KOÇ UNIVERSITY
College of Arts and Sciences
Department of Physics

Course: PHYS401 Quantum Mechanics I
Credits: 3
Semester: Fall 2003
Instructor: Professor **Tekin Dereli**

Final Exam: 26 January 2004, 12.15-14.00

Question: 1 Suppose $\hat{A}, \hat{B}, \hat{C}$ are linear operators. Prove the commutator identity

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}]] + [\hat{A}, \hat{C}]\hat{B}.$$

Using the canonical commutation relation $[\hat{x}, \hat{p}] = i\hbar\hat{1}$ and the above result show that

$$[\hat{x}^n, \hat{p}] = i\hbar n\hat{x}^{n-1}.$$

For any function $f(x)$ that can be expanded as a power series in x show that

$$[f(\hat{x}), \hat{p}] = i\hbar f'(\hat{x}),$$

where $'$ denotes differentiation.

Question: 2 Consider a quantum system which has only two linearly independent states

$$|1> = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad |2> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The most general state vector will a superposition state

$$|\psi(t)> = \alpha|1> + \beta|2> = \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix}$$

where α, β are complex coefficients that satisfy $|\alpha|^2 + |\beta|^2 = 1$.

i. What is the probability of finding the system at time t in the state $|1>$? In the state $|2>$?

Suppose the Hamiltonian matrix is given by

$$H = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

where A, B are real constants.

ii. Find the eigenvalues and the corresponding (normalized) eigenvectors of H.

The time evolution of the system is determined by the Schrödinger equation

$$H|\psi(t)> = i\hbar \frac{d}{dt}|\psi(t)> .$$

iii. Assume that the system starts out at $t = 0$ in the state $|1>$. What will be the state vector at some later time $t > 0$?

Question: 3 Consider an electron in the ground state of a hydrogen atom.

i. Determine the expectation values $< \frac{1}{r} >$, $< r >$, $< r^2 >$.

Express them in terms of the Bohr radius $a_0 = \frac{\hbar^2}{mke^2}$ where $k = \frac{1}{4\pi\epsilon_0}$ in MKS units.

A useful integral:

$$\int_0^\infty r^n e^{-\alpha r} dr = \frac{n!}{\alpha^{n+1}}.$$

ii. If r is measured in the ground state, what would be the most likely value to be found?

iii. What is the uncertainty in the measurement of r in the ground state?

iv. What is the probability of finding the electron inside the nucleus? (Do not try to evaluate the integral.)