty in pre-disaster planning



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Strengthening the links of the network

- Vulnerable structures, such as bridges, can be strengthened to reduce their vulnerability
- The **failure probability of a link** depends on
 - the disaster scenario that unfolds
 - whether the vulnerable components in the link have been upgraded (strengthened)
- The link failure probability can be **reduced through investment**
 - “upgrading cost” of the link
 - a budget limitation on investments for upgrading

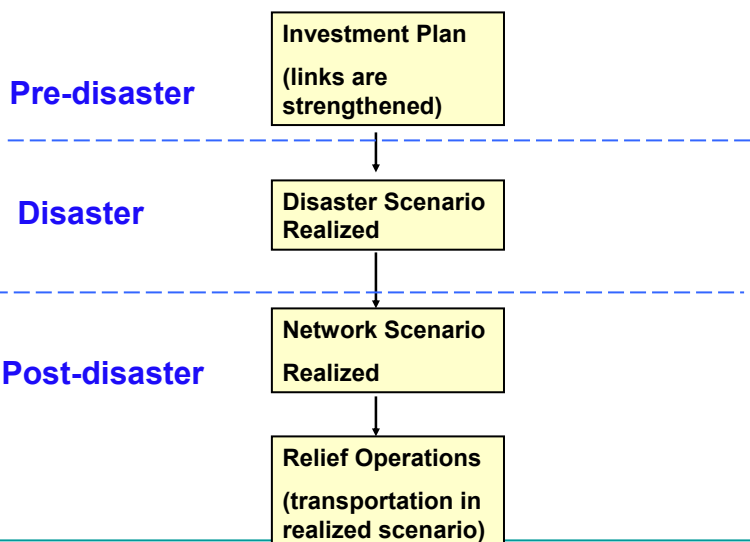
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Link Strengthening Problem

- **Decide on an investment plan**
 - for each link of the network the decision of whether the link is upgraded or not
- **Given an investment plan, a “surviving network” will be realized after the disaster event occurs**
- **Transportation problem in the realized network**

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Link Strengthening Problem



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Problem Definition

- **Make investment decisions**
 - in the **pre-disaster planning stage** under uncertainty about the surviving network
 - in order to improve the overall **post-disaster performance of the network**
- **Network performance**
 - connectivity measures
 - performability measures

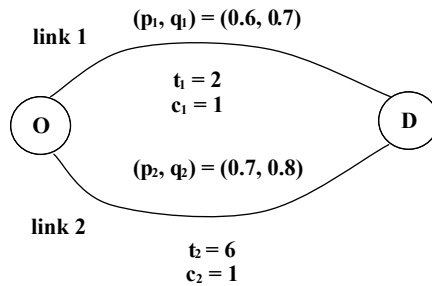
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Modeling approach

- **Plan for possible network realizations using a stochastic framework by optimizing expected post-disaster network performance**
- **First stage**
 - **Invest in the links of the network within the given budget**
- **Second stage**
 - **Optimize the performance of the network realized**
 - **Handle infeasibility using a penalty cost, M**
 - $(M > T_{\max})$

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Performance measures



Link invested in	O-D reliability (prob. of connectedness)	% Increase in reliability	Expected value of the least travel time over connected realizations	% Decrease in the expected value of the least travel time over connected realizations
1	0.91	3.409	2.923	10.683
2	0.92	4.545	3.391	-3.623

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Formulation

- Given a directed network $G = (N, E)$ with node set N and arc set E
 - index i denotes a node and
 - the index e denotes an arc in G .
 - Let O represent the origin node and D the destination node in G .
- Let p_e ($0 < p_e < 1$) denote the survival probability of link e
- This probability can be **increased** to q_e by investing an amount equal to c_e (>0)
- We are given a limited budget B for investment in the links to increase their survival probabilities.

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Formulation

- The investment decision vector is denoted by $y = (y_e)$, where y_e is binary-valued, taking the value 1 if there is an investment in link e , and 0 otherwise.
- After the occurrence of a disaster, each link would be either operational or non-operational.
- We use a binary-valued random variable ξ_e to denote the state of the link e .
- That is, $\xi_e = 1$, if link e is operational after the disaster, and $\xi_e = 0$, otherwise.

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Formulation

- Associated with link e is a non-negative traversal cost t_e , for all e in E .
- A unit flow is sent from O to D using the path with the least cost in the realized network.
- If there does not exist a path connecting O to D in this realization, a fixed penalty cost M ($< \infty$) is incurred.
- The flow vector from O to D is denoted by $x(\xi) = (x_e(\xi))$, where $x_e(\xi)$ is binary-valued, taking the value 1, if there is a unit flow through link e in the network realization ξ , and 0 otherwise.

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Stochastic programming formulation

$$Z = \min_y F(y) = \min_y E_{\xi|y}[f(\xi)]$$

subject to

$$\sum_{e \in E} c_e y_e \leq B \quad (1)$$

$$y_e = 0 \text{ or } 1 \quad \forall e \in E \quad (2)$$

$$\sum_{e \in \mathbf{H}(i)} x_e(\xi) - \sum_{e \in \mathbf{B}(i)} x_e(\xi) = \begin{cases} 1, & \text{if } i=O \\ -1, & \text{if } i=D \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in N \quad (3)$$

$$x_e(\xi) \leq \xi_e \quad \forall e \in E \quad (4)$$

$$x_e(\xi) = 0 \text{ or } 1 \quad \forall e \in E \quad (5)$$

where,

$$f(\xi) = \begin{cases} M, & \text{if } X(\xi) = \phi \\ \min_{\forall x(\xi) \in X(\xi)} \sum_{e \in E} t_e x_e(\xi), & \text{otherwise} \end{cases}$$

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Decision dependent probability distribution

$$P(\xi = \tilde{\xi} | y) = \prod_{\forall e \in E} P(\xi_e = \tilde{\xi}_e | y_e)$$

$$= \prod_{\forall e \in E} \{ \tilde{\xi}_e [(1-y_e)p_e + y_e q_e] + (1-\tilde{\xi}_e) [(1-y_e)(1-p_e) + y_e(1-q_e)] \}$$

$$F(y) = E_{\xi|y}[f(\xi)] = \sum_{\xi \in \Xi} P(\xi = \tilde{\xi} | y) f(\tilde{\xi})$$

$$= \sum_{\xi \in \mathcal{S}} P(\xi = \tilde{\xi} | y) f(\tilde{\xi}) + \sum_{\xi \in \mathcal{S}^c} P(\xi = \tilde{\xi} | y) f(\tilde{\xi})$$

$$= F_1(y)P(\mathcal{S} | y) + M P(\mathcal{S}^c | y)$$

\mathcal{S} : set of connected network realizations, $f(\cdot)$ second stage objective

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Difficulties

- Computation of $F(y)$ is not practical as the number of network realizations is exponential in the number of links
- In the current form it is not possible to obtain an explicit expression for the first stage objective function due to the decision-dependent probabilities

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Solution approach

- Path-based approach
- Compute **k-shortest paths**
- Consider the **least-cost-surviving path** in a network realization among the sample space of paths

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Equivalent deterministic program

$$F_1(y) = E_{\xi} [f_1(\xi)] = \sum_{\tilde{\xi} \in S} P(\xi = \tilde{\xi} | y) f(\tilde{\xi}) \quad (6)$$

$$= \sum_{\tilde{\xi} \in S} P(\xi = \tilde{\xi} | y) \left(\text{Min} \sum_{(i,j) \in E} t_{ij} x_{ij}(\xi) \right) = \sum_{\tilde{\xi} \in S} P(\xi = \tilde{\xi} | y) \left(\sum_{k=1}^{|K|} I\{k, \tilde{\xi}\} T(\pi_k) \right) \quad (7)$$

$$F_1(y) = \frac{1}{P(S|y)} \sum_{k=1}^{|K|} P(\pi_k | y) T(\pi_k) \quad (8)$$

$$F(y) = \sum_{k=1}^{|K|} P(\pi_k | y) T(\pi_k) + M P(S^c | y) \quad (9)$$

$$Z = \min_y \left\{ \sum_{k=1}^{|K|} P(\pi_k | y) T(\pi_k) + M P(S^c | y) \mid y \in Y \right\} \quad (10)$$

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Solution approach

- Relax the integrality of investment variables and apply **Taylor series expansion**

$$F(y) = F(y_0) + \sum_{e \in E} \frac{\partial F(y)}{\partial y_e} \Big|_{y=y_0} (y_e - y_0^e) + \sum_{q \in E} \sum_{e \in E} \frac{\partial^2 F(y)}{\partial y_e \partial y_q} \Big|_{y=y_0} (y_{e_1} - y_0^{e_1}) (y_{e_2} - y_0^{e_2}) + \dots$$

- Heuristic solution method: Use a first order approximation to F(y)**

$$F(y) \approx \bar{F}(y) = F(y_0) + \sum_{e \in E} \frac{\partial F(y)}{\partial y_e} \Big|_{y=y_0} (y_e - y_0^e)$$

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Characterize the objective function

- First order derivative for $F(y)$

$$\begin{aligned}\frac{\partial F(y)}{\partial y_e} &= \sum_{k=1}^{|K|} \frac{\partial P(\pi_k | y)}{\partial y_e} T(\pi_k) + M \frac{\partial P(S^c | y)}{\partial y_e} \\ &= E_{\xi | y} \left[\sum_{k=1}^{|K|} T(\pi_k) I(k, \xi) \frac{\xi_e [\Delta p_e] + (1 - \xi_e) [-\Delta p_e]}{P(\xi_e | y_e)} \mid \xi \in S \right] P(S | y) \\ &\quad - M E_{\xi | y} \left[\frac{\xi_e [\Delta p_e] + (1 - \xi_e) [-\Delta p_e]}{P(\xi_e | y_e)} \mid \xi \in S \right] P(S | y)\end{aligned}$$

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Results

- **Proposition 1:**

There exists a **monotone decreasing multilinear** function of the investment variables that coincides with the objective function at **integral** investment vectors.

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Results

- **Proposition 2:**

The coefficients in this function have direct economic interpretation, and denote the **marginal system-level benefits** from investing in the corresponding links.

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Results

- **Proposition 3:** Using the first order terms of the multilinear function leads to a 0-1 knapsack problem whose solution is a **local optimum** to the original problem
- We extend these results to the **multiple O-D case**, where we minimize total **weighted expected traversal time**

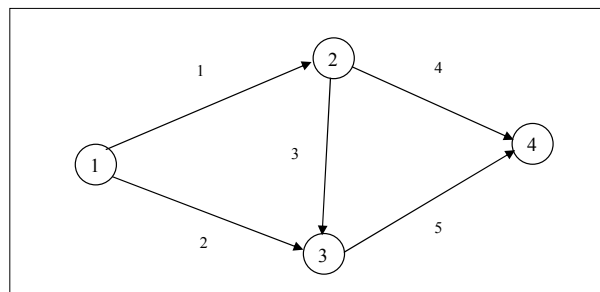
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Solution algorithm

- Calculate the first order derivatives
 - Scenario sampling may be used to reduce the computation
- Use the derivatives as objective function coefficients of a knapsack problem
- Solve the resulting 0-1 knapsack problem

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Insights from a numerical example



- Optimal or near optimal solution was obtained for most instances
- Trade-off: improvement in reliability and travel time
- Higher value for M to maximize the reliability

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Conclusions

- Introduced the problem of **investment under uncertainty** to **improve network performance**
 - We simultaneously consider **network reliability** and **performance** in making investment decisions
 - Application areas – disaster response in transportation networks
-

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Conclusions

- Two-stage stochastic programming model with **decision-dependent probability distribution**
 - Derived structural results for the model that characterize **system-level benefits of link investments**
 - We used the structural properties of the model to generate a **local optimal solution**
 - Insight from computations: trade-off between **reliability** and **functional performance**
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Ongoing Work

- Test the quality of the local optimal solution
- Meta-heuristics for possibly better solution
- Application to Istanbul metropolitan area considering most likely earthquake scenarios and possible damage under these scenarios

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A few words on application

Objectives:

- *Pre-disaster*: Strengthen critical structures in the transportation network to decrease the vulnerability of the system
- *Post-disaster*: Supply the required resources and commodities to the affected areas as soon as possible to enable emergency response, rescue and relief operations



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Operations Research Applications in Disaster Management

- Risk and vulnerability assessment
- Robust planning and strengthening of infrastructure networks
- Emergency response logistics
- Information systems for post-disaster coordination and decision support



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Questions and Comments

Thanks for listening.



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Implications of the penalty cost

$$\text{Let } y^* = \underset{\forall y \in Y}{\operatorname{argmin}} \{ F(y) \} \text{ and } y_1^* = \underset{\forall y \in Y}{\operatorname{argmin}} \{ F_1(y) \}$$

Claim 1

$$F(y^*) \leq F(y_1^*)$$

Minimizing $F(y)$ is not the same as minimizing $F_1(y)$

Claim 2

$$P(S^c | y^*) \leq P(S^c | y_1^*)$$

For the optimal investment decision in either case, the corresponding reliability is higher when infeasibility is considered

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Vulnerability for earthquakes

FIGURE 2.4 RELATIVE VULNERABILITY FOR EARTHQUAKES, 1980–2000

