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THE U.S. MARKET FOR HIGHER EDUCATION: A GENERAL EQUILIBRIUM ANALYSIS OF STATE AND PRIVATE COLLEGES AND PUBLIC FUNDING POLICIES

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ABSTRACT

We develop a new general equilibrium model of the market for higher education that captures the coexistence of public and private universities, the large degree of quality differentiation among them, and the tuition and admission policies that emerge from their competition for students. We use the model to examine the consequences of federal and state aid policies. We show that private colleges game the federal financial aid system, strategically increasing tuition to increase student aid, and using the proceeds to spend more on educational resources and to compete for high-ability students. Increases in federal aid have modest effects in increasing college attendance, with nearly half of the increased federal aid offset by reduced institutional aid and increased university educational expenditures. A reduction in state subsidies coupled with increases in tuition at public schools substantially reduces attendance at those universities, with mainly with poor students exiting, and with only moderate switching into private colleges.

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1. Introduction

The theoretical analysis of the provision of higher education is scarce.[1] Moreover, there is no compelling theoretical model that captures the coexistence of public and private universities and the tuition and admission policies that arise from their competition for students. One key unresolved theoretical challenge is to explore the different objectives of private and public universities and the different constraints they face within a general equilibrium model. Our approach builds on the insight that neither public nor private schools are likely to maximize profit. Private schools focus primarily on legacy or reputation. This motivates our approach of modeling private schools as maximizing quality, which depends on the measured abilities of their students and the educational resources they provide them.

Private universities are largely unconstrained in their policies beyond the limits imposed by technology and the market. They typically engage in third-degree price discrimination, conditioning tuition on measures of student ability and household wealth. An outstanding puzzle in the literature on higher education has been to provide a compelling theoretical explanation of the fact that even small private colleges that seem to have little market power can systematically engage in pricing by income and, therefore, extract significant additional revenues from their students. Previous papers have either ignored this fact or explained this type of price discrimination by appealing to a "serving-the-poor" motive, which can be justified if poor students provide important socio-economic diversity on campus.[2] This paper shows that we can obtain realistic pricing patterns without appealing to income diversity as an explicit objective of private schools, if students have idiosyncratic preference shocks for each college in the choice set. Pricing by income then naturally arises as part of the optimal behavior of private schools within a framework of monopolistic competition. Our modeling approach thus resolves a

[1]Exceptions are Chan and Eyster (2003), Loury, Fryer, and Yuret (2008), and Epple, Romano, and Sieg (2008) who study the impact of affirmative action on college enrollment. Epple, Romano, and Sieg (2006) provide a model of competition among private universities focusing on the determination of merit and need based aid. Sarpca (2010) studies specialization among colleges if students differ by a vector of different skills. Chade, Lewis, and Smith (2010) and Fu (2012) consider college choice under uncertainty about admission. Avery and Levin (2010) and Kim (2010) study early admission to select private universities. Epple, Romano, Sarpca, and Sieg (2006) consider bargaining over financial aid packages between a university and prospective applicants. De Fraja and Valbonesi (2010) model efficient and equilibrium policies in balancing teaching and research.

[2]This approach is taken in Epple, et.al. (2006).

longstanding puzzle in the literature.[3]

Public universities face state mandates to provide affordable education to in-state students. This suggests modeling state universities as maximizing the aggregate achievement of in-state students. Public schools also face regulated price caps and only have limited powers to set tuition and financial aid policies. Public universities, however, obtain direct subsidies from their state legislatures. Moreover, regulated tuitions generally differ between in-state and out-of-state students. Our model shows that state colleges optimally use minimum ability admission thresholds that differ between in- and out-of-state students. Our results suggest that out-of-state students provide two important functions for state schools. First, they provide important peer externalities since the admission standard for out-of-state students is typically higher than the admission standard for in-state-students. Second, out-of-state students provide important sources of revenue since they pay higher tuition rates and thus cross-subsidize the education of in-state-students.

Another open theoretical issue is to model and evaluate the impact of federal aid on equilibrium in the market of higher education. The federal government pursues a very different strategy than state governments in providing aid to higher education. Instead of providing higher education at subsidized rates, it provides direct aid to students and their families. The amount of available aid is basically determined by the difference between the tuition that is charged by the college and the federally determined expected family contribution, as long as the difference is below a maximum amount of aid. Federal aid, therefore, can benefit students at public and private universities while state subsidies are primarily targeted at in-state students that attend public schools. One surprising feature of the existing federal aid policy is that over some interval it provides a one-to-one offset to students for tuition increases since aid equals the difference between tuition and expected family contribution. Another key contribution of this paper is that we show that this type of aid policy provides some potentially undesirable incentives for private colleges to "game the system," strategically increasing tuition to increase student aid. These tuition increases are, however, used to increase spending on educational resources and to

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[3]There are many empirical papers that have documented that pricing by income is prevalent in the financial data. See, among others, Epple, Romano, and Sieg (2003).

compete for high-ability students.

Finally, we supplement the theoretical analysis with some quantitative counterfactual policy simulations. The equilibrium of the computational version of the model matches well the observed distribution of student types between state and private colleges in the U.S. It also matches the degree of need-based and merit-based aid provided by private colleges and the allocation of federal aid. As a consequence, we think that it can be used to evaluate recent policy changes that have been enacted in the U.S. While there is broad agreement among educators, policy makers, and economists that government should ensure affordable access to quality higher education, the functioning of the current aid system is not well understood.[4]

We evaluate the effects of two recent policy changes. First, the Obama administration has significantly increased the amount of federal aid available to students. We show that a one-third increase in the maximum federal aid from $6000 to $8000 has small effects on attendance and student cost, with virtually all the attendance increases occurring in state colleges. Private schools react with a mixture of increased tuition and expenditure on educational inputs, and by substituting high-ability middle-income students for some richer not-as-high ability students. Overall, the federal aid increase fails in significantly increasing college attendance with much of the increase instead bidding up college expenditures and tuition. We find that decreases in federal aid of the same magnitude have the opposite effects, but the predicted attendance decline would be much larger.

The second policy experiment is motivated by the reduced state subsidies coupled with increased tuition that have occurred in a number of states on the heels of the recent recession. We examine a revenue neutral reduction in the per student state subsidy of $2000 dollars accompanied by the same increase in tuition to in-state and out-of-state students. This policy change has large effects on attendance at state universities, mainly with poor students exiting, and with only moderate switching into private colleges. Average student cost at state colleges

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[4]Federal Student Aid, an office of the U.S. Department of Education, is the largest provider of student financial aid in the U.S. During the 2010-11 school year alone, Federal Student Aid provided approximately $144 billion in new aid to nearly 15 million post-secondary students. In addition, the provision of higher education is highly subsidized by state governments. According to the National Center for Educational Statistics, 70 percent of higher education students in the U.S. attend public universities and colleges operated by state governments. The total state aid to four-year institutions is $62.18 billion or $4,818 per student enrolled in public colleges in 2008 (Palmer, 2008).

rises by about \$1750, with the remainder of the \$2000 tuition increase being covered by increased federal aid for which students become eligible. Admission standards of state schools increase since they struggle to maintain quality and achievement of their students as they lose poor and highly able students. Moreover, state colleges admit more out-of-state students whose admission brings the school more tuition.

The rest of the paper is organized as follows. Section 2 develops our equilibrium model of the market for higher education. Section 3 defines equilibrium and provides a theoretical characterization of general equilibrium properties. In Section 4 we introduce federal financial aid policies as they are currently used in practice and study the impact of federal aid on student and college choices. We also introduced other extensions of the model that add more realism to its predictions. Section 5 introduces our quantitative model specification, describes the baseline equilibrium, and then analyzes recent policy changes. Section 6 offers some conclusions and directions for future research.

2. Private and Public Provision of Higher Education

We develop a new model of public and private competition in higher education. To understand the basic mechanisms we abstract from the existence of the federal aid program in this section. For expositional ease, we use “college” and “university” interchangeably.

2.1. Higher Education Alternatives. We consider a model with S regions or states. Normalize the student population in the economy to 1. Let π_s denote the student population proportions or size of each state and note that $\sum_{s=1}^{S}\pi_s = 1$. Students in each state differ continuously by after-tax income y and ability b. Let $f_s(b, y)$ denote the density of (b, y) in state s. Each state operates one public university. In addition to the S public universities, there are P private universities that operate nationwide and are also competing for students. There is an outside option referenced by 0 -- not attending university -- which is free and provides a given educational quality denoted by q_0 . The total number of alternatives is then $J = S + P + 1$.[5]

[5]We abuse notation for convenience by using S to denote both the number of state colleges and the set of them $\{1, 2, \ldots, S\}$, and likewise for P and J (which usage will be obvious by context). Also for expositional convenience, we refer to university or college j from the set of all alternatives J, distinguishing the outside option 0 only when it is important to do so. Likewise, the population of students consists of those that are feasibly college students, including those that chose the outside option in equilibrium.

2.2. Preferences. A student with ability b that attends a university of quality q_j has an achievement denoted by $a(q_j, b)$. Let $p_{sj}(b, y)$ denote the tuition that a student from state s with ability b and income y pays for attending college j. Let ε_j denote an idiosyncratic preference shock for school j, which is private information of the student.

Assumption 1 *The utility of student (s,b,y) for college j is additively separable in the idiosyncratic component and given by:*

$$U_j(s,b,y,\varepsilon_j) = \alpha\, U(y - p_{sj}(b,y), a(q_j,b)) + \varepsilon_j. \tag{1}$$

$U(\cdot)$ *is an increasing, twice differentiable, and quasi-concave function of the numeraire and educational achievement,* $a(\cdot)$. *Educational achievement is an increasing, twice differentiable, and strictly quasi-concave function of college quality and own ability; and* α *is a weighting parameter.*

Students choose among colleges to maximize utility as discussed further below. Let the optimal decision rule be denoted by $\delta(s, b, y, \varepsilon)$.

Assumption 2 *The vector* ε *satisfies standard regularity assumptions in McFadden (1974). Integrating out the idiosyncratic taste components yields conditional choice probabilities for each type:*

$$r_{sj}(b,y;P(s,b,y),Q) = \int 1\{\delta_j(s,b,y,\varepsilon) = 1\}\, g(\varepsilon)\, d\varepsilon, \tag{2}$$

where $1\{\cdot\}$ *is an indicator function,* $\delta_j(\cdot) = 1$ *means college j is chosen,* $P(s,b,y)$ *denotes the vector of tuitions that apply to student type (s,b,y), and Q denotes the vector of college qualities.*

2.3 Private Colleges. Private colleges attract students from all states of the country. Their objective is to maximize quality. We make the following assumptions about costs functions, private college endowments, and college quality.

Assumption 3 *College j has a cost function*

$$C(k_j, I_j) = F + V(k_j) + k_j I_j, \tag{3}$$

where k_j *denotes the size of college j's student body and* I_j *expenditures per student on educational resources in college j .*

The costs $F + V(k_j)$ are independent of educational quality, which we refer to as "custodial costs."

Assumption 4 *Let* E_j *denote the (exogenous) non-tuition income of college j. Private colleges can be ranked by these amounts:* $E_1 < E_2 < \ldots < E_P$.

Assumption 5 *Letting* θ_j *denote mean ability in college j's student body, college quality* $q_j = q_j(\theta_j, I_j)$ *is a twice differentiable, increasing, and strictly quasi-concave function of* (θ_j, I_j).[6]

We model private colleges as monopolistically competitive:

Assumption 6 *Private college j takes as given other colleges' tuitions and qualities when maximizing quality.*

Note that Assumptions 3 and 5 apply to state colleges as well. Under these assumptions we can write the quality optimization problem of private college j as follows:

$$\max_{\theta_j, I_j, k_j, p_{sj}(b,y)} q(\theta_j, I_j) \tag{4}$$

subject to a budget constraint

$$\iint \sum_{s=1}^{S} \pi_s \; p_{sj}(b,y)\, r_{sj}(b,y;P(s,b,y),Q)\; f_s(b,y)\, db\, dy + E_j \;=\; F + V(k_j) + k_j I_j \tag{5}$$

and identity constraints:

$$\theta_j \;=\; \frac{1}{k_j} \iint b \left(\sum_{s=1}^{S} \pi_s r_{sj}(b,y;P(s,b,y),Q) f_s(b,y) \right) db\, dy \tag{6}$$

$$k_j \;=\; \iint \left(\sum_{s=1}^{S} \pi_s r_{sj}(b,y;P(s,b,y),Q) f_s(b,y) \right) db\, dy. \tag{7}$$

Solving the private college's problem, we obtain the following result.

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[6]There is a large literature on educational peer effects. Methodological issues in identifying peer effects are discussed in Manski (1993), Moffitt (2001), and Brock and Durlauf (2001). Recent research on peer effects in higher education includes studies of college dormitory roommates (Sacerdote, 2001; Zimmerman 2003; Boisjoly, Duncan, Kremer, Levy and Eccles, 2006; Duncan, Boisjoly, Kremer, and Levy, 2005; Stinebrickner and Stinebrickner 2006; Kremer and Levy, 2008), dormitory residential groupings (Foster 2006), randomly formed groups in military academies (Lyle, 2007, 2009; Carrell, Fullerton, and West, 2009), structured assignments in military academies (Carrell, Sacerdote, and West, forthcoming), classroom peer effects (Arcidiacono, Foster, Goodpaster, and Kinsler, 2009), effects of high school peers (Betts and Morell,1999), and peer effects among medical students (Arcidiacono and Nicolson, 2005). See Epple and Romano (2011) for a more complete literature survey.

Proposition 1 *For any student (s,b,y) with* $r_{sj} > 0$, *tuition satisfies:*

$$p_{sj}(b,y) + \frac{r_{sj}(b,y;\cdot)}{\partial r_{sj}(b,y;\cdot)/\partial p_{sj}(b,y)} = V'(k_j) + I_j + \frac{\partial q(\theta_j, I_j)/\partial\theta}{\partial q(\theta_j, I_j)/\partial I}(\theta_j - b). \qquad (8)$$

All proofs are reported in the appendix.

The left-hand side of (8) is marginal revenue, reflecting the college's exercise of market power to extract rents from those who have a strong idiosyncratic preference for the college. As will become evident in our computational analysis, this proves to play a central role in accounting for the price discrimination by income that characterizes observed pricing in private colleges. The right-hand side is the "effective marginal cost" of student *(s,b,y)*'s attendance, which sums the marginal resource cost given by the first two terms and the marginal peer cost given by the last term. The marginal peer cost multiplies the negative of the student's effect on the peer measure (equal to $(\theta - b)/k$) by the resource cost of maintaining quality (equal to $\frac{\partial q/\partial\theta}{\partial q/\partial I}k$). Henceforth, we let:

$$EMC_j(b) \equiv V'(k_j) + I_j + \frac{\partial q/\partial\theta}{\partial q/\partial I}(\theta_j - b) \qquad (9)$$

denote the effective marginal cost of the student. Note that EMC varies with students in college j only with the student's ability, and that the peer cost is negative for students of ability exceeding the school's mean.[7]

2.4 Public Colleges. From the perspective of a state college a student is either an in-state student or an out-of-state student. We assume that the state legislature sets tuition rates, and we do not model this process.

Assumption 7 *Tuition charged to in-state students is fixed exogenously at* T_s *and to out-of-*

[7] It is interesting to compare this result to that for a profit-maximizing private college. We have shown a profit-maximizing college would have a tuition function that is of the exact *form* of (9). Given educational inputs, the quality maximizing college sets tuition to maximize profits, while taking account of the peer value effect, so as to have the maximum funds to increase quality. However, the quality maximizing college has stronger incentive to spend on educational inputs, implying the expenditure on inputs will differ between the profit and quality maximizers. Moreover, the latter implies the weight on the peer effect $(\theta - b)$ in (9) will differ, implying the quality maximizer has stronger incentives to attract higher ability students. Distinguishing the objectives empirically is then relatively subtle, as both objectives imply similar pricing, though merit aid should be steeper under quality maximization. Quality maximization also leads to use of revenues to enhance educational resources beyond their effects on increasing revenues.

state students at T_{so}*. The state also provides its college an exogenous per student subsidy of* z_s*, financed by a balanced budget state income tax denoted* t_s*.*

We assume a state college maximizes the aggregate achievement of its in-state students.

Assumption 8 *Letting* $\gamma_s(b,y)\in[0,1]$ *denote the fraction of in-state students of type* (b,y) *state college s admits and* $r_{ss}(b,y)$ *the fraction of those admitted that attend, the state college maximizes:* $\int\int a(q(\theta_s,I_s),b)\gamma_s(b,y)r_{ss}(b,y;P,Q)f_s(b,y)dbdy$.

To write a state college's optimization problem while taking account of the constraints, let $\gamma_{so}(b,y)\in[0,1]$ denote the proportion of out-of-state students of type (b,y) the college admits and $r_{ts}(b,y;P,Q)$ the fraction of those admitted from state $t\neq s$ that attend.[8] State college s solves:

$$\max_{\theta_s,I_s,k_s,\gamma_s(b,y),\gamma_{so}(b,y)}\iint a(q(\theta_s,I_s),b)\gamma_s(b,y)r_{ss}(b,y;P,Q)f_s(b,y)dbdy \tag{10}$$

subject to the identity constraints:

$$\begin{aligned}\theta_s &= \frac{1}{k_s}\int\int b\pi_s\gamma_s(b,y)r_{ss}(b,y;P,Q)f_s(b,y)dbdy\\ &+\frac{1}{k_s}\int\int b\gamma_{so}(b,y)\left(\sum_{t\neq s}\pi_t r_{ts}(b,y;P,Q)f_t(b,y)\right)dbdy\end{aligned} \tag{11}$$

and

$$\begin{aligned}k_s &= \int\int\pi_s\gamma_s(b,y)r_{ss}(b,y;P,Q)f_s(b,y)dbdy\\ &+\int\int\gamma_{so}(b,y)\left(\sum_{t\neq s}\pi_t r_{ts}(b,y;P,Q)f_t(b,y)\right)dbdy\end{aligned} \tag{12}$$

the budget constraint:

$$\begin{aligned}F+V(k_s)+k_sI_s-z_sk_s &= \int\int p_{ss}(b,y)\pi_s\gamma_s(b,y)r_{ss}(b,y;P,Q)f_s(b,y)dbdy\\ &+\int\int\gamma_{so}(b,y)\left(\sum_{t\neq s}\pi_t p_{ts}(b,y)r_{ts}(b,y;P,Q)f_t(b,y)\right)dbdy\end{aligned} \tag{13}$$

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[8]The value to college s of attracting an out-of-state student of type (b,y) does not vary with the state, implying it is optimal to admit out-of-state students of type (b,y) with the same frequency. The yield will vary in general, however.

the tuition regulation constraint:

$$p_{ts}(b,y) = \begin{cases} T_s \text{ for all students } (t,b,y) \text{ with } t = s \\ T_{so} \text{ for all students } (t,b,y) \text{ with } t \neq s \end{cases} \tag{14}$$

and the feasibility constraints:

$$\gamma_s(b,y), \gamma_{so}(b,y) \in [0,1] \text{ for all students } (t,b,y) \tag{15}$$

The following result summarizes optimal behavior of state colleges:

Proposition 2 *State college s admits all in-state students with* $b \geq b^s_{min}$*, all out-of-state students with* $b \geq b^o_{min}$*, and no other students, where*

$$a(q(\theta_s, I_s), b^s_{min})/\lambda + T_s + z_s - EMC_s(b^s_{min}) = 0 \tag{16}$$

$$T_{so} + z_s - EMC_s(b^o_{min}) = 0 \tag{17}$$

Since EMC(b) is a decreasing function, it is further implied that:

$$b^s_{min} < (=)(>) b^o_{min} \text{ as } a(q(\theta_s, I_s), b^s_{min})/\lambda + T_s > (=)(<) T_{so}. \tag{18}$$

Out-of-state students are admitted if and only if the revenue they generate covers their $EMC(b)$. Their value to the state school comes from their tuition and, perhaps, positive effect on in-state peers. In-state students have an additional marginal value of attendance, specifically their direct contribution to the school's objective of in-state achievement maximization. The term a/λ in (16) and (18) equals the monetized value of the increase in aggregate state achievement from the in-state student's attendance. While $T_s < T_{so}$ empirically, it is also likely that $a(q(\theta_s, I_s), b^s_{min})/\lambda + T_s > T_{so}$, implying a lower admission standard for in-state students.

2.5 Utility Maximization. Let $S_a(s,b,y)$ denote the subset of state colleges to which student *(s,b,y)* is admitted, and $J_a(s,b,y) \subset S_a(s,b,y) \cup P \cup O$ the options that provide positive utility available to the student. Taking as given tuitions, qualities, and non-institutional aid (introduced later), student *(s,b,y)* chooses among $j \in J_a(s,b,y)$ to maximize utility . By Assumption 2, the choice $\delta(s,b,y,\varepsilon)$ is generically unique, with choice probabilities for student type *(s,b,y)* given by (2).[9]

[9] The informational environment in our model implies students face no uncertainty in admissions, so we can abstract

2.6 <u>State Budget Balance</u>. To close the model, we assume that each state operates with a balanced budget. Letting Y_s denote aggregate pre-tax income in state s per potential college student in the economy, the state income tax satisfies:

$$t_s Y_s = z_s k_s \ \ for\ all\ s \in S. \tag{19}$$

3 Equilibrium

3.1 <u>Definition of Equilibrium</u>. We are now in a position to define equilibrium. Let P_{-j} denote the vector of price functions that omits college j, and likewise for qualities Q_{-j}. The exogenous elements of equilibrium are: (i) the student utility and achievement functions and the distribution on the idiosyncratic preference vector; (ii) the state student type distributions and proportions; (iii) the college cost and quality functions; (iv) the number of private colleges and their non-tuition revenues; (v) the number of states, their state subsidies, and in- and out-of-state tuitions; and (vi) the quality of the outside option.

Definition 1 *Given (i) - (vi), an equilibrium consists of a price and quality vector* (P,Q) *with corresponding college characteristics* (θ_j, I_j, k_j) *for all* $j \in J \setminus O$*; state admission criteria* $(\gamma_s(b,y), \gamma_{so}(b,y))$ *for all* $s \in S$*; and a set of student choices* $\delta(s,b,y,\varepsilon)$ *for all* (s,b,y) *and* $j \in J$ *with corresponding utilities* U_δ *and choice probabilities* $r_{sj}(b,y)$ *that satisfy:*

(a) private college quality maximization by all colleges $j \in P$*, taking as given* (P_{-j}, Q_{-j})*, the student choice probability functions, and public policies;*

(b) public college in-state achievement maximization by all state colleges $s \in S$*, taking as given* (P_{-s}, Q_{-s})*, the student choice probability functions, and public policies;*

(c) utility maximization by all students (s,b,y)*, taking as given* (P,Q) *and public policies including state admission criteria; and*

(d) state budgets balance.

The equilibrium notion is monopolistically competitive. In particular, colleges take as given

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from an application/admission game. See Chade, Lewis, and Smith (2011) and Fu (2012). We also abstract from the choice of a major. Arcidiacono (2005) and Bordon and Fu (2012) develop and estimate a dynamic model of choice of academic major under uncertainty.

other colleges' prices and qualities when choosing their own. Thus, a college does not consider that variation in their own pricing/admission policies will have an impact on other colleges' qualities through size and peer effects. This is reasonable if individual colleges are small in the market for students and vastly simplifies the analysis.[10]

3.2 Properties of Equilibrium One property of equilibrium regards the college quality hierarchy:

Proposition 3 *In an equilibrium, private school qualities are strictly ordered as are their non-tuition revenues.*

While one can order the qualities of private colleges generally, their qualities relative to state college qualities depend on state policies. To gain some additional insights we invoke some parametric assumptions.

Assumption 9 *The quality function is given by*

$$q_j = \theta_j^{\gamma} I_j^{\omega}, \ \gamma, \omega > 0 \tag{20}$$

The utility function is given by:

$$U_j(y - p_{sj}, a(q_j, b)) = \alpha \ln[(y - p_{sj})q_j b^{\beta}] + \varepsilon_j \tag{21}$$

The disturbances ε_j are independent and identically distributed with Type I Extreme Value Distribution.

Using (9), effective marginal costs are then given by:

$$EMC_j(b) = V'(k_j) + I_j + \frac{\gamma I_j}{\omega \theta_j}(\theta_j - b). \tag{22}$$

The probability that student (s,b,y) chooses college $j \in J_a(s,b,y)$ is:

$$r_{sj}(b,y;P(s,b,y),Q) = \frac{[(y - p_{sj})q_j]^{\alpha}}{\sum_{k \in J_a(s,b,y)}[(y - p_{sk})q_k]^{\alpha}}. \tag{23}$$

As a consequence, we have:

$$\frac{\partial r_{sj}}{\partial p_{sj}} = -\frac{r_{sj}(1 - r_{sj})\alpha}{y - p_{sj}}. \tag{24}$$

[10]The model also assumes students take as given college qualities as well as price and admission practices, while not taking as given other student college choices. This is relevant to proving Proposition 3 below, as discussed in the appendix.

Substituting this into equation (8) implies:

$$p_{sj}(b,y) = \frac{(1-r_{sj})\alpha}{1+(1-r_{sj})\alpha} EMC_j(b) + \frac{1}{1+(1-r_{sj})\alpha} y. \tag{25}$$

Tuition is a weighted average of $EMC(b)$ and student income.[11] As a consequence, our model can explain the combination of merit and need-based aid that colleges frequently provide. Holding r_{sj} constant, (25) implies that tuition declines with ability and increases with income. Making precise statements in the general equilibrium is difficult, because r_{sj} will vary with *(b,y)* due both to college j's choices and other colleges' choices. If private college j had a monopoly on higher education, with students having only the outside option as their alternative, then it is necessarily implied that tuition will decline with ability and increase with income.

Corollary 1 *If a private college has a monopoly on higher education, then tuition declines with ability and increases with income to attending students.*[12]

We will see this result typically holds as well with competing colleges and federal aid in our computational analysis.

4 Federal Aid, Price Caps, and Non-Tuition Costs

For clarity, we have thus far focused on the model without federal financial aid. In this section, we introduce a realistic version of financial aid into the model specification and explore pricing by a private school in the presence of federal aid. To obtain a better quantitative model, it is also desirable to account for price caps in private schools and non-student tuition costs. We discuss each of these extensions in this section of the paper.

4.1 Federal Aid. The federal government provides college students with aid through several programs. Broadly speaking, federal aid levels vary with student resources and with the cost of attending college. For students seeking aid, the federal government first computes a student's expected family contribution (EFC). This is the amount the federal government deems as appropriate for the family to pay out-of-pocket for a college education. In addition to the student's family income, this depends on a variety of factors, mainly family assets and family

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[11] Student types that attend private college j must have $y > EMC(b)$. Otherwise, by (25), $p_{sj} > y$, contradicting that college j is in the student's effective choice set.

[12] The proof of Corollary 1 is omitted from the appendix due to space constraints. It is available in the on-line appendix.

size.[13] For families with few assets and given household size, we can model EFC as an increasing function of student's after-tax family income.[14] Federal aid is then linked to the difference between the student's educational costs and their EFC. The idea is that aid should be made available only to the extent the student's educational costs exceed EFC.

Federal aid to student with income y and ability b in college j is denoted A_j. To economize on notation, we suppress the dependence of aid and tuition on (b,y) . Let $\bar{A}$ denote the maximum aid and $EFC(y)$ the federally determined expected family contribution. The discussion above suggests approximating the federal aid formula as follows:

Assumption 10 Federal aid is given by the following equation

$$A_j = min\{\bar{A}, max[p_j - EFC(y), 0]\}. \tag{26}$$

This assumption then implies that

$$\frac{\partial A_j}{\partial p_j} = \begin{cases} 0 & if \quad p_j < EFC(y) \\ 1 & if \quad EFC(y) < p_j < EFC(y) + \bar{A} \\ 0 & if \quad EFC(y) + \bar{A} < p_j \end{cases} \tag{27}$$

The modified demand for college $j \in J_a(s,b,y)$ is now given by:

$$r_j = \frac{[(y_t - p_j + A_j)q_j]^\alpha}{\sum_{k \in J_a(s,b,y)}[(y_t - p_k + A_k)q_k]^\alpha}. \tag{28}$$

The key result regarding private school pricing and federal aid is the following:

Proposition 4 *If $p_j < EFC(y)$, the tuition consistent with private school optimization must satisfy:*

$$p_j = \frac{(1-r_j)\alpha}{1+(1-r_j)\alpha} EMC + \frac{1}{1+(1-r_j)\alpha} y \tag{29}$$

and $A_j = 0$. If on the other hand $p_j \geq EFC(y_t) + \bar{A}$, the tuition satisfies:

[13] While federal aid is purely need based, in equilibrium it can also depend indirectly on ability through tuition.
[14] We make an adjustment for assets in the quantitative analysis below.

$$p_j = max\left\{ EFC(y) + \bar{A}, \frac{(1-r_j)\alpha}{1+(1-r_j)\alpha} EMC + \frac{1}{1+(1-r_j)\alpha}(y+\bar{A}) \right\} \qquad (30)$$

and $A_j = \bar{A}$. *Optimal tuition will never be in the range:* $p_j \in [EFC(y), EFC(y) + \bar{A}]$.

Equilibrium is thus characterized by a corner solution: Either a student gets no federal aid or she obtains the maximum possible aid.[15] This result is driven by the fact that the private school has no incentives to set prices in the middle range of price in (27). Increasing p_j just increases aid dollar for dollar, which is then optimal for the college.

To understand the gaming of the federal aid formula and how tuition varies with student type we will see in the quantitative analysis below, it is useful to examine price setting by private colleges graphically. Figure 1 shows the demand of an observed student type *(s,b,y)* to attend college j, taking account of aid. The $r_j(\cdot)$ in Figure 1 is from (28), holding constant q_j, and other college prices, qualities, and aid levels. The upper part of the demand function has aid equal to the maximum $\bar{A}$, so that student cost is $p_j - \bar{A}$. In the vertical range of demand, corresponding to the middle range of (27), student cost is constant at $EFC(y)$, and then demand is constant. In the lower range of demand, $p_j < EFC(y)$ and there is no aid.

To find the college's optimum, find the implied marginal revenue implied by the student's demand function. This is illustrated in Figure 2. Then introduce the student type's EMC. Marginal revenue (MR) is not defined at $r_j(EFC, y)$, while otherwise has the usual properties. If the upper portion of MR were crossed by the student's EMC (not shown in the figure), then the optimum would be in the upper range of demand with maximum aid. If EMC crossed between the holes in MR (also not shown), then the corner solution arises with tuition equal to $EFC + \bar{A}$, again with maximum aid. Finally, if EMC crosses the lower segment of MR as shown, then one must check if the corner solution is optimal or if tuition would equate MR and EMC.[16] If the latter is the optimum, no aid would arise. We return to the character of

[15]Our analysis abstracts from federal provision of, and subsidies to, college loans, only examining the effect of aid. We have also developed the theoretical model where both aid and subsidized loans are available from the federal government. For the purposes of the general equilibrium analyzed here, we simplify while focusing on the main effects.

[16]The payoff function is discontinuous and there are two local maxima in these cases.

pricing by private colleges in the quantitative analysis in Section 5.

To close the model, the federal budget must be balanced. The federal income tax, denoted t, satisfies:

$$t\sum_{s\in S}Y_s = \int\int\left(\sum_{s\in S}\pi_s\left(\sum_{j\in P}r_{sj}(\cdot)A_j(b,y)+\sum_{t\in S}r_{st}(\cdot)A_t(b,y)\right)f_s(b,y)\right)dbdy. \quad (31)$$

Letting y_p denote a student (s,b,y)'s pre-tax income, it is further implied that $y=(1-t_s-t)\,y_p$. Above we have specified the state income densities using after-tax income, simply because these are more convenient to work with.[17]

4.2 Price Caps. Next we discuss how to modify a private college's optimum when it has a self-imposed price cap or maximum tuition. The notation here suppresses the college and the student type, except student income is sometimes included to match some of the notation earlier in the paper. Though we do not have an explicit theory explaining commitment to a tuition maximum, in practice private colleges adopt a maximum and then provide some students with financial aid.[18] Let P^c denote the price cap and P^* the optimal tuition ignoring the price cap. To describe the optimum, we need to define some values of r, denoted below as $\tilde{r}$ and $\hat{r}$. Define $\tilde{r}$:

$$\tilde{r}=\begin{cases} r(P^c-\overline{A},y) & if \quad P^c-EFC(y)\geq\overline{A} \\ r(P^c-(P^c-EFC(y)),y) & if \quad \overline{A}\geq P^c-EFC(y)\geq 0 \\ r(P^c,y) & if \quad 0\geq P^c-EFC(y) \end{cases} \quad (32)$$

$\tilde{r}(\cdot)$ in (32) is the demand of the type to attend the college in question, written as a function of net tuition and student income. The demand in (32) assumes the tuition equals the price cap and then takes account of allowed financial aid. This will be the attendance level if the price cap is binding and the college wants to admit the type to the level of demand, as analyzed next.

Define $\hat{r}$ in $P^c=EMC(\hat{r}),$ provided there exists a solution, $\hat{r}>0.$. The right-hand side

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[17]The density of pre-tax income distribution is given by: $f_s^p(y_p,b) \equiv f_s(y_p(1-t-t_s),b).$ We assume that colleges take students' post-tax incomes as given, i.e., ignore any effects of their decisions on post-tax income operating through tax changes.

[18]Adoption of a price maximum is probably explained by marketing to students and society. A price maximum will prevent some wealthy and lower-ability students from buying their way into top colleges. In reality, no doubt exceptions to the latter occur. The model with price caps abstracts from these exceptions, while we have found that the model without price caps exaggerates this buying in.

is our standard expression for EMC, but now written as a function of the measure of the type. Since V' is increasing, this will have a unique solution if it has a solution at all. With price caps, it is straightforward to see that the private college optimum will have:

$$\begin{array}{llll} \text{If } P^c \geq P^* & \text{then} & & P = P^* \\ \text{If } P^c < P^* & \text{then} & & r = 0 \text{ if no } \hat{r} \text{ exists} \\ \text{If } P^c < P^* & \text{then} & r = \tilde{r} \text{ if} & P^c \geq EMC(\tilde{r}) \\ \text{If } P^c < P^* & \text{then} & r = \hat{r} \text{ if} & P^c < EMC(\tilde{r}). \end{array} \quad (33)$$

The top line is the case where the price cap is not binding. The remaining cases have a binding price cap. In the second line, the price cap does not cover EMC and no such types are admitted. The third line has the price cap cover EMC up to the level of demand at the price cap. The fourth line has the price cap equal to EMC before demand is exhausted at the price cap, so the college optimally limits admission of the type.

4.3 Non-Tuition Student Costs. Finally we introduce some non-tuition student costs for attending college that are not counted in determining aid. These costs can arise for various different reasons. Students face transportation costs, living costs in excess of room and board, and face opportunity costs of time. We introduce these costs to explain the fact that a large fraction of low income students that are eligible for need-based financial aid do not attend college.[19]

5 Quantitative Analysis

5.1 Quantitative Model Specification. To assess the performance of the model, we examine a numerical specification of it and then go on to compute equilibria under alternative public policies.[20] We consider a model with two states and thus state schools, each state having the same policies and distributions of potential student types. Table 1 summarizes the parameter values that we use.[21] The average in-state tuition in 2007-08 was \$6,200, and the average out-of-state tuition was \$15,100 for full-time undergraduates enrolled in public 4-year institutions.[22] The average public subsidy was \$8,495 per Full Time Equivalent (FTE) student.[23]

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[19]As we will see in Section 5 of the paper, there are reasonable values for the non-tuition costs such that the quantitative model well matches the data.
[20]The on-line appendix discusses how to compute equilibria.
[21]As we explain below, most of these parameters are estimated using a variety of different moments.
[22]Stats in Brief, US Dept. of Education, December 2010.
[23]The enrollment-weighted average is calculated from Figures S1 and S2 of Kirshtein and Hurlburt (2012).

To obtain values for private colleges, we rank colleges by SAT score and combine them into five groups (colleges) with an equal number of students in each group. Endowments per student are chosen to correspond to those in the NSF WebCASPAR data. We assume a draw of 2% per year from endowment is allocated to undergraduate education. Resulting endowment draws are \$155, \$243, \$386, \$755, and \$4,149 per student. Average “sticker prices” for private bachelor’s and private research universities in 2009 were 22.6 and 30.4 (in thousands of dollars henceforth).[24] We set the price caps for the five private universities as 26, 28, 30, 32, and 34, following the quality hierarchy.

We set the quality of the outside option equal to $q_0 = 2.794.$ The parameters of the utility-quality functions in (20) and (21) are set as $\gamma = .85,$ $\omega = .14,$ and $\beta = .85.$ The weight, γ, on peer quality in the utility function is a combination of a production function effect (more able peers give rise to favorable achievement spillovers) and a preference effect (highly ranked universities convey networking and prestige benefits.) These four parameters are set such that in the baseline equilibrium: (i) the average private tuition is 23.4 and share of private schools in total enrollment is 30 percent;[25] (ii) total enrollment is 40 percent of potential students;[26] and (iii) shadow prices on income and ability are consistent with financial aid regressions reported in the literature. The choice of α in the utility function largely affects the proportion of in-state students at state schools and the mark-ups over marginal cost in private schools. To match the former, we set $\alpha = 17.$[27] This implies a reasonable average mark-up of \$4,894 in the baseline

[24]Kirshstein and Hurlburt, Delta Cost Project, Revenues: Where Does the Money Come From, Figure 3.

[25]The average of tuition and fees at private universities in 2009 was \$23,400 (Stats in Brief, US Dept. of Education, December 2010, Table 1). In 2009 there were approximately 5.5 million FTE students enrolled in public colleges and universities and 2.3 million FTE students enrolled in private colleges and universities, suggesting a public enrollment share of 70 percent (NCES Digest of Education Statistics, FTE calculated weighting a part-time student as 40% of a full-time student).

[26]The narrowest and most straightforward measure of participation in higher education is the proportion receiving a degree, which is 33.87 percent according to the U.S. Census. This measure understates the product of higher education if benefits result to those who do not complete a degree. The average number of years attended per entering student is 3.53, and only 65 percent of entering freshmen graduate with a degree. Our measure takes account of the fact that the typical graduate takes more than four years to obtain a degree and also incorporates educational benefits to students who attend college for some number of years but never obtain a degree. Dividing the average years attended by the probability of obtaining a degree, we obtain the average years attended per degree earned, which equals 5.47. The typical graduate takes 4.6 years to take the prescribed number of courses. The number of degree-equivalents provided to a cohort is then (5.47/4.6)*.3387 = 40 percent. Thus, this measure accounts for the fact that a cohort obtains more college education than is reflected in the number of degrees awarded while also reflecting the reality that, on average, students take less than a full load.

[27]Using the empirical in- and out-of-state average state college tuitions, the average tuition of all public students (\$7,100, Stats in Brief, US Dept. of Education, December 2010) implies an 89.9 percent in-state student share; and

model. We set the non-tuition cost of attending college as \$3,000.

We specify the college cost function as $C(k,I) = F + c_1 k + c_2 k^2 + k\,I$. Epple, Romano, and Sieg (2006) estimate "custodial cost functions" (costs net of *kI*) using micro data for a large sample of colleges and discuss how to aggregate cost functions. Their analysis suggests that average cost functions initially decline quickly and then are fairly flat over a large range of values. Also, custodial costs amount to approximately 60 percent of total expenditures on average.[28] Given the values of utility function parameters and the number of state and private schools, the choice of cost function parameters also need to be consistent with school sizes in equilibrium. Based on these, we specify the cost function parameters as $F = 0.165$, $c_1 = 0$, and $c_2 = 40.$

To approximate the EFC function, we assume that the student is a dependent and follow the EFC formula guide worksheet A.[29] We set a student's income equal to zero and just consider parental income. We calculate the income protection allowance assuming a four-member family with one college student. We approximate the family's contribution from assets as 7% of gross income when gross income exceeds \$50,000, and denote the resulting *adjusted gross income* by $\tilde{y}$.[30] We then calculate the *EFC* function by using $\tilde{y}$ and Table 6 of worksheet A. A good approximation to the resulting mapping is $EFC(\tilde{y}) = max\{\tilde{y}/5.5 - 5{,}000,\ \tilde{y}/3.2 - 13{,}600,\ 0\}$.

We measure federal aid as a weighted sum of grants, work-study aid, and loans using the formula: Federal Aid = Grants + 0.33 Work-study + 0.1 Loan. The maximum Pell Grant in 2008 was \$4731. Subsidized federal loans are capped at \$3,500 and \$4,500 for the first two years, and at \$5,500 for each year after that. The upper limit on work-study earnings varies by the cost of living, with the average is on the order of \$2500. Combining these and weighting according to the above formula implies a maximum federal aid of very close to \$6,000.

We use data from the Current Population Survey (CPS) for 2009 to estimate the income

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Profile of Undergraduate Students: 2007-08 reports 90.5 percent. In the baseline model we then target 90 percent.

[28]See Table 1 in Epple, Romano, and Sieg (2006).

[29]This is available at https://studentaid.ed.gov/.

[30]Household assets are included in determining the level of aid in general. However: (a) assets of households with adjusted income below \$50,000 are not considered; (b) most standard assets like housing equity are not considered (Investment income, liquid assets, and education IRS's are counted, but not retirement savings or life insurance); and (c) eligible assets are first discounted by \$45,000 and then just .12 of the remainder is counted towards EFC.

distribution. We find that a lognormal distribution with a location parameter fits the data well. The parameter estimates are reported in Table 2. Ability is calibrated to IQ, normalized such that $ln(b) \sim N(1.0, 0.15)$. We follow Epple and Romano (1998, 2008) in setting the correlation of household income and student ability as 0.4.

5.2 The Baseline Equilibrium. The first two columns of Table 3 summarize the fit of our baseline model. We report the total enrollment, the fraction of students in state schools and the proportion of in-state students in state schools. We report the average tuition rates in each school type, the average of federal aid by school type and the fractions receiving such aid. Finally, we report the average institutional aid, equal to the average discount in private colleges from private colleges' tuition caps. Overall, the model does an excellent job of replicating U.S. average values. The only drawback is that the fraction of students that receive federal aid in private colleges is a bit higher than in the data.

Table 4 provides more detail on the baseline equilibrium. The upper part of the table shows values by college, with the first two rows for the two identical state colleges and the next five rows for the five private schools ordered by their quality and thus per student endowment. The state colleges are much larger than the private colleges and the private colleges shrink as their qualities improve. Resources per student, mean ability, average tuition, and the mean income of students all increase along the college quality hierarchy. Average federal financial aid is higher in private than state colleges, but almost flat and declines as one moves from the second highest to highest quality private college. While average tuition is higher in the top college, eligibility of students for federal aid declines due to a wealthier student body. Some averages across colleges are reported at the bottom of the table. By "average aid conditional," we mean the average conditional on receiving some federal aid. In private colleges, this average approaches the maximum federal aid of \$6000 as private schools increase tuition to game the system. Average student cost nets out federal aid and is much higher in private colleges. Note that the values for this reported in Table 4 do not include the additional \$3000 in non-tuition costs each college attendee bears.

The minimum ability thresholds for admission at state schools are also reported in Table 4. The in-state threshold for admission is lower than the out-of-state threshold. Thus, the higher tuition that state colleges get from out-of-state students is not enough to offset a state's focus on achievement of its own residents (as discussed in Proposition 2). The minimum ability admission threshold for in-state students is between the 4th and 5th ability decile and between the 5th and 6th ability decile for out-of-state students.

The panels of Table 5 show the attendance proportions in colleges of prospective students by income and ability deciles. The rows are delineated by income deciles and the columns by ability deciles. For example, in the upper panel titled "state colleges," the .076 entry in the lower right cell means that 7.6% of the highest-decile income and ability types attend a state college in equilibrium. The middle and lower panel report these proportions, respectively, for all private colleges and the highest quality private college. Looking at the upper two panels, one can see that attendance proportions in any college generally rise with income and ability. No poor and low-ability students attend college, and almost all very high-ability students attend college regardless of income.

Examining the upper and middle panels of Table 5 some patterns are apparent. First, private colleges are more selective than public colleges. One can see in the middle panel that, once ability and income are sufficiently high, the proportions attending a private college increase with income and ability. Comparing the middle panel to the upper panel, the proportion attending a private college exceeds the proportion attending a state college for sufficiently high income and ability, and the difference increases in income and ability. However, among the lowest-income deciles, students of moderately high ability that attend college are predominantly in state colleges. The columns of zeros in state colleges at low-ability deciles reflect, of course, the minimum admission ability thresholds.

The lower panel shows the attendance proportions at the most elite private college. We selected the top private college thinking readers might find its attendance pattern of most interest among the private colleges.[31] Relative to other private colleges, one can see it is more selective and richer. The 22% of the highest income and ability decile that attend this college is the highest among all colleges; e.g., this percentage is equal to 18.7% in the second highest quality college. Most of the students in the most elite college are in the highest ability decile, though a few with a

[31] This information for all colleges and the information we provide below for all colleges is available in the on-line appendix.

bit lower ability of the highest income buy their way in.

Figures 3 and 4 show, respectively, the ability and income distributions by college, scaled to show relative college sizes. The biggest distribution in each figure is for in-state public school students. The distribution for out-of-state public students is labeled, and the numbered distributions are for the private colleges following the quality hierarchy. In addition to illustrating relative college sizes, these figures illustrate the extent of income and ability heterogeneity within colleges and stratification by income and ability across colleges. The heterogeneity within colleges arises both from students' idiosyncratic preferences for colleges and from the cross-subsidization within colleges of high-ability, low-income students by lower-ability, higher-income students.

Table 6 shows by ability and income percentile the within cell average student tuition, federal aid, and total student cost (i.e., tuition minus federal aid plus non-tuition cost) conditional upon attendance in state colleges, and for the highest quality private college. For example, the highest-decile ability and lowest-decile income students that attend state college pay $6,200 in tuition on average (upper right entry in the 'Tuition' panel), receiving $6,000 in federal aid, with then average cost equal to the $3,000 non-tuition cost plus the $200 difference. For state colleges, since tuitions vary only by in- versus out-of-state status, the variation in tuition across the cells among attending students is only a consequence of variation in this status. The largest variation comes in moving from the 5th to 6th decile students, this due to the higher ability admission threshold for out-of-state students. Federal aid to state college students drops with income and varies a bit with the tuition variation that arises from the changing mix of in-state and out-of-state students. As one sees in the 'Total Cost to Student' panel, poor students attending a state college have their tuition fully funded with federal aid or nearly so.

Regarding private colleges, consider first tuition at the highest quality college. The central property of the tuition function is that it is increasing with income and decreasing with ability. Thus the model can explain the presence of merit- and need-based aid that private colleges provide. The combination of merit- and need-based aid is well documented empirically, but this has not been well explained theoretically. The model presented in Epple, Romano, and Sieg (2006) yielded such pricing, but the model assumed an income-based peer externality to explain need-based aid. The key difference here is that idiosyncratic preferences among students for attending particular colleges increase colleges' market power, permitting more price discrimination by student income. The decline in tuition by student ability is explained by the positive impact of ability on college quality, this manifest in lower effective marginal cost of higher ability students. The structure of private school pricing applies to all private colleges, with one exception, though tuition is generally less in lower-quality colleges (not shown in any table here). The exception is that in the second- and third-highest quality colleges, among the highest ability decile students, average tuition rises a little as income drops from the 9th to 8th decile (see the on-line appendix). This derives from gaming the federal aid formula. Those in the 8th income decile can get some federal aid, and these private colleges then raise their tuition. But the student cost in private schools always declines with ability and increases with income. *We view the findings about the tuition and student cost structure as a central contribution of the paper*. Many students in private colleges receive close to the maximum financial aid. This is, again, because private colleges game the system. The price caps prevent colleges from raising tuition enough to the richest students, so their federal aid is a limited.

Using the top private college to illustrate, Figure 6 shows tuition to three ability (percentile) types as income varies (the upper curves), and the federal aid they receive (the lower curves). This figure illustrates both the price discrimination by income practiced by private colleges and the greater financial aid they provide to attract more able students. Part of $EFC(y)+\bar{A}$ is also graphed to facilitate understanding. Consider the tuition function to the b = 97 percentile type, for whom all ranges of demand and the price cap come into play. For low incomes, demand and EFC are relatively low, and the optimum is on the upper portion of demand in Figure 2 with maximum aid received. As income rises the optimum initially stays in this range of demand and tuition rises with demand. When income and EFC gets high enough, the optimum is at the "corner solution," with tuition equal to $EFC(y)+\bar{A}$. As income and EFC rise further, the optimum remains at the corner and tuition then tracks $EFC(y)+\bar{A}$, on the left side of the spike in Figure 6. Eventually, the increased demand from increasing income induces the college to prefer the solution on the lower segment of demand (see Figure 2) with more students of the type served. Here tuition declines discretely and aid goes to 0. Then as income and demand increase further tuition rises until the price cap is reached. The progression as income rises is essentially the same for the b = 99 percentile type, but tuition is lower at all incomes until the price cap is reached (not shown), reflecting lower EMC of higher ability types. The tuition cap is reached as income rises before the corner solution would arise for the lowest-ability type shown. This type receives less than the full aid over a range where he qualifies for aid because the price cap prevents the university from increasing tuition to extract the maximum.

To measure the market power of private colleges, we compute the average tuition mark-ups over marginal cost along the quality hierarchy. These are 4.470, 4.761, 5.020, 5.356, and 5.711. As a proportion of average tuition, along the lines of the Lerner Index, the respective values are .2091, .2120, .2138, .2182, and .2115. While these values indicate substantial market power, they would be higher yet with no price caps.

5.3 Policy Analysis. The first policy change we examine is a change in the maximum level of federal aid, this motivated by the substantially increased aid implemented under the Obama administration. We consider an increase in the maximum federal aid from $6,000 to $8,000, and then further to $9,000. We focus the discussion on the increase to $8,000. Aggregate effects of this policy change are summarized in Table 3, with detailed enrollment effects provided in Table 7. The upper panel of Table 7 shows the absolute percentage attendance changes relative to the baseline equilibrium by ability and income decile aggregated over all colleges. The increase in federal aid results in only about a 1% increase in college attendance of the potential student population (see Table 3). Virtually all the increased attendance is at state colleges. Studying the upper panel of Table 7, the bulk of the increase in attendance is seen to be by relatively low-ability students and by middle-income students of very high ability. The former attend a state college and some of the latter gain access to private colleges. Consider the response of private colleges to the increase in federal aid. Average federal aid to students at private colleges rises by about $1,100. Average tuition at private colleges rises by about $440 (and private colleges decrease average institutional aid by $440). Hence, roughly 40% of the increased federal aid is offset by a reduction in institutional aid, with private colleges using those funds instead to increase expenditure on educational inputs. The private colleges also become a bit more selective, this entailing some substitution of high-ability middle-income students for lower-ability rich students. Hence, average income of students at private colleges declines somewhat (see the on-line appendix). The proportion receiving any aid in private colleges rises substantially from 45% to 50%. The average change in student cost at private colleges (Table 3) drops by about $630. Overall, access to private colleges does not change, but there is a substitution toward higher-ability and lower-income students.

As noted, the small increased net attendance occurs at state colleges and by lower-ability students. With more access to aid, more students attend out-of-state colleges, this increasing average tuition a little at public colleges. Given more federal funding and their mission to educate in-state students, state colleges reduce somewhat the ability cutoff for admission of in-state students and increase it to out-of-state students. Average federal aid at state colleges rises modestly by about $240, and average student cost declines by about $50 (Table 3). *If the policy change is intended to increase attendance by the poorest students, it does so, but the effects are very small. Cost saving to state college students is small and only moderate at private colleges.* Increasing the maximum federal aid further to $9,000 has effects in the same directions, and with roughly proportional magnitudes (see the fourth column of Table 3).

Figure 7 uses again the top college and the 97th percentile ability student to illustrate why the cost saving to students in private schools is not greater as a result of the increased federal aid. The college increases tuition to any student that is not at the price cap. Cost to student (CTS), also shown before and after the policy change, declines for students not at the price cap after the change, but not as much as the aid they receive. The "Full Passthrough" student cost shows what cost would be if all the post-change equilibrium aid were used to reduce cost (or, equivalently, if the private college did not increase tuition).

Equivalent dollar decreases in the maximum federal aid have larger effects. The equilibrium attendance effects of reducing federal aid from $6,000 to $4,000 are reported in Table 7 and, again, aggregate effects in Table 3. Table 3 shows that total college enrollment drops by 2.3% of the potential student population, with over 90% of the decrease at state colleges (not shown). *As one can see in the middle panel of Table 7, comparing the enrollment changes to the baseline values in Table 5, the effects on the poorest two deciles of students are drastic.* The funding reduction drives out virtually all of the lowest income decile of students and about two-thirds of the second lowest income decile. An interesting effect is that state colleges increase the admission threshold to in-state students to try to maintain quality and thus student achievement, while lowering it some to out-of-state students from whom they get more tuition. On net, a substantial proportion of the lowest ability students also exit state colleges.

Students at private colleges receive an average of $1,010 less in federal aid, but private colleges reduce tuition by an average of $630. Thus, the average cost of private college attendees increases by about $414. Private colleges increase institutional aid by about $700, and they lower expenditure on educational inputs some. There is some substitution of lower-ability richer students for higher-ability students, with most of the latter switching to state colleges. Average federal aid at state college declines by more than a third, from $1500 to $920. The proportion receiving any aid in state colleges drops substantially from 37.8% to 32.3%. Average student cost rises by $521 in state colleges.

Figure 5 shows the effects on the ability distributions in state colleges and in all private colleges, scaled to reflect their sizes, of the $2000 federal aid reduction. Some lower ability students are closed out of state colleges as discussed above. Some richer moderately high ability students switch into private colleges, who become willing to accept them for their high tuition payment.

Recently, states have cut funding to state colleges with offsetting tuition increases. This motivates the last policy experiment we conduct. We consider a $2,000 ($3,000) decrease in per student state funding, accompanied by a $2,000 ($3,000) increase in tuition to both in- and out-of-state students. Table 7 shows that enrollment of poor students is drastically affected, with pattern very similar to the effect of decreasing maximum federal aid by $2,000. Overall, enrollment in all colleges drops substantially by 3.2% of the potential student population, or 8% of the pre-change college student population. All the reduction in attendance is at state colleges; private college attendance increases slightly. The increased expense of attending state colleges increases demand to private colleges. Private colleges substitute some higher-ability middle-income students for lower-ability high-income students, and slightly increase tuition and expenditures on inputs.

Of course, tuition at state schools increases, an average of $2,070. In an effort to maintain quality, state colleges increase the ability admission threshold to in-state students and decrease the higher threshold to out-of-state students (though still above the in-state threshold), and the proportion of out-of-state students rises a little. An increase in average federal aid to students at state colleges of $310 buffers the tuition increase some, but average student cost at state colleges rises by $1,768. The last column of Table 3 shows aggregate effects if tuition (state funding per student) were increased (decreased) by another $1,000. *Attendance and cost effects, especially for the poor, of these state policy changes are dire.*

6 Conclusions

This paper provides a comprehensive model of the market for higher education that includes competing state and private colleges with alternative objectives, students that differ by income, ability, and unobserved idiosyncratic preference for colleges, and a realistic characterization of federal aid. The model provides an appealing set of theoretical predictions, including provision of need- and merit-based aid at private colleges, minimum ability admission standards at state colleges that vary across in- and out-of-state students, and gaming of the federal aid formula by private colleges. The quantitative version of the model does an excellent job of matching aggregates as well as predicting patterns of attendance, private college tuition, and student costs.

Utilizing the model for policy analysis, we find small overall enrollment effects of increased federal aid, but with some benefits to lower-income students and potential students. Increased federal aid leads private colleges to substitute lower income and higher ability students for somewhat less able students with higher income. Predicted effects of decreased federal aid are more significant, with a large exodus of low-income students from higher education. Attendance changes are concentrated in state colleges. Decreased subsidies at state colleges coupled with higher tuition, as has characterized many states of late, has dire effects on attendance by poorer students and on student costs.

Our theoretical and computational findings exhibit the benefits of modeling the distinctive features of the market for higher education. Scope clearly remains for further generalizations, such as extending the analysis to consider heterogeneity across states and investigation of alternative approaches to provision of federal aid. A perhaps more difficult extension would be to make endogenous the state subsidy and tuition policies. An issue here is whether one would assume the state regulator’s objective differs from the college objective. Finally, expanding the dimensions of student heterogeneity such as race is also of interest.

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Appendix. Proof of Proposition 1. Given Assumptions 5 and 6, the quality maximization problem is a strictly quasi-concave programming problem with unique solution under the condition described next. Substitute (7) into (5) and (6). Then (5) and (6) define an implicit mapping from $p_{sj}(b,y)$ into (θ_j, I_j) say $(\tilde{\theta}_j(p_{sj}(b,y)), \tilde{I}(p_{sj}(b,y)))$. If the latter is a convex set, the problem is strictly quasi-concave, which we then assume. To derive (8), write out the Lagrange function for the problem presented in (4) - (7). Suppressing the j subscript and the functional arguments, the Lagrange function is:

$$L = q + \lambda\left[\left(\iint \sum_{s=1}^{S} \pi_s p_s r_s f_s dbdy\right) + E - F - V - kI\right] + \eta\left[k\theta - \iint b\sum_{s=1}^{S} \pi_s r_s f_s dbdy\right] + \Omega\left[k - \iint \sum_{s=1}^{S} \pi_s r_s f_s dbdy\right]. \tag{34}$$

Compute the derivatives with respect to θ, I, and k, and the first variation with respect to $p_s(b,y)$.

$$L_\theta = q_\theta + \lambda\iint \sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial q} q_\theta f_s dbdy + \eta\left[k - \iint b\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_\theta f_s dbdy\right] - \Omega\iint \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_\theta f_s dbdy = 0. \tag{35}$$

$$L_I = q_I + \lambda\left[\iint \sum_{s=1}^{S} \pi_s p_s \frac{\partial r_s}{\partial q} q_I f_s dbdy - k\right] - \eta\iint b\sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_I f_s dbdy - \Omega\iint \sum_{s=1}^{S} \pi_s \frac{\partial r_s}{\partial q} q_I f_s dbdy = 0. \tag{36}$$

$$L_k = -\lambda[V' + I] + \eta\theta + \Omega = 0. \tag{37}$$

$$L_{p_s(b,y)} = \lambda\pi_s f_s(r_s + p_s\partial r_s / \partial p_s) - \eta b\pi_s f_s\partial r_s / \partial p_s - \Omega\pi_s f_s\partial r_s / \partial p_s = 0 \ \forall\ p_s(b,y). \tag{38}$$

From (35) and (36), one obtains:

$$\frac{q_\theta}{q_I} = -\frac{\eta}{\lambda}. \tag{39}$$

Divide (37) and (38) by λ, yielding respectively:

$$-[V' + I] + \frac{\eta}{\lambda}\theta + \frac{\Omega}{\lambda} = 0. \tag{40}$$

$$\pi_s f_s\left[r_s + p_s \partial r_s / \partial p_s - \frac{\eta}{\lambda} b \partial r_s / \partial p_s - \frac{\Omega}{\lambda} \partial r_s / \partial p_s \right] = 0 \ \forall \ p_s(b,y). \quad (41)$$

Substituting (39) and (40) into (41), after dividing through by $\partial r_s / \partial p_s$, completes the derivation.

Proof of Proposition 2. From the first-order conditions, one can write the first variation with respect to admission of in-state and out-of-state students as:

$$L_{\gamma_s} = \lambda \pi_s r_s f_s(b,y)[a(\cdot)/\lambda + T_s + z - EMC_s(b)] \quad (42)$$

$$L_{\gamma_{so}} = \lambda \left(\sum_{t \neq s} \pi_t r_{ts} f_t(b,y) \right) [T_{so} + z - EMC_s(b)] \quad (43)$$

where $\lambda > 0$ is the Lagrange multiplier associated with the budget constraint (13). From (42) and (43), using the feasibility constraints, one obtains the results.

Proof of Proposition 3. Suppose to the contrary that private schools i and j have qualities $q_i \leq q_j$ given $E_i > E_j$. Then college i could credibly provide a quality higher than q_j by spending $I_i = I_j$ and adopting a tuition function that offers students somewhat lower tuition than does college j for those with $b > \theta_i$ and somewhat higher to those with $b < \theta_i$, these tuitions so that $k_j = k_i$. Quality is higher for college i because the tuition function improves the peer group relative to college j. College i can afford to charge lower tuition to the higher ability students than college j because it has higher non-tuition revenue; while increasing tuition to those of lower ability permits maintenance of size and thus cost and is feasible as it relaxes college i's budget constraint. The fact that college i could increase quality implies it has not maximized quality, a contradiction.[32]

Proof of Proposition 4. If $p_j < EFC(y)$, then (26) implies $A_j = 0$. The college's first-order condition for optimization must be satisfied, which, with no aid, is (29). If $p_j \geq EFC(y)$, then

[32] Models with peer effects sometimes admit a multiplicity of equilibria that allows a reversal of the type of ordering in Proposition 3. For example, with two ability types and two schools, a less efficient school might be better in equilibrium because all the high-ability types believe their high-ability peers will attend the less efficient school and all the low ability types will attend the more efficient school. But such equilibria require students to take as given the school choices of other students in equilibrium. Our model has students take as given the equilibrium qualities of schools, while schools maximize quality. If a school can credibly offer a higher quality – one consistent with optimizing choices by students – then the higher quality must result in equilibrium.

$p_j < EFC(y) + \bar{A}$ cannot be optimal. If the latter, then student cost of attending college j is: $EFC(y) = p_j - A,$ implying demand is invariant for any such tuition. By increasing tuition, the college could then increase quality by having more to spend on educational resources; a contradiction to quality maximization. Thus $p_j \geq EFC(y)$ implies $p_j \geq EFC(y) + \bar{A}$ and thus $A = \bar{A}.$

Now consider what tuition must be whenever $p_j \geq EFC(y) + \bar{A}.$ There are two possibilities. If $p_j \geq EFC + \bar{A},$ then $\partial A / \partial p_j = 0,$ the quality maximization problem is locally differentiable, and the first-order condition (25) must be satisfied but with effective income of $y + \bar{A}$ rather than $y.$ Then tuition must be given by the second value on the right-hand side of (30). If $p_j = EFC(y) + \bar{A},$ then the quality maximization problem is not locally differentiable, and $p_j = EFC(y) + \bar{A}$ might be optimal. If

$$Max\{ EFC(y) + \bar{A}, \frac{(1-r_j)\alpha}{1+(1-r_j)\alpha} EMC(y) + \frac{1}{1+(1-r_j)\alpha}(y + \bar{A})\} = EFC(y) + \bar{A}, \tag{44}$$

then adopting the lower tuition contradicts $p_j \geq EFC(y) + \bar{A}.$ This is the case of the corner solution with $p_j = EFC(y) + \bar{A}.$ If,

$$\begin{aligned} &Max\{ EFC(y) + \bar{A}, \frac{(1-r_j)\alpha}{1+(1-r_j)\alpha} EMC(y) + \frac{1}{1+(1-r_j)\alpha}(y + \bar{A})\} \\ &= \frac{(1-r_j)\alpha}{1+(1-r_j)\alpha} EMC(y) + \frac{1}{1+(1-r_j)\alpha}(y + \bar{A}), \end{aligned} \tag{45}$$

then increasing tuition from the lower to the higher value must be optimal, as quality is differentiable over $p_j \in (EFC(y) + \bar{A}, \frac{(1-r_j)\alpha}{1+(1-r_j)\alpha} EFC(y) + \frac{1}{1+(1-r_j)\alpha}(y + \bar{A})).$

<u>Proof of Corollary 1</u>. See the on-line appendix.

Table 1: Parameter Values

State College Tuition and Subsidy	
In-state Students	6.2
Out-of-state Students	15.1
Per-student Subsidy	8.5
Private College Endowments and Price Caps	
Endowments	.155, .243, .386, .755, 4.149
Price Caps	26, 28, 30, 32, 34
Ability & Income Distribution	
Ability	$ln(b) \sim N(1.0, 0.15)$
Income	$ln(y + 41,536) \sim N(11.46, 0.402)$
Ability-Income Correlation	0.40
Utility Function	
α	17
γ	0.85
ω	0.14
β	0.85
q_0	2.794
Non-tuition Costs	3
Cost Function	
F	0.165
$v1$	0
$v2$	40
Federal Aid	
Maximum Aid	6

Table 2: Parameter Estimates: Income Distribution

	Coefficient	Std. Error
mean	11.46	0.126
std deviation	-0.402	0.044
location parameter	41,536	11,867

Table 3: Aggregates: Baseline and Policy Changes

	Data	Baseline	MaxFed=8	MaxFed=9	MaxFed=4	MaxFed=3	StChg=2**	StChg=3**
Total Enrollment	40%	40%	40.96%	41.4%	37.7%	36.62%	36.8%	35.16%
Share of state schools	70%	69.53%	70.15%	70.4%	68.1%	67.4%	66.75%	65.2%
Proportion of in-state at state	90%	90%	87.96%	86.8	90.7%	91.02	89.1%	88.6%
Average aid (state schools)	1.25-1.5	1.50/3.98*	1.74/4.48*	1.86/4.77*	0.92/2.84*	0.65/2.23*	1.81/4.36*	1.93/4.56*
Average aid (private schools)	2-2.5	2.44/5.44*	3.55/7.17*	4.12/8.01*	1.43/3.68*	0.98/2.81*	2.51/5.37*	2.53/5.33*
Average price cap	30	29.5***	29.5	29.5	29.5	29.5	29.5	29.5
Average institutional aid	6.6	6.04	5.6	5.36	6.7	6.98	5.79	8.19
Private tuition average	23.40	23.46	23.90	24.14	22.83	22.52	23.71	21.31
State tuition average	7.09	7.09	7.27	7.37	7.02	7.0	9.16	10.22
Fraction Receiving Aid (state)	30-40%	37.8%	38.8%	39.0%	32.3%	29.1%	41.4%	42.4%
Fraction Receiving Aid (private)	30-40%	44.9%	49.5 %	51.4%	38.8%	34.9%	46.7%	47.5%
Chg. in Avg. St. Cost (state)			-49	-74	521	766	1,768	2,703
Chg. in Avg. St. Cost (private)			-630	-962	414	554	218	330
Tax Rates (Federal/State)		1.17/3.86	1.52/3.98	1.71/4.04	0.67/3.57	0.45/3.42	1.23/2.61	1.23/2.06

*Conditional on positive aid.
**State tuition increases and state subsidy decreases by the same amount.
***The price caps are 26, 28, 30, 32, 34, following the quality hierarchy.

Table 4: Baseline College Values

j	k_j	θ_j	I_j	q_j	Ave.Tuit.	Ave.Aid	Ave.Inc
1	0.1391	2.923	8.84	3.376	7.09	1.51	73.16
2	0.1391	2.923	8.84	3.376	7.09	1.51	73.16
3	0.0288	3.365	14.66	4.085	21.38	2.46	101.72
4	0.0274	3.380	15.57	4.135	22.46	2.47	104.76
5	0.0260	3.396	16.47	4.185	23.48	2.47	107.90
6	0.0238	3.425	17.41	4.248	24.55	2.49	111.04
7	0.0160	3.550	20.21	4.471	27.00	2.24	123.65

Average Tuition	state= 7.09;	private= 23.43
Average Student Cost*	state= 5.58;	private= 20.98
Average Aid	state= 1.51;	private= 2.44
Average Aid (conditional)	state= 3.98;	private= 5.44
Fraction Receiving Aid	state= 0.38;	private= 0.45
Minimum Ability in-state	2.641	
Minimum Ability out-of-state	2.725	

*Does not include the non-tuition cost.

Table 5: College Attendance Proportions

		Ability Deciles*									
		1	2	3	4	5	6	7	8	9	10
		State Colleges									
Income Deciles**	1	0	0	0	0	0.032	0.042	0.046	0.061	0.046	0.037
	2	0	0	0	0	0.347	0.523	0.530	0.523	0.543	0.488
	3	0	0	0	0	0.422	0.617	0.621	0.626	0.621	0.474
	4	0	0	0	0	0.358	0.540	0.545	0.541	0.539	0.324
	5	0	0	0	0	0.331	0.495	0.498	0.499	0.477	0.177
	6	0	0	0	0	0.411	0.592	0.598	0.594	0.510	0.165
	7	0	0	0	0	0.466	0.685	0.686	0.654	0.444	0.114
	8	0	0	0	0	0.611	0.766	0.771	0.646	0.345	0.121
	9	0	0	0	0	0.605	0.842	0.843	0.656	0.354	0.130
	10	0	0	0	0	0.596	0.903	0.907	0.547	0.192	0.076
		Private Colleges									
	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0.106
	3	0	0	0	0	0	0	0	0	0.000	0.227
	4	0	0	0	0	0	0	0	0	0.003	0.405
	5	0	0	0	0	0	0	0	0.002	0.044	0.644
	6	0	0	0	0	0	0	0	0.012	0.148	0.721
	7	0	0	0	0	0	0	0	0.050	0.353	0.834
	8	0	0	0	0	0	0	0	0.157	0.552	0.845
	9	0	0	0	0	0	0	0	0.223	0.581	0.846
	10	0	0	0	0	0	0	0	0.395	0.786	0.917
		Highest Quality Private College									
	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0.001
	3	0	0	0	0	0	0	0	0	0	0.008
	4	0	0	0	0	0	0	0	0	0	0.023
	5	0	0	0	0	0	0	0	0	0	0.041
	6	0	0	0	0	0	0	0	0	0.001	0.052
	7	0	0	0	0	0	0	0	0	0.004	0.102
	8	0	0	0	0	0	0	0	0	0.015	0.169
	9	0	0	0	0	0	0	0	0	0.058	0.172
	10	0	0	0	0	0	0	0	0	0.103	0.220

*The ability deciles are 2.245 , 2.395, 2.513, 2.611, 2.717, 2.825, 2.942, 3.082, and 3.287.
**The income deciles are 14.86, 25.91, 35.17, 44.47, 53.81, 63.81, 76.42, 92.98, and 120.32.

Table 6: Student Cost, Tuition, and Federal Aid by Ability and Income Percentile*

TOTAL COST TO STUDENT**

State College									
-	-	-	-	3.20	3.20	3.20	3.20	3.20	3.20
-	-	-	-	3.20	3.20	3.20	3.20	3.21	3.20
-	-	-	-	4.31	4.43	4.34	4.25	4.34	4.50
-	-	-	-	6.22	6.48	6.37	6.48	6.50	6.36
-	-	-	-	8.35	8.87	8.92	8.93	9.01	8.98
-	-	-	-	9.20	9.89	9.93	9.93	9.93	9.92
-	-	-	-	9.20	9.86	9.89	9.89	9.89	9.88
-	-	-	-	9.20	9.93	10.01	9.99	10.01	10.04
-	-	-	-	9.20	10.40	10.47	10.47	10.47	10.48
-	-	-	-	9.20	11.09	11.25	11.20	11.10	11.21
Highest Quality Private College									
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	7.51
-	-	-	-	-	-	-	-	-	11.59
-	-	-	-	-	-	-	-	-	15.37
-	-	-	-	-	-	-	-	-	18.56
-	-	-	-	-	-	-	-	32.47	22.99
-	-	-	-	-	-	-	-	33.95	26.53
-	-	-	-	-	-	-	-	37.00	30.98

TUITION

State College									
-	-	-	-	6.20	6.20	6.20	6.20	6.20	6.20
-	-	-	-	6.20	6.20	6.20	6.20	6.20	6.20
-	-	-	-	6.20	6.23	6.22	6.22	6.22	6.23
-	-	-	-	6.20	6.48	6.48	6.50	6.51	6.47
-	-	-	-	6.20	7.44	7.55	7.56	7.61	7.59
-	-	-	-	6.20	8.20	8.28	8.28	8.26	8.29
-	-	-	-	6.20	7.12	7.15	7.13	7.14	7.11
-	-	-	-	6.20	6.93	7.01	6.99	7.01	7.04
-	-	-	-	6.20	7.40	7.47	7.47	7.47	7.48
-	-	-	-	6.20	8.09	8.25	8.20	8.10	8.21
Highest Quality Private College									
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	8.79
-	-	-	-	-	-	-	-	-	14.18
-	-	-	-	-	-	-	-	-	17.76
-	-	-	-	-	-	-	-	-	20.91
-	-	-	-	-	-	-	-	33.99	25.10
-	-	-	-	-	-	-	-	34.00	26.36
-	-	-	-	-	-	-	-	34.00	27.98

FEDERAL AID

State College									
-	-	-	-	6.00	6.00	6.00	6.00	6.00	6.00
-	-	-	-	6.00	6.00	6.00	6.00	5.99	6.00
-	-	-	-	4.89	4.80	4.88	4.97	4.88	4.73
-	-	-	-	2.98	3.00	3.11	3.02	3.01	3.11
-	-	-	-	0.85	1.58	1.63	1.63	1.60	1.61
-	-	-	-	0.00	1.31	1.35	1.35	1.33	1.37
-	-	-	-	0.00	0.26	0.26	0.25	0.25	0.23
-	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00
-	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00
-	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00
Highest Quality Private College									
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	4.28
-	-	-	-	-	-	-	-	-	5.59
-	-	-	-	-	-	-	-	-	5.39
-	-	-	-	-	-	-	-	-	5.35
-	-	-	-	-	-	-	-	4.52	5.12
-	-	-	-	-	-	-	-	3.05	2.83
-	-	-	-	-	-	-	-	0.00	0.00

*All values are measured in thousands. The columns are delineated by ability percentiles and the rows by income percentiles.

**Total student cost includes the non-tuition costs.

Table 7: Changes in Total Enrollment

		Ability Deciles*									
		1	2	3	4	5	6	7	8	9	10
		Max Aid=$8,000									
Income Deciles**	1	-	-	-	-	0.01	0.01	0.01	0.02	0.01	0.01
	2	-	-	-	-	0.10	0.03	0.03	0.03	0.03	0.05
	3	-	-	-	-	0.04	-0.01	-0.01	-0.01	-0.01	0.02
	4	-	-	-	-	0.05	0.00	0.01	0.01	0.01	0.04
	5	-	-	-	-	0.05	0.03	0.04	0.04	0.05	0.05
	6	-	-	-	-	0.03	0.00	0.01	0.01	0.04	0.03
	7	-	-	-	-	0.10	-0.01	-0.01	-0.02	0.02	0.01
	8	-	-	-	-	0.06	-0.01	-0.01	-0.02	0.01	0.00
	9	-	-	-	-	0.07	-0.01	-0.01	-0.02	0.00	0.00
	10	-	-	-	-	0.14	-0.01	0.00	-0.02	0.00	0.00
		Max Aid=$4,000									
	1	-	-	-	-	-0.03	-0.04	-0.04	-0.06	-0.05	-0.04
	2	-	-	-	-	-0.27	-0.37	-0.37	-0.37	-0.37	-0.35
	3	-	-	-	-	-0.14	-0.10	-0.11	-0.13	-0.11	-0.08
	4	-	-	-	-	-0.07	0.04	0.04	0.04	0.03	-0.01
	5	-	-	-	-	-0.05	0.02	0.02	0.02	0.01	-0.03
	6	-	-	-	-	-0.08	0.01	0.01	0.01	-0.01	-0.03
	7	-	-	-	-	-0.11	0.04	0.03	0.03	0.00	-0.01
	8	-	-	-	-	-0.13	0.03	0.03	0.03	-0.01	0.00
	9	-	-	-	-	-0.11	0.02	0.02	0.03	0.00	0.00
	10	-	-	-	-	-0.11	0.01	0.01	0.03	0.00	0.00
		State Change=$2,000									
	1	-	-	-	-	-0.03	-0.04	-0.04	-0.06	-0.05	-0.04
	2	-	-	-	-	-0.25	-0.36	-0.37	-0.36	-0.37	-0.32
	3	-	-	-	-	-0.09	-0.09	-0.11	-0.13	-0.11	-0.06
	4	-	-	-	-	0.00	0.04	0.04	0.04	0.04	0.02
	5	-	-	-	-	0.02	0.02	0.02	0.02	0.02	0.00
	6	-	-	-	-	-0.07	-0.10	-0.10	-0.10	-0.07	-0.02
	7	-	-	-	-	-0.10	-0.07	-0.07	-0.08	-0.04	0.00
	8	-	-	-	-	-0.10	-0.06	-0.06	-0.06	-0.01	0.00
	9	-	-	-	-	-0.05	-0.03	-0.03	-0.04	0.00	0.00
	10	-	-	-	-	-0.04	-0.01	-0.01	-0.02	0.00	0.00

*The ability deciles are 2.245 , 2.395, 2.513, 2.611, 2.717, 2.825, 2.942, 3.082, and 3.287.
**The income deciles are 14.86, 25.91, 35.17, 44.47, 53.81, 63.81, 76.42, 92.98, and 120.32.

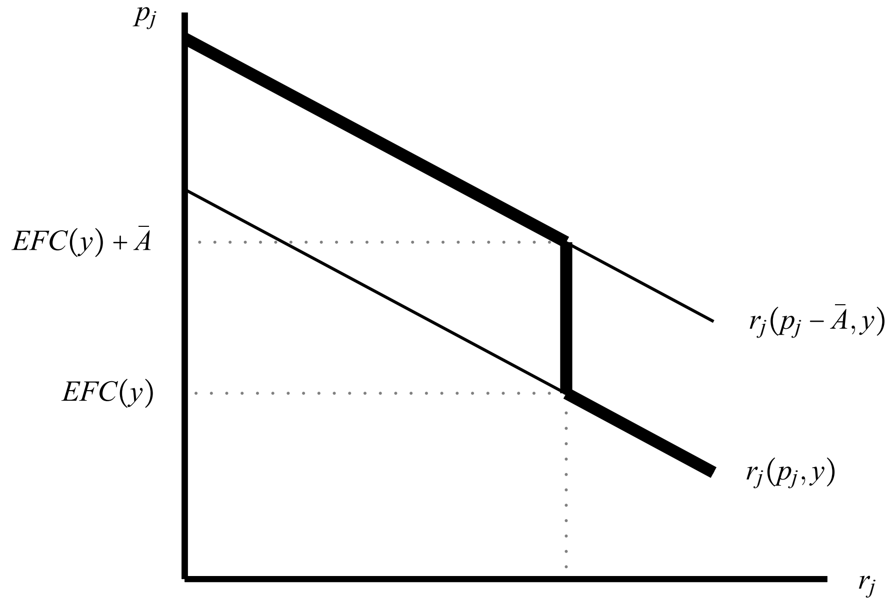


Figure 1: Demand

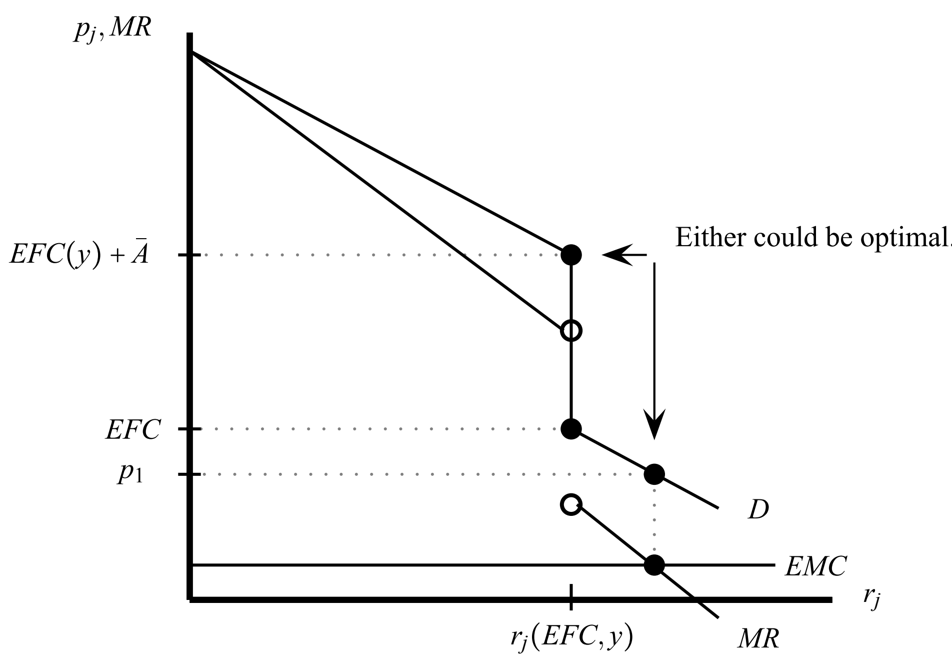


Figure 2: Optimal Pricing

Figure 3

College Ability Distributions in Baseline Equilibrium

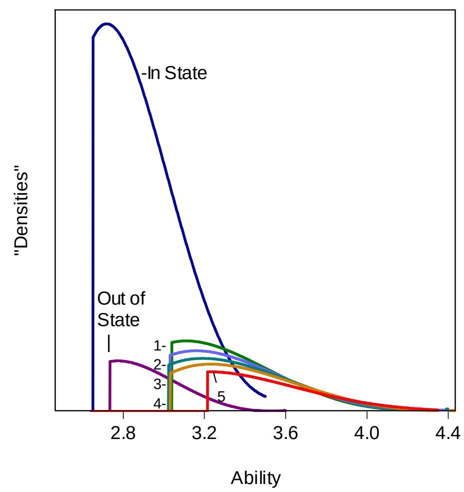


Figure 4

College Income Distributions in Baseline Equilibrium

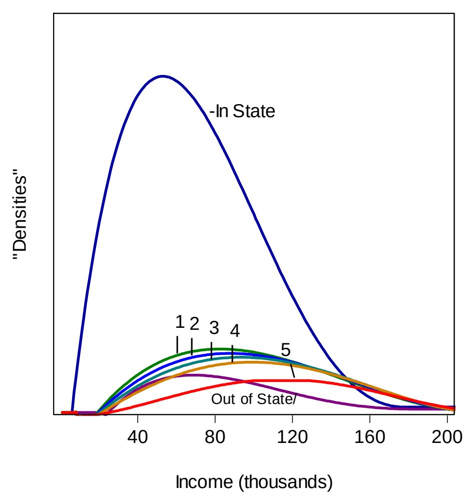


Figure 5

Effect of Reduced Federal Aid on Distributions of
Ability in Public and Private Universities

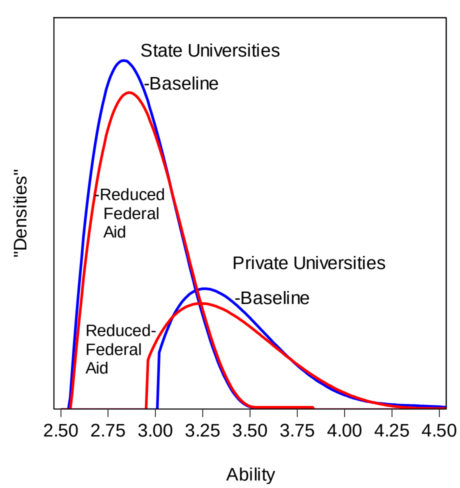
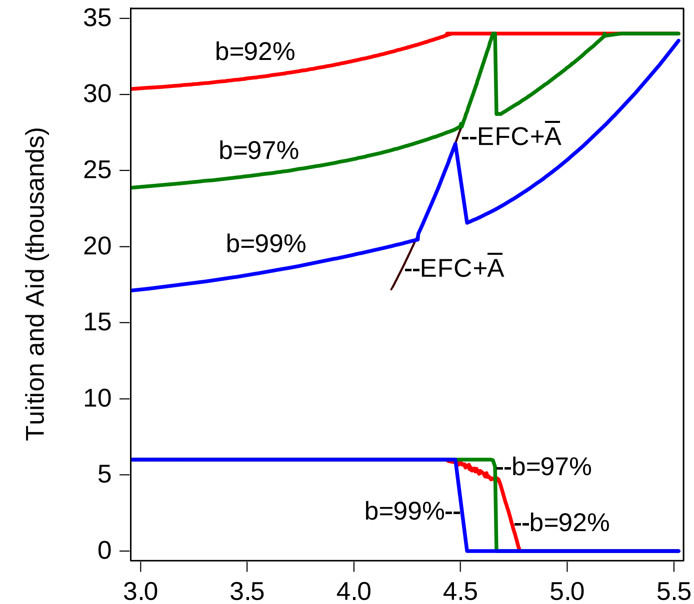


Figure 6

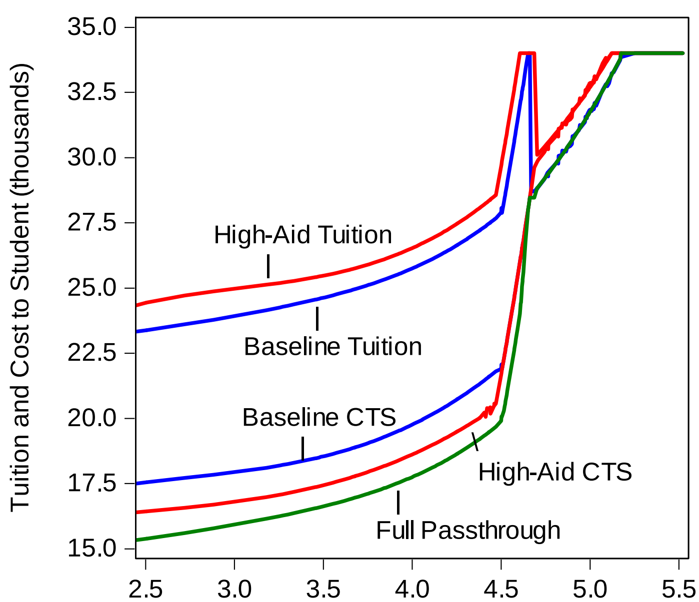
Tuition and Aid at Top Ranked School
for Three Ability Levels



Logarithm of Income

Figure 7

Effect of Increased Federal Aid on Tuition
and Cost to Student



Logarithm of Income