Public Housing Units vs. Housing Vouchers:
Accessibility, Local Public Goods, and Welfare*

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Abstract

A perennial debate worldwide over housing aid policy focuses on whether the government should provide housing vouchers or subsidized public housing units. To complement the empirically-dominated literature, this paper builds a general equilibrium model that merges urban land use (monocentric city) and Tiebout frameworks. In our model, public housing units or housing vouchers are “rationed” and some lower-income people have to compete with those with higher incomes in the private rental market. We discuss how location of public housing units is an essential policy variable in addition to the numbers and sizes of units, and argue why housing vouchers may be preferable to public housing.

Keywords: Public housing, housing vouchers, lottery, welfare.

JEL Classification: H40, D60, H82, R13

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“Yet there are interesting problems that a theory of urban land must consider. There is, for instance, a paradox in American cities: the poor live near the center, on expensive land, and the rich on the periphery, on cheap land. On the logical side, there are also aspects of great interest, but which increase the difficulty of the analysis. When a purchaser acquires land, he acquires two goods (land and location) in only one transaction, and only one payment is made for the combination. He could buy the same quantity of land at another location, or he could buy more, or less land at the same location.” (Alonso, 1960, p.149)

“The public housing units themselves have frequently become slums and hotbeds of crime, especially juvenile delinquency. The most dramatic case was the Pruitt-Igoe public housing project in St. Louis—a massive apartment complex covering fifty-three acres that won an architectural prize for design. It deteriorated to such an extent that part of it had to be blown up. At that point only 600 of 2,000 units were occupied and the project was said to look like an urban battleground. We well remember an episode that occurred when we toured the Watts area of Los Angeles in 1968. We were being shown the area by the man who was in charge of a well-run self-help project sponsored by a trade union. When we commented on the attractiveness of some apartment houses in the area, he broke out angrily: “That’s the worst thing that ever happened in Watts. That’s public housing.” He went on to say, “How do you expect youngsters to develop good character and values when they live in a development consisting entirely of broken families, almost all on welfare?” (Friedman and Friedman, 1990, p.110)

1 Introduction

Housing markets are typically intervened by governments all around the world. The forms of these interventions differ significantly both across and within economies. For instance, while the United States tends to provide cash subsidies, European countries are more inclined to directly provide physical structures, even when the spending are comparable: In 2001, the United States spent slightly above 1.5% of its GDP on public aid on housing, while the counterpart in France is similar, slightly above 1.7% of the GDP. Yet the construction-subsidized rental sector (mainly the habitation à loyer modéré, or HLM in short) accommodates 17% of households in France, with less than 2% for the U.S. counterpart. Even a simple policy like a cash subsidy can lead to very different outcomes when institutional details differ. For instance, Priemus (2001) finds that, while in the USA, 100% of the additional rent is paid by the tenants; the Netherlands tenants will only pay 25%. The government will be responsible for the rest. Moreover, in the Netherlands, there is no waiting list and the rent subsidy is perceived as a “right.” As a result, the number of applicants

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1 See Smeeding et al. (1993) for cross country comparisons. Olsen (2003) comments on the large number of programs in the U.S.
2 Priemus (2000).
rose from 348,000 in 1976, to 922,000 in 1996. In 1998, there were more than a million households receiving a housing allowance, out of more than 6 million households in Netherlands.

Housing aid policies are among most expensive welfare programs in many countries. Olsen (2003) comments on the amount of research on the effects of housing assistance programs in U.S. as “shockingly small.” The existing research on public housing is mainly empirical, evaluating the effects of certain programs. Endogenous variables and program specific details can burden such analyses at times. In addition, housing assistance programs may have a larger impact on the economy than a few policy outcome variables of direct interest. For instance, Banerjee (1997) discusses how government interventions, in general, may introduce new distortions (e.g., rationing) which may outweigh the original benefits of interventions. Documenting such general equilibrium effects is certainly interesting. Given the enormous costs of “experiments” that will provide clear answers to such questions, there are large potential benefits from studying a formal model that enables a thorough comparison of the effects of alternative policy proposals. Identifying the general equilibrium effects, endogeneity problems, etc., can also help with improving empirical research strategies on housing aid policies.

We study the effects of two of the most common housing aid policies, subsidized units and housing vouchers, using a general equilibrium model that merges urban land use (monocentric city) and Tiebout frameworks. The land is differentiated by both distance and local public goods, and the housing aid policies are financed by general income taxes. Households differ in their incomes and preferences for the local public good, education. The quality of education in a neighborhood depends on peer quality and educational expenditures. The expenditures are determined by property taxes, rates of which are chosen by majority voting. We pay particular attention to equilibrium outcomes such as rents, spatial distribution of households, neighborhood compositions, school qualities, and welfare.

Two attributes households care about when making residential choices, accessibility and local public goods and services, have traditionally been studied separately. Urban land use theory, based
on pioneering works of Alonso (1964), Muth (1969), and Mills (1972), focused on the implications of the trade-off between accessibility and space, while Tiebout models considered the implications of local public goods and services (Tiebout (1956); Ellickson (1971); Epple, Filimon, and Romer (1984, 1993); Nechyba (1998, 1999, 2000); Fernandez and Rogerson (1996)).

Urban land use frameworks predict households locating in rings around the center according to their types, whereas Tiebout models predict strong stratification based on income and preferences for the local public goods. There are a few recent attempts to merge the two lines of research and obtain a more realistic description of urban location patterns (de Bartolome and Ross (2003), Hanushek and Yilmaz (2007)).

Our approach, based on Hanushek and Yilmaz (2007), allows us to account for both key issues in residential decision making - accessibility and local public goods- simultaneously, providing us with a rich model that captures many essential features of urban spatial structure.

We start by discussing the benchmark equilibrium, with no government intervention in the housing market. We then introduce public housing into the model and study the effects both for the hosting and neighboring communities. Public housing programs exhibit large variations from one market to another, and sometimes even within the same market. We combine the most common elements of the widespread applications to construct our program, and investigate the effects on rents, sorting of households, school qualities, and welfare of different types of households as well as overall welfare. Building public housing in a neighborhood causes some households to relocate in a way that results in a stronger sorting across neighborhoods. Rents increase in general, and so does the education quality gap between neighborhoods, decreasing overall welfare. Then we investigate the effects of location by relocating the public housing units. We find that in fact the location matters. Our results suggest that household sorting increases as public housing units move further from the city center, because the fiscal burden problem created by public housing gets more serious as the spatial distribution and of households deviates further from that of the

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5The observed household sorting is stronger (weaker) than urban (Tiebout) models predict. See Glaeser, Kahn, and Rappaport (2000).

6See Nechyba and Epple (2004), and Nechyba (2006) for a review of alternative modeling approaches.
“no-intervention” equilibrium. This further increases the school quality gap across neighborhoods. As an alternative, we consider providing housing vouchers to the same participants instead of subsidized units and provide comparisons. We find housing vouchers to be superior to subsidized units in the presence of peer group externalities as well as the spatial characteristics (hence with a non-convex consumption set). Thus, the intuition behind basic microeconomic textbook discussion for “in-cash versus in-kind” may apply to a more general environment than it seems.

The organization of the rest of this paper is as follows: Section 2 develops the theoretical framework, and discusses the calibration and results of the computable model without government intervention. We discuss our models of the two housing assistance programs and their results in Sections 3 and 4. Section 5 concludes.

2 The Model

We incorporate a Tiebout problem with peer externalities into a basic land use framework (e.g., Alonso (1964)). It may be useful to provide an informal description before going into its details: Households in a monocentric city work at the Central Business District (CBD) and choose residences in the surrounding area. The distance of home to the workplace matters because of pecuniary and time costs of commuting to work. A straight line that goes through the CBD (e.g., a river) divides the city into two jurisdictions, East (e) and West (w). Each jurisdiction provides its residents a local public good, education. The provision level (quality) of education in a district is endogenous, and depends on the composition of the households through the peer group effect as well as the local spending - property tax revenues, rates of which are determined by majority voting. Therefore the jurisdictions may differ in their tax and expenditure policies. The populations and boundaries of the districts are endogenous. Agents are allowed to move across jurisdictions and this creates a form of competition à la Tiebout (1956).
2.1 Households

Households need to choose a neighborhood (East or West, which also determines the public school that the offspring can attend) and a location in that neighborhood (distance to CBD), amount of land to reside on, leisure, and consumption. Each household has one school-age child. The preferences of households are defined over consumption of the composite consumer good $z$, land consumption (the size of the residential lot) $s > 0$, leisure $l \in [0, 24]$, and quality of education $q$ at the school the offspring attends. The preferences of a household can be represented by the utility function:\footnote{Single-crossing assumptions on preferences are consistent with empirical regularities on household sorting, and are commonly adopted in the related literature. (See Epple and Romer (1991), Fernandez and Rogerson (1998) for discussions, among others.) Single peakedness assumption ensures a voting equilibrium. The specification we adopt satisfies these two properties and proves quite useful in terms of tractability in our model.} $U(q, s, z, l) = q^{\alpha} s^{\eta} z^{\gamma} l^{\delta}$. A member of every household supplies labor to earn an exogeneously determined hourly wage $W$. Households differ according to the wages they earn, as well as in their preferences for education. We name the higher income types \textit{skilled} workers (earning $W_S$), and the lower income types \textit{unskilled} workers (earning $W_U < W_S$). Skilled workers value education more than unskilled do ($\alpha_S > \alpha_U$).\footnote{Hanushek and Yilmaz (2007), and Hanushek, Sarpka, and Yilmaz (2007) allow for heterogeneity in tastes for education at every income level. The focus of this study is public housing rather than education, and the specification we adopt does not affect related results significantly.}

The city has a dense radial transportation system. The further one lives away from CBD, the higher commuting costs he/she will face. In particular, if a household lives $r$ miles away from CBD, the cost of daily roundtrip commute will be $ar$ dollars (pecuniary cost, $a > 0$) and $br$ hours (time cost, $b > 0$), which converts to $bWr$ dollars given the opportunity cost of time. Let $l \in [0, 24]$ denote the number of non-work or leisure hours. We normalize the price of the composite consumption good to one and denote the unit rent of land $r$ miles away from CBD by $R(r)$. Households pay a property tax with rate $\tau$ on value of land. The budget constraint of a household can be written as:

$$z + (1 + \tau)R(r)s + Wl = Y(r) = 24W - (a + bW)r.$$  

(1)
The term on the RHS of the above equation is household income net of transportation costs.\textsuperscript{9}

Given the market rent curves \( \{ R_e(r), R_w(r) \} \) and quality-tax packages \( \{ (q_e, \tau_e), (q_w, \tau_w) \} \) for each jurisdiction, type \( i \in \{ S, U \} \) household solves the problem:

\[
\max_{s,z,l,j \in \{e,w\}} U(q, s, z, l) = q_{ij}^\alpha s^\eta z^\gamma l^\delta
\]

\text{s.t. } z + (1 + \tau_j)R_j(r)s + W_i l = Y_i(r).

\textbf{2.2 Market Rent Curves and Allocation of Land}

Land is owned by absentee landlords and auctioned off to the highest bidders. The reservation price of the landlord, \( R_a \), is determined by an alternative use of land, such as agriculture, and is independent of the location. For a given utility level \( \bar{u} \) we can find the maximum rent a household is willing to pay per unit of land that is \( r \) miles away from CBD by solving the problem \( \Psi(r, u, q, \tau) = \max_{s,z,l} \left\{ (Y - z - wl)/(1 + \tau) \right\} \) to obtain the bid rent function:

\[
\Psi(r, u, q, \tau) = \frac{k^{1/\eta}}{(1 + \tau_j)^{\omega^{\delta/\eta}}} q_{ij}^{\alpha_i/\eta} Y_i(r)^{(\eta + \gamma + \delta)/\eta} u^{1/\eta}
\]

where \( k = \frac{q_{ij}^{\eta - \gamma - \delta}}{(\eta + \gamma + \delta) \eta + \gamma + \delta} i \in \{ S, U \}, \) and \( j \in \{ e, w \} \).\textsuperscript{10} At an auction for a particular location \( r^* \), the winner will be the type with the highest bid rent curve at that location. Given the two types in the model, in each jurisdiction there are two bid rent curves.\textsuperscript{11} The equilibrium rent curve \( R_j(r) \) is the upper envelope of the bid rent curves of two types and the agricultural rent \( R_a \). Because all bid rent curves are convex and decreasing, the equilibrium rent curve \( R_j(r) \) will be decreasing up to a distance \( r^*_{jf} \), the fringe distance, and will stay constant from that point on. In our computational

\textsuperscript{9}The number of schools in a typical city exceeds the number of employment centers, so the average distance to a school is considerably less than the average distance to the downtown area. Also, travel within the downtown area is very quick and inexpensive compared to the average daily commute. So we ignore non-commute transportation costs, and travel within the CBD.

\textsuperscript{10}For derivations and a detailed discussion of the properties of these bid-rent functions see Hanushek and Yilmaz (2007).

\textsuperscript{11}If as a result of a policy, the number of types increases, so will the number of bid-rent curves.
analysis we check and verify that two different types’ bid rent curves can intersect at one point at most. This single-crossing property of bid rent curves proves quite useful for our analysis. Households with steeper bid rent curves will locate closer to the CBD. Higher income increases the demand for land consumption and attracts households further away from CBD, but it also increases the opportunity cost of commuting time.

Our city is a closed city, population is given exogenously. This helps us to pin down the population density at the equilibrium. Some more notation is necessary to clarify this: Let \( L(r) \) represent the land density \( r \) miles away from CBD, and \( n_j(r) \) the equilibrium density function of the household population in jurisdiction \( j \in \{e,w\} \). Suppose in equilibrium the residents of the land at distance \( r \) in jurisdiction \( j \) are type \( i \) households. If the equilibrium level of utility of the type \( i \) agent, \( i \in \{S,U\} \) is \( u^*_i \), then \( n_j(r) = \frac{L(r)}{s(r,u^*_i)} \). Let \( \bar{N}_S, \bar{N}_U \) denote the populations of the respective types. The population constraint for each type can then be stated as:

\[
\int_0^\infty \frac{L(r)}{s_w(r)} I[t^*_w(r) = i] dr + \int_0^\infty \frac{L(r)}{s_e(r)} I[t^*_e(r) = i] dr = \bar{N}_i \tag{4}
\]

where \( t^*_j(r) \) is a function showing the type of the occupant at distance \( r \) in jurisdiction \( j \), and \( I[.] \) is an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0 otherwise. The population constraint implicitly assumes the land market clears in each jurisdiction (\( \forall r \leq r^*_j, s_j(r)n_j(r) = L(r) \)).

### 2.3 Neighborhoods

The two neighborhoods differ only in the quality of education and property tax rate \((q_j, \tau_j)\) packages they provide. When choosing a community, households take the tax-expenditure packages as given. They move in and vote for the property tax rate. The households are myopic when voting; they do not consider the implications on migration patterns and the composition of neighborhoods. For a type \( i \) household most preferred tax rate \( \tau^* = \frac{\alpha_i}{\eta - \alpha_i} \) is the solution to the indirect utility
maximization problem:

\[ \tau^* = \arg\max_{\tau} \ V(.) = \frac{k}{(1 + \tau_j)^\eta R_j(r)^\omega \delta} q_j \eta R_j(r)^\eta + \gamma + \delta \quad \text{s.t.} \quad q_j = \Pi_j E_j \]

\[ \text{and } E_j = \tau_j R_j. \]  

We focus on the stationary equilibrium, which is attained when no one has an incentive to relocate in response to the voting results.

There is one public school in each jurisdiction.\(^{12}\) Public schools are neighborhood schools, enrollment is open to residents of the community only. Admission is free, schools are financed by property taxes on residential land. The quality of education \(q(\Pi, E)\) in a school is determined by (per-student) instructional expenditures \(E\) and the peer quality \(\Pi\). For a given group of students, an increase in the instructional expenditures increases the quality of education \((\partial q / \partial E \geq 0)\).\(^{13}\) An equilibrium property of our model is that in each jurisdiction there is a distance \(r_{fj}^*\) called the fringe distance beyond which no households reside. In each community entire revenue from property taxes is spent on education. Given the equilibrium rent function \(R_j(r)\), and equilibrium tax rate \(\tau_j\), we can calculate the tax base, and total tax revenues to find the per-student expenditure in the public school system:

\[ E_j = \frac{1}{N_j} \tau_j \int_0^{r_{fj}} R_j(r) L(r) dr \]  

for \(j \in \{e, w\}\) where \(N_j\) denotes the number of students in neighborhood \(j\).

Different groups of students may benefit differently from a given amount of instructional expenditures. That is what the peer quality (or efficiency) component captures \((\partial q / \partial \Pi \geq 0)\). Some parents value education more than others, and as a result they may spend more time helping with the kid’s homework, provide a nicer study environment at home, be more involved in how schools

\(^{12}\)If there are multiple schools in the neighborhood, and if one’s quality exceeds that of another, students will keep on switching to the better school until quality equalizes among the two schools. So we consider the neighborhood school system as one big school as in Epple and Romano (1998) or Nechyba (2000).

\(^{13}\)There is a debate on the effectiveness of monetary inputs on student’s achievements (see Burtless (1996)), however, it is reasonable to assume that households would value an increase in educational expenditures. And if they do, equilibrium implications would be identical.
operate, etc. Recall that type $S$ agents value education more than type $U$ counterparts, and as a result having more students from type $S$ families may bring in a higher level of positive externality through the peer group effect. The following formulation has been proved to be very tractable and captures the idea that the peer quality is increasing in the proportion of $S$ types:

$$\Pi = c_0 + c_1 \exp \left( -c_2 \frac{N_U}{N_S} \right), \quad c_0, c_1, c_2 \geq 0.$$  

(7)

Clearly, when $c_1$ or $c_2$ equals zero, the peer quality $\Pi$ becomes a constant.

The timing of events is as follows: At the beginning of each period, households make residential choice decisions, expecting last period’s quality-tax rate packages to prevail. They move in and vote for the tax rate. The quality tax rate package may be different from what they expected, however they are stuck until the beginning of next period. Then they update their expectations and the events start over again. We solve for the stationary equilibrium, which is attained when no one has an incentive to relocate in response to the voting results.

**Definition:** An equilibrium is a set of utility levels for each type $\{u^*_S, u^*_U\}$, market rent curves for each jurisdiction $\{R_e(r), R_w(r)\}$, quality of education and property tax rate pairs $\{(q_e, \tau_e), (q_w, \tau_w)\}$ for each neighborhood, household population distribution functions $\{n_e(r), n_w(r)\}$, and type functions $\{t^*_e(r), t^*_w(r)\}$ that show the equilibrium locations of the occupants at distance $r$ in community $j$ such that:

- **Households’ choices are determined by solving (2),**
- **The market rent function $R_j(r)$ in each jurisdiction is determined through a bidding process among different types of households,**
- **Same types of households obtain the same level of utility regardless of their choices,**
- **The tax rates in each jurisdiction are determined by majority voting by myopic voters,**
- **Local governments’ budgets balance in each jurisdiction, (6),**

$^{14}$Alternative specifications give similar results. See, for example, Hanushek and Yilmaz (2007) and Hanushek, Sarıçaba, and Yilmaz (2007).
2.4 Parameters for the Computational Model

The equilibrium of our model can be calculated only numerically. We specify parameter values to match certain statistics from mid-size U.S. cities in 2005. Normalizing the sum \( \eta + \gamma + \delta \) to 1, the solution to the household problem gives the optimal budget shares for leisure, consumption, and lot size as \( \delta \), \( \gamma \), and \( \eta \), respectively. In U.S. average hours of work per week in full time jobs is 40 hours, and average annual earnings of workers are $30,104 for high school graduates and $58,114 for college graduates.\(^{15}\) Accordingly, we set the hourly wages for unskilled and skilled types as \( W_U = 14 \) and \( W_S = 27 \). In a 168 (= 24 * 7) hour week, 40 hours of work implies a 0.762 budget share for leisure. The data on household expenditures suggest that expenditures on shelter constitute about 20% of the budget of an average household.\(^{16}\) This implies budget shares of 4.76% for housing and 19% for consumption. There are two possibilities for the most preferred property tax rate according to (5). We set these most preferred tax rates equal to 1.97% (1.04%) for the high (low) valuation types.

The average population density in a city with population 1 to 2.5 million is 2901 people per square mile.\(^{17}\) The utility function parameters consistent with all these are \( \alpha_L = 0.014 \), \( \alpha_H = 0.021 \), \( \delta = 0.762 \), and \( \gamma = 0.19 \).\(^{18}\)

We calculate the commuting costs assuming the households drive to work. The pecuniary cost can be calculated based on the cost of owning and operating an automobile. In 2004 pecuniary cost per mile was $0.56, and we set \( a = 1.1 \). Assuming the commuting speed in the city is 20 miles per hour, we set \( b = 0.13 \). We assume 1.5 million households populate the city. When computing the equilibrium, we target for a (endogenous) fringe distance (city radius) around 15 miles in each

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\(^{16}\)U.S. Statistical Abstract.

\(^{17}\)US Census Bureau, Census 2000 Summary File 1.

\(^{18}\)Property taxes are paid over property value, whereas the model is written for a day. Therefore the property tax rates are converted to daily values, which are consistent with these parameters. The conversion formulas are available upon request from the authors.
jurisdiction. The proportion of college graduates in U.S. is about 30%. We expect this proportion to be slightly higher in a city. Hence, we set the proportion of skilled households to 40%. We set the parameters of the school quality function to $c_0 = 0.1, c_1 = 1.3677, \text{ and } c_2 = 0.05743$ to match some related empirical observations.

### 2.5 Equilibrium without Public Housing

The equilibrium contains a spatial distribution of agents as well as of rents. It also includes the tax-quality packages that neighborhoods offer. Rather than providing all the details in the text, we use Figure 1 to provide a visualization—a map—of the city. As a complement, Table 1 provides some important summary statistics.\(^{19}\) Several observations are immediate: (1) Both household types exist in each community; (2) There is (partial) Tiebout sorting across neighborhoods; (3) Low income types choose smaller lots closer to the city center in each neighborhood. The first two observations highlight an advantage of our approach: The partial sorting across heterogeneous communities is consistent with empirical findings on household sorting (e.g., Davidoff (2005)).

(Figure 1, Table 1 about here)

The results are intuitive. Costly commuting causes the market rents in each jurisdiction to decrease as one moves away from the CBD. Higher income increases the land demand and attracts households further away from CBD where the land is cheaper. High income also increases the opportunity cost of commuting time, but this effect is dominated by the former. As a result, in each community lower income U types occupy a semi circle around to the CBD. The S types reside in a semi-ring that surrounds the semi-circle of U types. The intuition behind heterogenous communities is also straightforward: Households choose the lot size, and the community (with a given quality of education and a property tax rate) simultaneously. So lower taxes or lower rents in

\(^{19}\)Details on computation of equilibrium can be found in the Appendix.
community (hence larger lots) can compensate for lower quality of education and vice versa.\textsuperscript{20} The outer end of the S semi-ring is the fringe distance, the land beyond which is left for agricultural use.\textsuperscript{21}

Notice that about seventy percent of high income types live in the same neighborhood. They constitute a fifty five percent majority of the population there. As a result, the taxes are higher (1.97\% vs 1.04\%). Without loss of generality, we refer to the higher tax neighborhood as West school district throughout this paper. Because the two neighborhoods have comparable populations, the higher taxes mean higher educational expenditures per student in West ($3653 vs $2027). Also, the peer quality is higher in the West neighborhood, thus quality of education exceeds that in East.\textsuperscript{22} This difference in quality of education is capitalized into rents; Rents in West are about twenty-five percent higher on average.\textsuperscript{23}

3 Public Housing

We study two housing aid policies in this and the following sections: subsidized units and housing vouchers. These programs are financed by income taxes with rate $\theta$ on earnings of all city residents (plus participant contributions in the case of public housing units). Only low-income (type $U$) households are eligible. The government sets the maximum number of participants $N_P$ exogenously. If the number of program applicants exceed $N_P$, participants are selected by a lottery. For comparison purposes, we keep the sizes of two programs same.

\textsuperscript{20}Hanushek and Yilmaz (2007) have obtained a similar heterogenous communities result by allowing preferences for education to vary at different income levels. This paper obtains the same result with a simpler framework, with preferences fixed for a given income level. Even though increasing the proportion of higher income households is empirically irrelevant, we tried it for sensitivity purposes and the heterogeneous communities result still holds.

\textsuperscript{21}This “rings” structure dates back to von Thunen’s model of land use (1826), and is one of the building blocks of Urban Land Use Theory. In our models we have semi-rings instead of full ones because of the jurisdiction boundaries, or the local public goods problem. As a result of this, the widths of rings may differ between neighborhoods. The ordering of households around CBD, however, does not change.

\textsuperscript{22}The higher tax/expenditure community providing a higher quality of education with higher per-student-expenditure could easily mislead one to overemphasize the role of expenditures on school quality. Hanushek, Sarpça, and Yılmaz (2007) show the existence of a private sector for education breaks the link between expenditures and school quality.

\textsuperscript{23}The average rent per acre for S type (U type) households in west is $2,203 ($5,416). The average rents is east are $1,433 and $3,367.5 for S and U types.
Our main focus in this paper is how location of public housing may matter through its effects on rents, household sorting, and neighborhood outcomes. Although questions such as the optimal lot size, optimal program size, etc. are extremely interesting, they are beyond the scope of this paper. Instead, we construct a program that combines the common principles underlying widespread applications, that is also compatible with our framework.

Public housing units are typically clustered rather than spread out, mostly in the form of several apartment buildings at the same location, and all residents in those buildings are program participants. So we choose the public housing area as a connected range. The semi-ring structure is a natural candidate for several reasons, among which two most important are: This structure minimizes inequality in commuting costs of public housing residents, and it is consistent with our framework, in particular, how same type of households locate in a semi-ring in benchmark equilibrium.

It is possible to build public housing in both neighborhoods, such as a full-ring on the map, and guarantee certain outcomes such as desegregation exogenously. In the equilibrium of such a model we would obtain results similar to the benchmark equilibrium with slightly weaker sorting, and slightly less inequality in quality of education across jurisdictions. Typically, however, public housing is not built in the best neighborhoods, because of both costs and political reasons. Moreover, building public housing in a neighborhood may cause some residents to relocate, altering the composition and the desirability of the neighborhood. So keeping one of the neighborhoods free from public housing gives us a chance to observe the effects of such a program on its hosting neighborhood, resulting inbound-outbound migration, and the effects of this migration on neighborhood composition/characteristics as well as on local public good provision in both hosting and neighboring localities.

An essential feature of a public housing unit is its subsidized rent, and a commonly adopted policy (especially in the U.S.) is to collect a certain proportion of the resident’s income as rent. The applicants have to satisfy some low-income criteria, but still often demand exceeds supply. Then
the units can be allocated by a lottery.\textsuperscript{24} The principle behind most in-kind transfers is to increase the consumption of the relevant good. Public housing units are typically more desirable than the resident household would get on its own. The distance and lot size are the two characteristics in our model, and distance being exogenously determined, we present the results of a program that offers public housing units that is slightly larger than the average unit in that area in benchmark equilibrium.\textsuperscript{25}

3.1 Public Housing Model

More formally, the government buys the land between \(r_P\) and \(r_P + w_P\) miles from CBD in the east neighborhood. The rents are determined by the same auction process described above, but now bidders exclude the public housing recipients. For any location between \(r_P\) and \(r_P + w_P\), the government pays the maximum rent households are willing to pay, had this land been available to them. This land is then divided into equally sized lots \(s_P\) and rented out to \(N_P\) program participants, both determined exogenously. A program participant household pays a fixed price \(R_{PsP}\) as their program contribution, independent of location of the lot in the band. This policy reduces a public housing unit resident’s problem to leisure-consumption choice only:

\[
\max_{z, l} U(q, s, z, l) = q^\alpha U_s q^\eta z^\gamma l^\delta \quad \text{s.t.} \quad \tag{8}
\]
\[
z + (1 + \tau_P) R_{PsP} + (1 - \theta) W_U l = Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r,
\]

whereas others solve the original problem (2) in the presence of income taxes:

\[
\max_{s, z, l, j \in \{e, w\}} U(q, s, z, l) = q^\alpha U_s q^\eta z^\gamma l^\delta \quad \text{s.t.} \quad \tag{9}
\]
\[
z + (1 + \tau_j) R_j(r) s + (1 - \theta) W_i l = Y_i(r) = 24(1 - \theta)W_i - (a + b(1 - \theta)W_i)r.
\]

\textsuperscript{24}Alternatively, the units can be allocated on a first-come-first-served basis. If households receive information and act on it in a random way, this can be thought as a lottery too for our purposes.
\textsuperscript{25}Results with benchmark-size and smaller units are also available from the authors.
taking rents and neighborhood quality-tax pairs as given.

The cost of public housing is financed by income tax revenues and participant contributions. The time constraint of a household gives the labor supply as all the time except leisure and commute time: \( n = 24 - l - br \). The solution to (9) gives the optimal leisure of household residing at distance \( r \) as \( l^*_i(r) = \frac{\delta Y_i(r)}{(1-\theta)W_i} \) for \( i \in \{S,U\} \). For a household that lives in public housing, optimal leisure is \( l^*_P(r) = \frac{\delta Y_P(r) - (1+\tau)R_P}{(1-\theta)W_U} \). We can define a \( t^*_j(r) \) function that shows the type of the occupant at distance \( r \) in jurisdiction \( j \) (whether \( S \), or \( U \), or \( P \), i.e., \( U \) in public housing). Let \( I[.] \) be an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0 otherwise.

The program budget constraint is:

\[
\int_{r_p}^{r_p+w_p} R_e(r)L(r)dr - N_P R_P S_P = \theta \sum_{j \in \{e,w\}} \left[ \int_{0}^{r_j} \frac{L(r)}{s_j(r)} \left( \sum_{i \in \{S,U,P\}} I[t^*_j(r) = i](24 - l^*_i(r) - br)W_i \right) \right] dr
\]

(10)

The inside summation in the RHS identifies which household type resides at a particular location and calculates their labor income at equilibrium. The integral calculates the total labor income using the households' density at that location. The outside summation adds labor income of two communities. A fraction \( \theta \) of this gives us the income tax revenues. Program’s total cost (LHS) is the price of land minus the contributions. Equilibrium now also requires program budget constraint (10) holds.

### 3.2 Equilibrium

We first study a model in which the government provides public housing on the land that is between 4 and 6 miles away from CBD in the East neighborhood. This causes most high income types to reside in west, which then becomes the higher rent/tax/school quality neighborhood. In the benchmark model U types lived in this band in east, and the lot sizes ranged from 4,739 to 6,691 sq feet (at 4 and 6 miles away from CBD) with an average lot size of 5,575 sq feet. The average monthly rent was $3,520 per mile square. Each public housing unit measures 6,970 sq feet.
now, regardless of its location within the band. This is about twenty-five percent larger than the
average unit within that band in benchmark equilibrium. This band then accommodates about
fifteen percent of all low income types.\textsuperscript{26} Some lots may be closer to city center than others, so
the equilibrium utility of public housing residents may vary depending on their lot location, an
outcome of the lottery. A determinant of rent in HUD’s public housing program is 10\% of monthly
income.\textsuperscript{27} We set the rent of a subsidized unit to $239 and the income tax rate to 0.57\%, so that
utility increase from public housing is equivalent to that from a 10\% income subsidy in equilibrium
and (10) holds.

Figure 2 and Table 2 present some highlights of the new equilibrium. Sorting is stronger com-
pared to the benchmark: Seventy-five percent of high income types live in West (up from 69\% in
benchmark) and constitute the majority there. Rents are higher in both neighborhoods. The intu-
ton is straightforward: The public housing policy removes a substantial amount of land from the
private market. The non-recipients, who are either the skilled or the less lucky unskilled workers,
compete for the land remaining. However, the composition of the demand changes, and so does
the equilibrium price. To fix the idea, let us assume that originally the total land supply is $L$. The
amount of skilled and unskilled workers are $N_S$ and $N_U$ respectively. Without any public housing
policy, the ratio of skilled relative to the unskilled is $N_S/N_U$. Now the government removes an
amount of land $L_P > 0$, and the remaining amount of land available for the market has decreased
to $L - L_P$. At the same time, a portion $N_P > 0$ of unskilled workers receive public housing units
and leave the market. Thus, among those remaining in the market, the ratio of skilled relative to
the unskilled becomes $N_S/(N_U - N_P)$, which is clearly larger than the original ratio $N_S/N_U$. Other
things being equal, as the skilled naturally demand more land than the unskilled, such change in the
composition of demand would have a tendency to generate a higher rent. But other things are not
equal. First, there is a fiscal burden issue here. To finance the public housing project, the govern-
ment needs to impose taxes, that weakens the incentives to work. In addition, the public housing

\textsuperscript{26}In France, HLM accommodates about 17\% of all households.
\textsuperscript{27}http://www.hud.gov/renting/phprog.cfm.
residents’ effective rents are below market values. As they only pay property taxes as a proportion of these subsidized rents, the neighborhood’s property tax revenues and spending on education decrease. Moreover, the skilled workers may “vote on foot” and move to another neighborhood, because of their stronger preferences for education. They would prefer to live in West where the marginal effect of a tax dollar on per-student expenditures are higher. As a result, the difference in quality of education is higher between two neighborhoods compared to the benchmark, since both the spending and the peer quality in West (East) are higher (lower) than before. Our numerical implementation of the model simply attempts to capture this chain of effects quantitatively.

(Figure 2, Table 2 about here)

As a side observation, we also see that public housing residents on average decrease their labor supply by about 3 hours a week, as a result of both substitution (participant contribution as a percentage of income and income taxes change the relative price of leisure) and income (decrease in rent allows for increased consumption of leisure) effects. Sensitivity analysis shows the presence of such a strong effect with other reasonable parameters. This is consistent with the empirical literature that housing subsidies would actually have negative effect on labor supply, and such negative effect tends to be larger under public housing scheme than that under housing voucher system.\textsuperscript{28}

(Table 3 about here)

We calculate the change in household welfare according to their types in second row of Table 3. We present the necessary change in rents to provide households their benchmark utility level. A negative number means the household type is worse off now, since rents need to be decreased to keep them indifferent to benchmark allocation. The welfare gain to public housing residents is equivalent to a gain from an eighty-six percent decrease in rents. The welfare loss to the rest

\textsuperscript{28}Clearly, it is beyond the scope of this paper to review this literature. Among others, see Bingley and Walker (2001), Hulse and Randolph (2005), Olsen et al (2005), and the reference therein.
is equivalent to a 5.5 percent increase in rents. Our overall welfare measure, Aggregate Expected Utility (a normalized sum of household utilities), is lower.

3.3 Does Within-district Location of Public Housing Units Matter?

We further exploit the spatial features of our model by conducting additional analysis that compares the economic outcomes with public housing units provided at different locations. To facilitate the comparison, we keep the size and the number of subsidized units constant, and vary only their distance to CBD. Recall that in the above model the public housing units are built in the belt that is between 4 and 6 miles from the CBD. In this section, we also study when public housing occupies the land between 3 and 5.38 miles, and between 5 and 6.71 miles away from CBD.

When public housing moves from 3 to 4 and then to 5 miles, we see a stronger sorting. More and more S types choose to reside in West, which means larger lots and less population density. Most significant implications are on quality of education and welfare. As public housing moves away from CBD, the education quality gap increases: The quality keeps on increasing in West, and decreasing in East. Both per student expenditure and peer quality affect this, but the role of expenditures is larger. Government provided band houses a fixed number of households ($N_P$) that pay less than full taxes over subsidized rents, but are fully entitled to educational expenditures. Notice that the area of the band stays constant as we move it around. As this band moves out, and since lot sizes increase in distance from CBD, the band replaces less and less people with public housing residents, increasing the population density in the neighborhood. Rents also decrease in distance to CBD, so the tax revenue from public housing residents decrease too.

The overall welfare decreases as public housing band moves away from its original location (4 miles) in each direction (Table 3, rows 1 and 3). At 4 miles, the public housing unit size is slightly larger than a typical lot at the same location in benchmark model. When we move the public housing units, we keep their sizes same. Now as we move towards CBD, the public housing unit becomes much larger compared to a typical lot at a similar location. As we move away from CBD,
it becomes smaller than a typical unit at the neighboring locations. These deviations from efficient lot sizes affect community compositions and local public finance in a way that decreases overall welfare.

Our analysis suggests that location of public housing units is an important policy variable in addition to size and number of units. Location matters through its distortion on population density and composition, which in turn change the magnitude of the fiscal burden problem.

4 Housing Vouchers

4.1 Voucher Model

Rather than providing physical units with subsidized rents, the government can simply redistribute the income tax revenues to the low income households in the form of housing vouchers. Under this scheme, each of the $N_P$ program participants gets a voucher towards rent with amount $\nu_P$. For government budget to balance, income tax revenues must equal the total amount of these vouchers:

$$N_P \nu_P = \theta \sum_{j \in \{e,w\}} \left[ \int_0^{r_j^*} \frac{L(r)}{s_j(r)} \left( \sum_{i \in \{S,U,P\}} I[l_i^*(r) = i](24 - l_i^*(r) - br)W_i \right) dr \right]$$  \hspace{1cm} (11)

A household’s problem is same as the one in (9), but a voucher recipient’s housing expenditures are those exceeding $\nu_P$ only. This is equivalent to creating a third household type (say type $P$) with the same preferences as type $U$, and with the kinked budget constraint:

$$z + \max\{0, (1 + \tau_j)R_j(r) - s - \nu_P\} + W_U l = Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r.$$  \hspace{1cm} (12)

Since household utility increases in lot size, no household will spend less on housing than the voucher amount. Whether the household will spend more depends on the model parameters. The leisure choice of a voucher recipient is $l_P^* = \frac{Y_P(r) + \nu_P}{(1 - \theta)W_U}$ if the household spends on top of the voucher amount, and $l_P^* = \frac{Y_P(r)}{\gamma + \delta (1 - \theta)W_U}$ otherwise. The land is allocated according to the competitive
auction mechanism described in Section 2. An additional equilibrium condition is that the program budget (11) holds.

4.2 Equilibrium

For comparison, we keep the (number of) recipients and the tax rate same as the public housing policy. This implies a housing voucher with amount $7575 (annual). Again, table 3 provides a summary of the results. Vouchers shift land demand, increasing rents in both neighborhoods. The equilibrium rents, however, are lower than those under public housing policy, since more land is available in the private market now. Sorting of households is stronger than benchmark, equivalent to the public housing policy levels: Seventy-four percent of high income types live in West (as opposed to 69% in benchmark) and constitute a fifty-six percent majority there. The major cause of this is the increase in land demand in East: All voucher recipients reside in East (where low income types are a majority) because of their weaker preferences for education. Figure 3 gives a map of the city, and Table 4 gives some statistics. The quality of education in West is slightly higher than both the benchmark and public housing models, because of both the sorting and higher expenditures. A policy maker concerned with education of the poor may prefer vouchers over public housing: The quality in East, the poorer neighborhood, is higher than that in public housing model.

(Figure 3, Table 4 about here)

Unlike public housing, housing vouchers do not reduce labor supply of recipients. A common property of most public housing programs is that participant contribution increases with the income of the participant. This changes the relative price of leisure, hurting work incentives. Housing vouchers do not alter relative prices. Also, the fiscal burden problem we described in public housing units section is no concern here, since voucher recipients still pay property taxes at the market rent level. However, households without vouchers are hurt by the higher rents and income taxes. The equilibrium utility levels of non recipients (both S and U types) are higher than under public housing, but lower than benchmark models. The (average) utility level of voucher recipients falls
below public housing levels. We present the change in household welfare according to their types in the last row of Table 3. The welfare gain to voucher recipients is equivalent to a gain from a fifty-nine percent decrease in rents. The welfare loss to the rest is about 3.3 percent increase in rents, about half of that under public housing policy. Unlike public housing, the change in total welfare (as measured by AEU) is positive under this policy.

4.3 Can Housing Vouchers Alter Household Sorting?

Above we discuss several reasons why a housing voucher program may be preferred over a public housing program, given a program size and an income tax rate. Another potential benefit of housing vouchers over public housing is that the vouchers do not impose restrictions on location choice of households. Hence, a policy-maker with concern over the extent of household sorting across neighborhoods may be particularly interested in how the housing vouchers can influence this sorting.

The equilibrium neighborhood compositions under the two programs are, however, almost identical in the above analysis. West community provides higher quality of education at the cost of higher taxes on land consumption. The S type households have stronger preferences (and willingness to pay) for education compared to voucher recipients, so they outbid the voucher recipients on West land away from CBD. On the other hand, U type households without vouchers value proximity to CBD and outbid voucher recipients on West land close to CBD. The U type households demand smaller lots compared to voucher recipients, and therefore are not affected by larger taxes as much as voucher recipients who demand relatively larger lots. As a result, voucher recipients are not observed residing in West.

These observations suggest that whether a voucher program and a public housing program with the same size and income tax rates will have the same impact on household sorting may depend on the combinations of some parameter values. This encourages us to further explore other parameterizations. In particular, we conduct a series of numerical experiments. First, we study a
set of parameterizations in which policy maker increases the number of recipients without changing the income tax rates. This of course means a lower voucher amount for every recipient. Second, we study the effects of increasing tax rates while keeping the number of recipients the same as in the previous section. Neither attempt causes enough increase in bids of voucher recipients in West to overcome the effects summarized in the previous paragraph.\textsuperscript{29} The intuition is straightforward: Increasing number of recipients decreases the number of U types without vouchers, weakening their competition. It also lowers the bids by voucher recipients since the amount of the voucher decreases in the number of recipients. On the other hand, increasing voucher amount for the same number of recipients just help them afford larger lots in East instead of relocating, pushing some U types to West instead.

For the policymaker that is interested in influencing the community composition via housing vouchers, we do have some good news. More specifically, we find that it is possible to induce voucher recipients residing in West with both a larger program size and a larger budget. The increasing population and effective income of voucher recipients may increase their land bids in both neighborhoods and allow them to outbid some households in West. We present the equilibrium of one such model in Table 5 in which 25\% of unskilled types receive housing vouchers with amount $17900, and about one fifth of these recipients live in West. This program is financed by an income tax rate of 1.5\%, about twice the income tax rate in the earlier sections. The housing voucher recipients occupy a semi ring between the U types’ semi-circle around CBD and the S types’ semi-ring. The sorting in equilibrium is much weaker than programs we considered earlier. About 45\% of U types (with and without vouchers) live in West and constitute 49\% of the population. The reduction in sorting comes at the expense of a low quality of education in West, and lower rent differences between neighborhoods. Other aspects of the equilibria remain the same qualitatively, so we skip a detailed discussion here.

\textsuperscript{29}Detailed results are available from the authors.
5 Concluding Remarks

After surveying a vast literature on the housing market and housing policies in the U.S., Green and Malpezzi (2003, p.94) argue that “Most economists like vouchers because they are generally more efficient than other programs. (...) But in the United States, political support is generally stronger for programs tied more closely to the consumption of specific goods (housing, food, and medical care) than for income support.” This paper attempts to contribute to the related debate. In particular, this paper explicitly highlights the importance of location of public housing on equilibrium outcomes such as rents, neighborhood compositions, schooling opportunities, labor supply decisions, and social welfare. We explain the channels through which such location effects work. Using a rich general equilibrium model that combines land use theory with Tiebout framework, we provide a comparison of public housing and housing vouchers policies, and discuss several reasons why vouchers may be preferred over subsidized units. Our findings are consistent with previous “in kind vs. cash” discussions.

In addition, we also find that public housing policy tends to discourage labor supply, especially for the unskilled workers who reside in public housing, as some empirical literature has suggested. This seems to strengthen the in-cash rather than in-kind arguments even further.

The analysis we present in this paper can be adapted to compare public housing and/or vouchers to other housing aid programs, or to compare the outcomes of any single program under different institutional details.

Several directions for extension seem natural. First, this paper only considers a monocentric city. Future research may allow for a private sector for education as well as a multi-centric city and re-examine the desirability of alternative public housing policies. Second, city population does not stay constant in practice. What policy is more desirable in an environment with stochastic population flow? This seems to be another topic worth exploring. Third, recent research in urban economics emphasize the importance of dynamics. Future research should consider a dynamic extension for the public housing policy analysis. These research efforts would definitely deepen our
understanding and enrich the debate on the government involvement in the housing market.
References


Burtless, G. ed. (1996), Does Money Matter?: The Effect of School Resources on Student Achievement and Adult Success, Brookings Dialogues on Public Policy.


von Thunen, J.H. (1826), Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalekonomie, Hamburg.
Appendix: Computing the Equilibrium

1. The Algorithm

The following algorithm is based on the sequence of events described in the paper. Also see Figure A1 below. The school district j could be East or West. If we know the bid-rent curve of a household type in one district, we can draw the bid-rent curve for the same type in the other district as well, since identical households obtain the same level of utility regardless of where they live.

1. Define model parameters and discretize the space.

2. Equation 3 (and some algebra) suggests that bid-rent curve Ψ₁ of a household is steeper than that Ψ₂ of another if and only if for all r, \((a + bW_1)Y_2(r)/(a + bW_2)Y_1(r) > 1\). Using this, check if single crossing property holds and determine the spatial order of households. This ratio is always larger than 1 for any \(r^*\) and the spatial order happens to be first U-types followed by S-types as r increases.

3. Randomize the initial tax rate/quality of education package in each district.

4. Initialize fringe distance, \(r_f^j\). Find \(u_{S}^*\) by using equation 3 in the paper and the fact that at \(r_f^j\), the rent is \(R_a\). Use this information to calculate bid-rent and lot sizes for S-type households in both districts.

5. Calculate the rent at \(r_{SU}^*\) by using the bid-rent function of U-type households. Then find \(u_{U}^*\) by using equation 3. Calculate bid-rent and lot sizes for U-type households in both districts.

6. Determine the land area that S-type households outbid U-type households. By using lot sizes, calculate the population of U-type households.

7. If it is larger (smaller) than the target value, go back to 6 and increase (lower) \(r_{SU}^*\). If it equal to the target value, the land occupied by either household type are determined. Move to step 8.

8. Find majority winner property tax rates, tax bases, and quality of education in each school district.

9. Go back to step 3 and update tax rate/education package. Repeat until the current period tax rate/education packages are equal to those in the last period.

(Figure A1 about here)
2. On Path Dependency

Here’s an example that illustrates why some equilibrium cannot be attained. Consider a Giffen good characterized by:

**Demand:** $y = x$

**Supply:** $y = 0.25^*\exp(x)$

Where $y$ is the quantity and $x$ is the price.

It is easy to show there are two equilibria, $\epsilon_1$ and $\epsilon_2$ (See Figures A2 and A3 below)

**Example 1** Consider the following order of events: The supplier is myopic and produces the quantity based on yesterday’s price. The current price is determined by the demand curve, i.e., $y_t = 0.25^*\exp(x_{t-1})$ and $x_t = y_t$. Hence, $x_t = 0.25^*\exp(x_{t-1})$. If the initial point, is between $\epsilon_1$ and $\epsilon_2$, the system converges to $\epsilon_1$. If it is smaller than $\epsilon_1$, again we end up at $\epsilon_1$. If it is larger than $\epsilon_2$, it diverges. In conclusion, $\epsilon_1$ is stable while $\epsilon_2$ is unstable, and we never see $\epsilon_2$ in equilibrium.

**Example 2** Consider the following order of events: Agents place order based on last period’s price. The current price is determined by the supply side, i.e., $y_t = x_{t-1}$ and $x_t = \ln(4^*y_t)$. Hence, $x_t = \ln(4^*x_{t-1})$. If the initial point, is between $\zeta_1$ and $\zeta_2$, the system converges to $\zeta_2$. If it is larger than $\zeta_2$, again we end up at $\zeta_2$. If it is smaller than $\zeta_1$, it diverges. In conclusion, $\zeta_1$ is unstable while $\zeta_2$ is stable, and we never see $\zeta_1$ in equilibrium.

For more information, see:

(Figures A2, A3 about here)
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<th>Type S</th>
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Table 1: Benchmark Model

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<td>tax rate</td>
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distribution of types across neighborhoods

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<td>U</td>
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neighborhood population breakdown

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<td>U</td>
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Table 2: Public Housing Model

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distribution of types across neighborhoods

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neighborhood population breakdown

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(14% in public housing)
Figure 3: Voucher Model

Table 4: Voucher Model

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distribution of types across neighborhoods

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<td>U</td>
<td>39%</td>
<td>61% (14% in public housing)</td>
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neighborhood population breakdown

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<tr>
<td>U</td>
<td>44%</td>
<td>78% (18% in public housing)</td>
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**Figure 4: Desegregation via Vouchers**

**Table 5: Desegregation via Vouchers**

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<thead>
<tr>
<th></th>
<th><strong>west</strong></th>
<th><strong>east</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>ave mo rent per acre</td>
<td>2553</td>
<td>2123</td>
</tr>
<tr>
<td>tax rate</td>
<td>1.97</td>
<td>1.04</td>
</tr>
<tr>
<td>quality of education</td>
<td>10.28</td>
<td>6.3</td>
</tr>
<tr>
<td>exp per pupil</td>
<td>2697</td>
<td>1795</td>
</tr>
</tbody>
</table>

Distribution of types across neighborhoods:
- **S** types: 68% (4.7% vouchers) vs. 32% (5% vouchers)
- **U** types: 45% (20% vouchers) vs. 55% (26% vouchers)

Neighborhood population breakdown:
- **S** (50%): 50%
- **U** (5% vouchers): 72%
Figure A1:
Figure A2:

![Graph A2](image)

Figure A3:

![Graph A3](image)