Time-Optimal Control of Automobile Test Drives with Gear Shifts

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Introduction

Mixed-Integer Optimal Control (MIOC)

- Optimization of dynamic processes,
- Nonlinear stiff/non-stiff ODE/DAE models,
- Discrete and continuous controls,
- Nonlinear constraints.

Tasks

- Reduce infinite-dimensional MIOCP to NLP.
- Want to avoid MINLP: How to treat discrete controls ?

Applications

- Chemistry, Bioinformatics, Engineering, Economics, ...

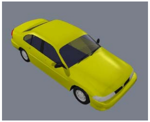
Introduction

Today's Application

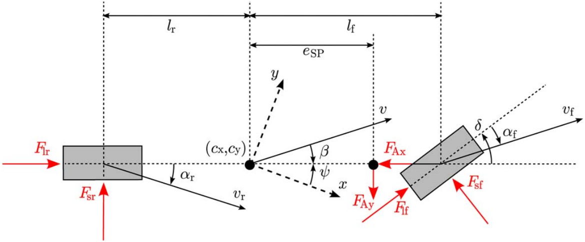
- Driver shall complete a prescribed track:

  Time optimal, energy optimal, pareto, periodic, ...

- ODE model: Vehicle dynamics.
- Continuous decisions: Acceleration, brakes, steering wheel ?
- Discrete decisions: When to select which gear ?
- Constraints: Stay on track, control bounds, engine speed, ...



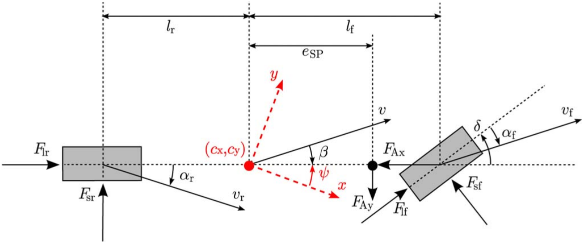
Forces



- F_{sf}, F_{lf}, F_{sr}, F_{lr}: Side and lateral forces at front and rear tyre (Pacejka),
- F_{Ax}, F_{Ay}: Accelerating forces attacking car's c.o.g.

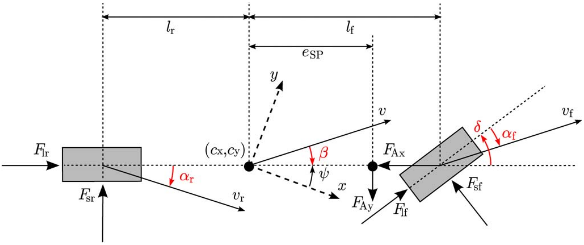
Source: M. Gerdts, *Solving mixed-integer optimal control problems by branch&bound: A case study from automobile test-driving with gear shift.* Opt. Contr. Appl. Meth. 2005; 26:118.

Coordinates



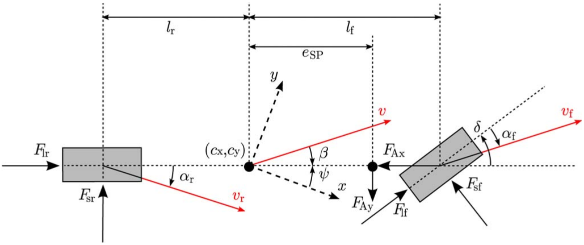
- x, y Global coordinate system,
- e_{SP} Displacement of car's center of gravity,
- c_x, c_y Car body's geometric center,
- ψ Angle of longitudinal axis against global ordinate.

Angles



- α_f Front wheel's direction of movement against longitudinal axis,
- α_r Rear wheel's direction of movement against longitudinal axis,
- β Car's direction of movement against longitudinal axis,
- δ Steering wheel angle against longitudinal axis.

Velocities



- v_f, v_r Front and rear wheel's velocity into directions α_f, α_r,
- v Car's velocity into direction β.

Controls

- $\dot{\delta}$ in $[-0.5, 0.5] \subset \mathbb{R}$: Time derivative of steering wheel angle,
- ϕ in $[0, 1] \subset \mathbb{R}$: Pedal position, translates to engine torque M_{eng},
- F_{brk} in $[0, 1.5 \cdot 10^4] \subset \mathbb{R}$: Braking force.

- μ in $\{1, 2, 3, 4, 5\} \subset \mathbb{Z}$: Selected gear, translates to gearbox transm. ratio i_{g}^{μ}.

Model part relevant for μ: Rear wheel drive

$$F_{\text{lr}}^{\mu} := \frac{i_{\text{g}}^{\mu} i_{\text{r}}}{R} M_{\text{eng}}^{\mu}(\phi, w_{\text{eng}}^{\mu}) - F_{\text{Br}} - F_{\text{Rr}},$$

$M_{\text{eng}}^{\mu}(\phi, w_{\text{eng}}^{\mu}) :=$ some nonlinear function of ϕ and engine speed w_{eng} in gear μ

Optimal Control Problem Class

Optimal Control Problem

$$
\begin{aligned}
\min_{t_\mathrm{f}, x(\cdot), u(\cdot), p} \quad & \phi(t_\mathrm{f}, x(t_\mathrm{f}), p) & \\
\text{s.t.} \quad & \dot{x}(t) = f(t, x(t), u(t), p) & \forall t \in [t_0, t_\mathrm{f}] \\
& 0 \leq c(t, x(t), u(t), p) & \forall t \in [t_0, t_\mathrm{f}] \\
& 0 = r^{\mathrm{eq}}(t_0, x(t_0), \ldots, t_m, x(t_m), p) & \\
& 0 \leq r^{\mathrm{in}}(t_0, x(t_0), \ldots, t_m, x(t_m), p) & \\
& u(t) \in \mathcal{U} \subset \mathbb{R}^{n_\mathrm{u}} & \forall t \in [t_0, t_\mathrm{f}]
\end{aligned}
$$

- ODE states trajectory $x(\cdot)$, control functions $u(\cdot)$,
- Free final time t_f and global parameters p.

Bock's Direct Multiple Shooting Method: Controls

Discretization Grid

Select a partition of the time horizon $[t_0, t_f]$ into $m-1$ intervals

$$t_0 < t_1 < \ldots < t_{m-1} < t_m = t_f.$$

Control Discretization

Select n_q base functions $b_j : \mathbb{R} \longrightarrow \mathbb{R}^{n_u}$. Using control parameters $q \in \mathbb{R}^{n_q}$, let for all $0 \le i \le m-1$

$$u_i(t) := \sum_{j=1}^{n_q} q_{ij} b_{ij}(t) \qquad \forall t \in [t_i, t_{i+1}]$$

Choices: Piecewise constant/linear/cubic splines, continuity by external constraints.

Bock's Direct Multiple Shooting Method: States

State Discretization

Introduce initial states s_i for $0 \leq i \leq m-1$ and solve m IVPs

$$\begin{aligned} \dot{x}_i(t) &= f(t, x_i(t), q_i, p) \qquad \forall t \in [t_i, t_{i+1}] \\ x_i(t_i) &:= s_i \\ s_{i+1} &= x(t_{i+1}; t_i, s_i, q_i, p) \end{aligned}$$

Advantages

- Existence of solution of IVP,
- Improve condition of BVP,
- Distribute nonlinearity,
- Supply additional a-priori information using the s_i,
- Use state-of-the-art adaptive ODE/DAE solver with IND.

Bock's Direct Multiple Shooting Method: Discrete NLP

Optimal Control NLP

$$
\begin{aligned}
\min_{t_\mathrm{f}, s_i, q_i, p} \quad & \phi(t_\mathrm{f}, s_m, p) \\
\text{s.t.} \quad & \dot{x}_i(t) = f(t, x_i(t), q_i, p) && \forall t \in [t_i, t_{i+1}]\ \forall i \\
& 0 = s_{i+1} - x_i(t_{i+1}; t_i, s_i, q_i, p) && \forall i \\
& 0 = r^{\mathrm{eq}}(t_0, x_0, q_0, \ldots, t_m, x_m, q_m, p) \\
& 0 \leq r^{\mathrm{in}}(t_0, x_0, q_0, \ldots, t_m, x_m, q_m, p)
\end{aligned}
$$

$x_i(t_{i+1}; t_i, s_i, q_i, p)$ denotes end point of solution of IVP i depending on initial values of t_i, s_i, q_i, and p.

Bock's Direct Multiple Shooting Method: Solution of NLP

Exploiting Structure

- Partial separability of objective,
- Can evaluate intervals in parallel,
- Block sparse jacobians and hessians,
- High-rank updates to hessian (modified L-BFGS).

Solution of NLP by structured SQP method

- Reduce NLP to size of single shooting system,
- Dense active-set QP solvers: QPSOL, QPOPT, qpOASES, ...

Mixed-Integer Optimal Control Problem Class

Optimal Control Problem

$$\min_{t_f, x(\cdot), u(\cdot), \omega(\cdot), p} \quad \phi(t_f, x(t_f), p)$$

$$\begin{aligned}
\text{s.t.} \quad \dot{x}(t) &= f(t, x(t), u(t), \omega(t), p) && \forall t \in [t_0, t_f] \\
0 &\leq c(t, x(t), u(t), \omega(t), p) && \forall t \in [t_0, t_f] \\
0 &= r^{\text{eq}}(t_1, x(t_1), \ldots, t_m, x(t_m), p) \\
0 &\leq r^{\text{in}}(t_1, x(t_1), \ldots, t_m, x(t_m), p) \\
u(t) &\in \mathcal{U} \subset \mathbb{R}^{n_u} && \forall t \in [t_0, t_f] \\
\omega(t) &\in \Omega \subset \mathbb{R}^{n_\omega} && \forall t \in [t_0, t_f]
\end{aligned}$$

$\Omega := \{\omega^1, \omega^2, \ldots, \omega^{n_w}\} \subset \mathbb{R}^{n_\omega}$ is a finite set of control choices, $|\Omega| < \infty$.

Inner Convexification for Integer Controls

Inner Convexification

Let Ω be the finite set of all control choices.

Relax $\omega(t) \in \Omega$ to $w(t) \in \operatorname{conv} \Omega \subset \mathbb{R}^{n_\omega}$.

Effects

+ Same number of controls n_ω.
+ Dense QPs solvers faster, less active set changes.
– Model must be evaluatable & valid for potentially nonintegral $w(t)$.
– How to reconstruct integral choice $\omega^\star(t)$ from relaxed $w^\star(t)$?

Outer Convexification for Integer Controls

Outer Convexification

For all t and for each member $\omega^i \in \Omega \subset \mathbb{R}^{n_\omega}$ introduce $w_i(t) \in \{0, 1\}$. Let then

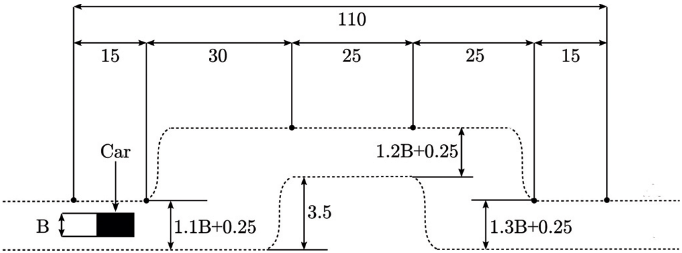
$$\omega(t) := \sum_{i=1}^{n_\text{w}} \omega^i \; w_i(t), \qquad\qquad 1 = \sum_{i=1}^{n_\text{w}} w_i(t) \quad (SOS1)$$

Relax all $w_i(t) \in \{0, 1\}$ to $w_i(t) \in [0, 1] \subset \mathbb{R}$ to obtain choice $\omega(t)$.

Effects

- − Increased number of controls $n_\text{w} = |\Omega|$ instead of n_ω.
- \+ Model can rely on integrality of the fixed evaluation points ω^i.
- \+ Relaxed solution often bang-bang in $w_i(t)$, thus integer.
- \+ If not, SUR-0.5 as ε-approximative scheme.

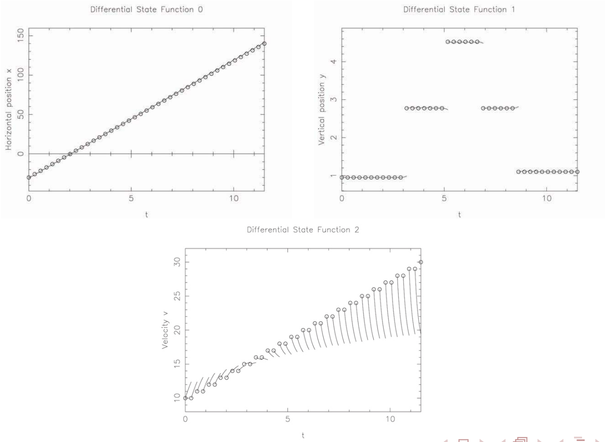
Avoiding an Obstacle



- Start to the left, driving straight ahead at 10 km/h.
- Complete track in a time-optimal fashion.
- Predefined evasive manoeuvre to avoid obstacle.

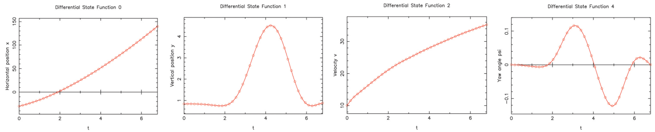
Avoiding an Obstacle: Initialization

- Example: 40 multiple shooting nodes.

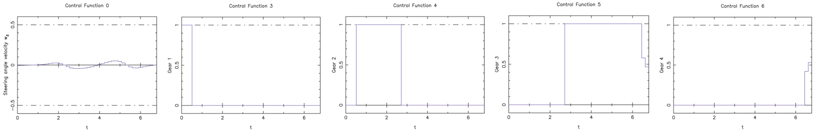


Avoiding an Obstacle: Solution

- Example: 40 multiple shooting nodes.
- Differential state trajectories:

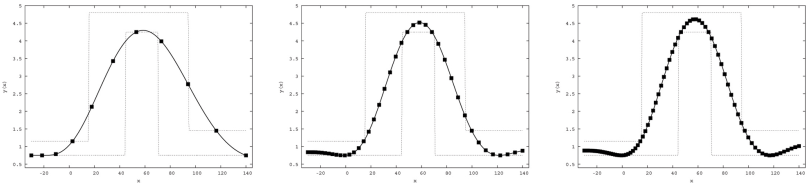


- Control trajectories:



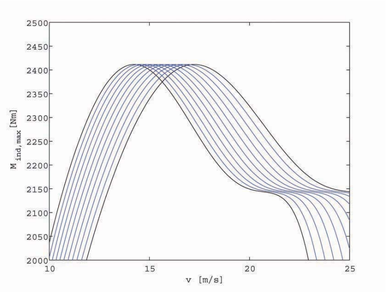
Avoiding an Obstacle: Constraint Discretization

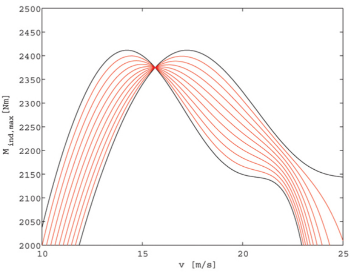
- Example: 10, 40, and 80 multiple shooting nodes.



Why is it integer ?

- Maximum indicated engine torque depending on velocity.





Computation Times

Branch & Bound

N	t_f	hh:mm:ss
10	not given	
20	6.779751	00:23:52
40	6.786781	232:25:31
80	-	-

[M. Gerdts, 2005] on a P-III 750 MHz

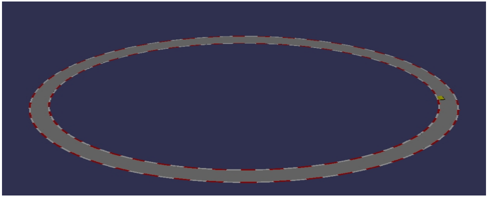
Outer Convexification

N	t_f	hh:mm:ss
10	6.798389	00:00:07
20	6.779035	00:00:24
40	6.786730	00:00:46
80	6.789513	00:04:19

[K. et al., 2008] on an Athlon 2166 MHz

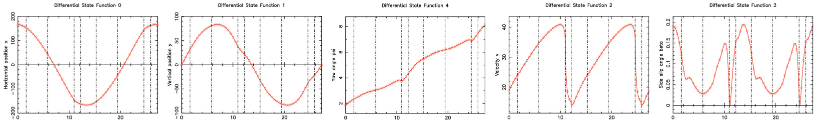
Racing on an ellipsoidal track

- Ellipsoidal track of 340m x 160m,
- Width of 5 car widths,
- Find time-optimal periodic solution.

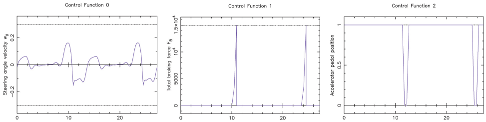


Racing on an ellipsoidal track: Solution

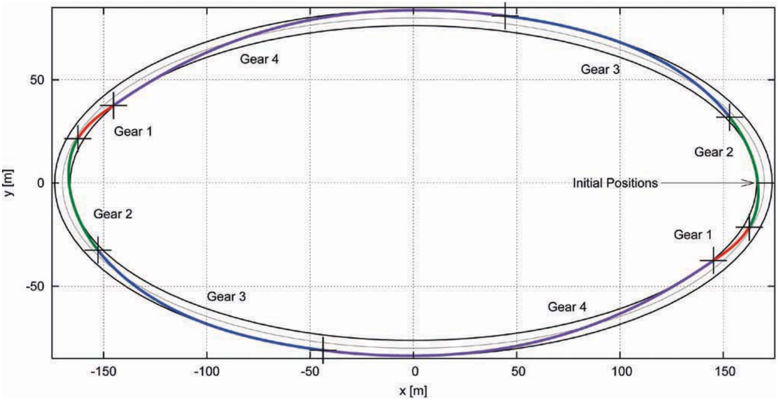
Differential state trajectories:



Control trajectories:



Racing on an ellipsoidal track: Solution



Future Work

More complicated tasks

- More complicated circuits (think Istanbul Park, Hockenheimring, ...); requires slight modification of model & coordinate system.
- More detailed modelling of integer decision effects.

More sophisticated techniques

- For longer tracks: use a moving horizon optimization technique.
- Closed-loop offline optimization.
- Closed-loop online optimization with an industry partner.

Real-time Feasibility

- Computation for reasonable discretization quite fast.
- Active set QP can solve a sequence of related problems at cheap additional cost.

References

References

- C. Kirches, S. Sager, H.G. Bock, J.P. Schlöder. *Time-optimal control of automobile test drives with gear shifts.* Opt. Contr. Appl. Meth. 2008; (submitted).
- M. Gerdts. *Solving mixed-integer optimal control problems by branch&bound: A case study from automobile test-driving with gear shift.* Opt. Contr. Appl. Meth. 2005; 26:1-18.
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- S. Sager. *Numerical methods for mixedinteger optimal control problems.* Der andere Verlag: Tönning, Lübeck, Marburg, 2005. iSBN 3-89959-416-9. `http://sager1.de/sebastian/downloads/Sager2005.pdf`.

Thank you for your attention.

Questions ?