Math 503, Fall 2009

Tentative Course Outline

Week 1: Review of Linear Algebra: Real and complex vector spaces, subspaces, span, linear independence, basis, dimension; A basis for the vector space of polynomials; Finite dimensional and infinite dimensional vector spaces; Examples.

Week 2: Linear operators and their properties, derivative and integral operators, matrix representation of a linear operator, zero and identity operator, vector space of linear operators.

Week 3: Invertible linear operators, characterization of finite-dimensional vector spaces, null space, examples; Inner product spaces; Orthonormal bases.

Week 4: Symmetric, skew-symmetric, and orthogonal matrices; Diagonalization; Quadratic forms.

Week 5: Review of Calculus of several variables: partial derivatives, differential, gradient and its applications and interpretations, differentiability and linear approximation;

Week 6: Taylor series expansion, quadratic approximations. Hessian, stationary points, local minimum, maximum, and saddle points.

Week 7: Double and triple integrals; Surfaces, tangent plane and surface normal, surface integrals; Continuous and differentiable vector-valued functions,

Week 6: Review of Vector Calculus: divergence and curl of a vector field; Line integrals, path independence, conservative fields; Green's theorem and applications.

Week 8: Divergence theorem in plane (with proof), Divergence theorem in R^3 (without proof), Green's identities, uniqueness theorem for Poisson equation.; Stokes' theorem and applications.

Week 9: First order ODEs: Initial value problem; Separable ODEs, Bernulli Equation, Modeling, exact linear differential operators and linear ODEs, general solution of 1st order linear ODEs, Initial value problems, existence and uniqueness theorem for the solution of the Cauchy problem for ODEs.

Week 10: General solution of second order linear ODE's. Solving second order linear homogeneous ODEs with constant coefficients ; Differential Operators; Modeling: Free Oscillations.

Week 11: Euler – Cauchy equation; Existence and uniqueness of of solution to the Cauchy problem for second order ODEs, Wronskian; Solving non-homogeneous second order linear ODEs. Method of variation of parameters; Modeling: Forced oscillations. Resonance.

Week 12: Power series solution of 2nd order linear ODEs: convergence interval and radius of convergence of power series, operations on power series, Legendre's equation, Legendre Polynomials, Frobenius method, Bessel equation , Bessel functions.

Week 13: Boundary-value problems; Sturm-Liouville problem, orthogonal functions, orthogonality of eigenfunctions, orthogonal eigenfunction expansion, completeness of orthonormal system

Week 14: Systems of ODEs: Systems of ODEs as models; conversion of higher order ODE to a system; Basic theory of systems of ODEs, general solution and Wronskian; Non-homogeneous linear systems.

Week 15: Qualitative methods for nonlinear systems: Linearization of nonlinear systems; Stability: stable and unstable critical points; Lotka –Volterra model.