# CALL CENTER OUTSOURCING CONTRACT DESIGN AND CHOICE

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# Call Center Outsourcing Contract Design and Choice

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**Abstract**

This paper considers a call center outsourcing contract design and choice problem, faced by an outsourcing vendor and a service provider. The service provider receives an uncertain call volume over multiple-periods, and is considering outsourcing all or part of these calls to an outsourcing vendor. Each call brings in a fixed revenue to the service provider. Answering calls requires having service capacity, thus implicit in the outsourcing decision is a capacity decision. Insufficient capacity implies that calls cannot be answered, which in turn means there will be a revenue loss. Faced with a choice between a volume-based and a capacity-based contract offered by an outsourcing vendor who has pricing power, the service provider determines optimal capacity levels. The optimal price and capacity of the outsourcing vendor together with the optimal capacity of the service provider determine optimal profits of each party under the two contracts being considered. Each party will prefer the contract that leads to higher profits. The paper characterizes optimal capacity levels, and partially characterizes optimal pricing decisions under each contract. The impact of demand variability and economic parameters on contract choice are explored through numerical examples. It is shown that no contract type is universally preferred, and that operating environments as well as cost-revenue structures have an important effect in outsourcing contract design and choice.

**Keywords:** call center; outsourcing; contract design; contract choice; capacity investment; exogeneous and endogeneous price.

## 1 Introduction

A growing number of companies outsource their call center operations. According to International Data Corporation (1999), the worldwide call center services market totalled $ 23 billion in revenues in 1998, and is estimated to double to $ 8.6 billion by 2003. Outsourcing constitutes 74 % of this market and is projected to be $42 billion in 2003. Datamonitor (1999) expects call center outsourcing to boom in Europe, where the \$ 7 billion market in 1999 is expected to grow to \$15 billion by 2004. In terms of outsourced agent positions, this constitutes a growth from 74,000 in 1999 to 126,500 in 2003.

While for some companies outsourcing the entire call center operation constitutes the best option, many are hesitant to hand over their most important source of customer contact to another firm. This has led to the emergence of the practice known as co-sourcing (Fuhrman, 1999), where some calls are kept in-house while others are outsourced. The decision of how to share calls in a co-sourcing setting is an important one. In practice, this can take many forms where for example certain types of calls are outsourced and others are kept in-house, or overflow calls in all functions are outsourced. Whatever the chosen form of sharing, detailed outsourcing contracts that specify requirements, service levels, and price are deemed necessary for success (Lacity and Willcox, 2001).

This paper is motivated by the call center outsourcing problem of a major mobile telecommunications service provider in Europe. The overall objective is to evaluate two types of contracts made available to this company by an outsourcing vendor (contractor). In making this comparison, detailed contracts that specify capacity (and implicitly service levels) and price within each type of contract will be considered. Like in most call center outsourcing situations, the outsourcing vendor operates a much larger call center operation compared to the one by the operator. This allows the vendor to have an advantageous cost structure and power in negotiating prices. The telecommunications service provider opted for a co-sourcing solution. Accordingly, some call types would be kept in-house while others would be shared with an outsourcing vendor. This paper is only concerned with the latter sharing of the calls. The vendor proposed two forms of sharing. The first type, which we label as *Contract 1* or as *subcontracting the base*, is a form of capacity reservation whereby the company reserves enough capacity for a steady level of calls at the outsourcing vendor for a given fee. All calls in excess of this level are considered for treatment in-house. The second type of contract, labeled as *Contract 2* or as *subcontracting the fluctuation*, stipulates that the telecommunication service provider answers all calls up to a specified level in-house, beyond which calls are diverted to the outsourcing vendor. In other words, in this type of contract overflow calls are outsourced. The vendor charges a fee per call treated. These two cases are illustrated in Figure 1. In both settings, the outsourcing vendor has pricing power. Both parties try to maximize their own profits.

While we are motivated by this particular instance, we note that the problem is common to call center outsourcing in general. As also noted by Gans and Zhou (2004), call center outsourcing

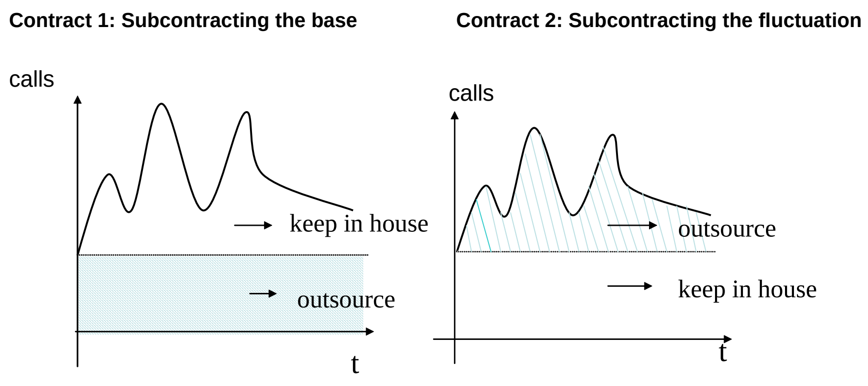


Figure 1: The two contracts offered by the outsourcing vendor

contracts are typically volume-based or capacity-based. The former type of contract implies a per-call pricing as in our Contract 2 and the latter implies a per-agent-per-hour type of pricing which resembles Contract 1. In order to model the contract choice problem, we specify these contracts in more detail, using our motivating instance to do this.

For the service outsourcing problem, we analyze the optimal capacity and pricing decisions under each contract type. We then explore the telecommunication service provider's basic question, namely which contract type they should prefer. Current practice at the company is to co-source with a contract of the form *subcontracting the base*. While basic operations management intuition would suggest that keeping the less variable portion of the demand in-house and outsourcing the high variation overflow would be more beneficial, we illustrate that both contract types may be preferred, depending on economic parameters and demand characteristics. In the following section, we provide a brief literature review. Section 3 formulates the model. This is then analyzed in order to determine optimal service capacities in Section 4 and the optimal prices in Section 5. Section 6 presents a numerical study to illustrate the relationships between contract parameters, demand characteristics, and contract preferences of each party. We provide concluding remarks in Section 7.

## 2 Literature Review

The problem of outsourcing or subcontracting has been studied in the economics literature in the context of vertical integration. This literature does not consider capacity constraints. Kamien et al. (1989), Kamien and Li (1990) first model capacity constraints, either implicitly or explicitly, in the context of subcontracting production. The supply chain literature provides a rich set of models that address supply contract design and analysis, where capacity is explicitly taken into account as a decision variable. See for example Tsay et al. (1998), Lariviere (1998), Anupindi and Bassok (1998), and references therein. As also noted by Van Mieghem (1999), these models typically consider only one party's capacity investment decision. Similar to Van Mieghem (1999), the capacity investment of both the call center and the outsourcing vendor are decision variables in our setting. We further consider the price as a decision variable for the outsourcing vendor.

Van Mieghem (1999) and many of the papers in the supply contracts literature consider the outsourcing problem in a single-period setting. The nature of the call center outsourcing problem requires a multi-period analysis, as is the case in this paper. A continuous time analysis of subcontracting, in a manufacturing context, can be found in Tan (2002), which addresses the demand variability inherent in a multi-period problem. Subcontracting is considered as a capacity option whose value is evaluated in the presence of demand uncertainty. Contract parameters are assumed to be exogenously specified, and the problem is analyzed from the standpoint of a single decision maker. Atamturk and Hochbaum (2001) provide a multi-period treatment of subcontracting, and use call centers as one of their motivating examples. In their paper, demand is deterministic, price is determined exogenously, and the capacity investment decision is once again only considered for a single party.

We consider a multi-period model of outsourcing, with multiple decision makers, in the presence of uncertain demand. Capacity investment levels of each firm and the outsourcing price are the decision variables. As such, our model combines various features found separately in previous subcontracting models in the literature. Our analysis demonstrates that for *Contract 1*, in a single period setting, the optimal decision is one where the call center outsources all calls to the outsourcing vendor. Given this result, in such a setting, our model can be analyzed as a single party capacity investment decision problem, like in Atamturk and Hochbaum (2001) and Lariviere and Porteus (2000).

Cachon and Harker (2003) is the only other study that considers outsourcing contracts in a service setting. The authors consider a queueing game between two service providers. The option of outsourcing to a contractor is one of the alternatives considered in comparing different supply chain designs for the service providers. In their outsourcing contract, the contractor charges a price per customer and ensures a service level, while the service providers guarantee a minimum demand rate to the contractor. This resembles Contract 2, though service levels and demand sharing is implicitly determined through each party's optimal capacity decisions in our setting. The Cachon and Harker (2003) analysis is concerned with the competition between two service providers, while we are interested in the specifics of the relationship between a single service provider and contractor.

In addition to differences in modelling assumptions, we also ask a different question compared to earlier papers in the operations literature. Our ultimate objective is to answer the contract choice problem faced by the telecommunications service provider, as described in the Introduction. The issue of contract choice is one which has been dealt with extensively within contract theory in economics. Rather than reviewing this literature, we point out some recent papers that deal with IT outsourcing problems. Gopal et al. (2003) perform an empirical analysis of offshore software development, where the choice between fixed price and time-and-materials type contracts are explored. It is shown that among other things, this choice is driven by requirements uncertainty. This resembles the demand uncertainty in our case. Kalnins (2004) explores empirically the role of firm relationships in choosing between these types of contracts, again in an information technology outsourcing setting. In our analysis, we will explore the role of demand uncertainty and economic parameters on such a contract choice, using a modeling approach.

Unlike most call center models, as reviewed in detail by Gans et. al. (2002), we do not model the call center as a queue in this paper. This choice is in part driven by tractability concerns, since embedding a queueing model in the games we analyze would not allow us to pursue the contract design and choice problem fully. However, we feel that this choice is not inappropriate since these types of outsourcing contract decisions are strategic decisions which would not be affected fundamentally by waiting or abandonment behavior of customers. Queueing models become necessary to analyze detailed implementations of call center outsourcing contracts, as in Gans and Zhou (2004). Viewing the call center outsourcing problem at the operational level, they analyze the routing problem faced between a service provider and outsourcing vendor. We do not consider this aspect of the problem herein.

# 3 Formulation of the Model

We consider a call center operator ($B$) which can send part or all of the calls it receives to an outsourcing vendor ($A$). Calls arrive to the operator during $N$ different periods, which represent parts of a typical day, like half-hour intervals. We assume that the number of calls in period $t$ is a real random variable $D_t$, characterized by the density $f_t(.)$, for $t \in \{1, ..., N\}$. $F_t(.)$ denotes the corresponding cumulative function, and has an inverse $F_t^{-1}(.)$. We will also use $G_t(.) := 1 - F_t(.)$

and $\tilde{G}_t(.) := \sum_{\tau=1}^{t} G_\tau(.)$, where $t \in \{0 \dots N\}$ with $\tilde{G}_0(x) := -\infty$.

A contract specifies how the arriving calls are distributed between the operator and the contractor. Calls that are not answered by either the operator or the contractor are lost. On the other hand, an answered call brings a revenue of $r$ per unit to the operator, even when that call was handled by the contractor. In the following, we analyze two types of contracts.

The operator chooses a service capacity level $K_t^B$ for each period $t$. Similarly $K_t^A$ denotes the service capacity of the contractor for period $t$. We define $c^B$ and $c^A$, the unit costs of the service capacity level per period for the operator and the contractor respectively. It is assumed throughout that $c^A < c^B$. In the first type of contract, all the $K_t^A$'s, $t \in \{1, ..., N\}$ are equal to a unique capacity $K^A$ which is fixed by the operator, while in the second one, these levels are chosen by the contractor. We denote by $K_t := K^A + K_t^B$, the total service capacity of the system in Period $t$. All parameters are common knowledge.

### 3.1 Contract 1: Subcontracting the Base

In this contract, the operator specifies the capacity level $K^A$ of the contractor. This capacity level will remain constant during the day, namely $K_t^A = K^A$ for $t \in \{1, ..., n\}$. In turn, the contractor charges a capacity reservation price $\gamma$ per unit of capacity and per period. This contract is only attractive to the operator if $c^B > \gamma$. Otherwise the operator keeps all capacity in-house.

The operator chooses $K^A$ and $K_t^B$, $t \in \{1, ..., N\}$ in order to maximize its total profit $\pi^B$, which is equal to,

$$\pi^B = \sum_{t=1}^{N} \pi_t^B \tag{1}$$

where $\pi_t^B$ is the profit for period $t$ given by:

$$\pi_t^B = rE[\min(D_t, K_t)] - c^B K_t^B - \gamma K^A. \tag{2}$$

The corresponding total profit of the contractor is equal to,

$$\pi^A = N(\gamma - c^A)K^A. \tag{3}$$

For the time being $\gamma$ can be regarded as an exogenously determined contract parameter. Section 5 focuses on the case where the contractor sets $\gamma$ in order to maximize its profit $\pi^A$.

### 3.2 Contract 2: Subcontracting the fluctuation

In this contract, the operator sends all calls it cannot answer to the contractor. The contractor charges a unit price $p$ per answered call. Calls that are not handled by the contractor do not incur any payment. This contract will lead to the outsourcing of some calls by the operator only if $r > p$ since the firm has no interest to outsource calls otherwise.

Hence, in period $t$, the operator first tries to saturate his service capacity $K_t^B$, before sending calls to the contractor. In other words, the number of calls $D_t^B$ received by the operator is equal to $\min(D_t, K_t^B)$. The corresponding number $D_t^A$ received by the contractor is then equal to $\max(0, D_t - K_t^B)$.

The operator chooses $K_t^B$, $t \in \{1, ..., N\}$ in order to maximize its total profit, which is equal to,

$$\pi^B = \sum_{t=1}^{N} \pi_t^B \tag{4}$$

where $\pi_t^B$ is the given by:

$$\pi_t^B = rE[\min(D_t, K_t)] - c^B K_t^B - pE[\min(D_t^A, K_t^A)]. \tag{5}$$

For this type of contract, the contractor chooses $K_t^A$, $t \in \{1, ..., N\}$ in order to maximize its total profit,

$$\pi^A = \sum_{t=1}^{N} \pi_t^A \tag{6}$$

where $\pi_t^A$ is given by:

$$\pi_t^A = pE[\min(D_t^A, K_t^A)] - c^A K_t^A. \tag{7}$$

Once again, as far as the capacity decisions are considered $p$ can be viewed as an exogenously determined parameter. Section 5 focuses on the more complicated pricing problem where the contractor selects $p$ in order to maximize $\pi^A$.

# 4 Optimal Service Capacities

In this section, we derive the optimal service capacity levels for both contracts. For the time being, we assume an exogenously set price $\gamma$ or $p$.

### 4.1 Contract 1: Subcontracting the Base

The following theorem provides the optimal capacity levels that the operator should set.

**Proposition 1** *The capacity levels $K^{A*}$ and $K_t^{B*}$, $t \in \{1, ....N\}$, which maximize the operator's profit function can be characterized as follows:*

1. *Determine $\phi_t$ for $t \in \{1, ....N\}$, where $\phi_t := F_t^{-1}(1 - c^B/r)$,*
2. *Re-index the periods such that $\phi_1 \leq ... \leq \phi_t \leq ... \leq \phi_N$,*
3. *Compute $\kappa^t := \tilde{G}_t^{-1}(\frac{tc^B - N(c^B-\gamma)}{r})$ for $t \in \{1, ....N\}$,*
4. *Define $t^*$ the time period such that, $t^* = \max(t, t \in \{1, ....N\}; \phi_t \leq \kappa_{t-1})$.*
5. *Compute $K^{A*}$ and $K_t^{B*}$ as follows,*

$$\begin{aligned} K^{A*} &= \max(\kappa_{t^*}, \phi_{t^*}) \\ K_t^{B*} &= \max(0, \phi_t - K^{A*}), \text{ for } t \in \{1, ....N\} \end{aligned}$$

**Proof:** Assume that the first four steps have been completed. The derivative of $\pi^B$ with respect to $K_t^B$ is equal to,

$$\frac{\partial \pi^B}{\partial K_t^B} = rG_t(K_t) - c^B \tag{8}$$

and $\pi^B$ is concave with respect to $K_t^B$. It follows that $\frac{\partial \pi^B}{\partial K_t^B} \geq 0 \Leftrightarrow K_t \leq \phi_t$. Hence, if $K^A \leq \phi_t$, $\pi^B$ reaches its maximum for $K_t^B = \Phi_t - K^A$ when all the other capacity levels are fixed. If not, the profit function is strictly decreasing and its maximum is obtained when $K_t^B = 0$. In short, $K_t^B$ is equal to $\max(0, \phi_t - K^A)$. It remains to compute the value of $K^A$ which maximizes $\tilde{\pi}^B(K_A) := \pi^B(K_A, (\max(0, \phi_t - K^A))_{t\in\{1,....N\}})$.

For a given period $\tau$, consider values of $K^A$ in the interval $[\phi_\tau, \phi_{\tau+1}]$. For all $t \leq \tau$, $K_t^B = \max(0, \phi_t - K^A) = 0$ and $K_t = K^A$. $K_t^B$ and $K_t$ are respectively equal to $\phi_t - K^A$ and $\phi_t$ otherwise. Hence the profit function is equal to:

$$\begin{aligned} \tilde{\pi}^B(K^A) = & \sum_{t=1}^{\tau} \left( r \int_0^{K^A} x f_t(x) dx + (rG_t(K_A) - \gamma) K^A \right) \\ & + \sum_{\tau+1}^{N} \left( r \int_0^{\phi_t} x f_t(x) dx + (rG_t(\phi_t) - \gamma - c^B) \phi_t \right) \\ & + (N - \tau)(c^B - \gamma) K^A. \end{aligned} \tag{9}$$

This function is twice differentiable for $\phi_\tau \leq K^A \leq \phi_{\tau+1}$, and we have for the first and second order derivatives, (we assume in the rest of the proof that derivatives computed on the lower-bound (resp. upper-bound) of an interval correspond to the right (resp. left) hand derivatives)

$$\begin{aligned}\tilde{\pi}^{B'}(K^A) &= r\tilde{G}_\tau(K^A) - \gamma\tau + (N-\tau)(c^B - \gamma) && (10)\\ \tilde{\pi}^{B''}(K^A) &= -r\sum_{t=1}^{\tau} f_t(K^A) - c^B. && (11)\end{aligned}$$

Hence, over the interval $[\phi_\tau, \phi_{\tau+1}]$, $\tilde{\pi}^B$ is a concave function, increasing if and only if $K^A \leq \kappa_\tau$. Furthermore for all $x \in [\phi_\tau, \phi_{\tau+1}]$, and $y \in [\phi_{\tau+1}, \phi_{\tau+2}]$, $\tilde{\pi}^{B''}(x) \geq \tilde{\pi}^{B''}(y)$, and since $\tilde{\pi}^B$ is also continuous, $\tilde{\pi}^B$ is concave.

From the definition of $t^*$, the right-hand derivative in $\phi_{t^*}$ is positive. It follows from the concavity of $\tilde{\pi}^B$ that the right-hand and left-hand derivatives (left-hand derivative only for $\phi_{t^*}$) are also positive for all $K^A \leq \phi_{t^*}$. Similarly, the left-hand derivative in $\phi_{t^*+1}$ is strictly negative so that the right-hand and left-hand derivatives are also negative for all $K^A \geq \phi_{t^*}$. There are then two cases depending on the sign of the left-hand derivative in $\phi_{t^*}$. If this value is strictly positive, that is if $\kappa_{t^*} > \phi_{t^*}$, then $\kappa_{t^*} \in [\phi_{t^*}, \phi_{t^*+1}]$ and $K^{A*} = \kappa_{t^*}$ from (10). If not, the left-hand derivative in $\phi_{t^*}$ is negative while its right-hand derivative is positive, so that $K^{A*} = \phi_{t^*}$. □

Since the objective function (1) is separable in the time periods, the capacity decision is independent of the pattern of $F_j$s over time. That is, different orderings of $j$ will lead to the same optimal capacity levels. The optimal capacity investment set by the contractor is the $t^\star$-th order statistic of the maximum of $\phi$ and $\kappa$.

### 4.2 Contract 2: Subcontracting the Fluctuation

For this contract, the profit functions of both the operator and the contractor are separable into $N$ profit functions $\pi_t^B$ and $\pi_t^A$ respectively. Each party specifies its service capacity for the entire horizon independently, but simultaneously. A and B act strategically, taking the other's decision into account. For each period, the contractor specifies its own service capacity, which impacts the operator's profit. Similarly, the operator's choice modifies the contractor's profit. This situation creates then a game between the operator and the contractor, whose final profits are determined by a Nash Equilibrium in each period. The following proposition specifies the capacity levels at the equilibrium for a given price $p$.

**Proposition 2** *In Period $t$, the unique Nash equilibrium is reached for the following capacity levels:*

$$K_t^{B*} = F_t^{-1}\left[\left(\left(\frac{r-c^B}{p}\right) - \left(\frac{r-p}{p}\right)\left(\frac{p-c^A}{p}\right)\right)^+\right]$$

$$K_t^{A*} = \left(F_t^{-1}\left[\frac{p-c^A}{p}\right] - K_t^{B*}\right)^+ .$$

**Proof:**

In each period, the game between the operator and the contractor is equivalent to the game between a manufacturer and a subcontractor described by Van Mieghem (1999), where the market demand for the subcontractor is zero. More precisely, consider the production-subcontracting subgame in Section 3.1 of Van Mieghem (1999) and the notations within. When the market demand for the subcontractor is zero ($D_S = 0$), the supplier does not produce goods for this market ($x_S = 0$) but produces at capacity for the manufacturer ($x_t^S = K_S$). It follows that the manufacturer outsources the surplus of his market demands ($x_t = min([D_M - K_M]^+, K_S)$) and the capacity investment game described in Section 3.2 of Van Mieghem (1999) (the choice of the capacities $(K_M, K_S)$) is equivalent to our operator-contractor game (determination of $(K_t^B, K_t^A)$). Van Mieghem (1999) shows that a unique Nash equilibrium exists for the capacity investment game. We can hence deduce a similar result for the operator-contractor capacity game. Furthermore, noting that

$$\pi_t^A = p\int_{K_t^B}^{K_t}(x - K_t^B)f(x)dx + pK_t^A[1 - F(K_t)] - c^A K_t^A$$

the best-response curve of the contractor is given by the positive solution of the following first order condition

$$p(1 - F(K_t)) - c^A = 0 \tag{12}$$

for a given $K_t^B$ (where $K_t = K_t^A + K_t^B$), and with $K_t^A = 0$ if (12) does not admit a solution. Similarly, the best-response curve of the operator is given by the following first order condition, after some simplifications,

$$\frac{\partial \pi_t^B}{\partial K_t^B} = r[1 - F(K_t)] - c^B + p[F(K_t) - F(K_t^B)] = 0. \tag{13}$$

Plugging (12) in (13), we obtain

$$F(K_t^B) = 1 - \frac{c^A + c^B}{p} + \frac{rc^A}{p^2}, \tag{14}$$

with $K_t^B = 0$ if (14) does not admit a solution. The result follows then from (12) and (14). □

## 5 Pricing decision

So far, we have assumed that the prices of the different contracts (the capacity reservation price $\gamma$ and the price per call $p$) are set exogenously by the market. In this section we assume that their values are determined by the contractor. By setting a price, the contractor may change the capacity level decisions of the operator, which may in turn impact the contractor's profit. Hence, when prices are endogenous, both contracts induce a game, in which the contractor is a Stackelberg leader. In Contract 1, given the price $\gamma$ set by A, B optimizes $K^A$ and $K_t^B$. In Contract 2, once A sets the price $p$, A and B play a Nash game to determine the equilibrium levels of $K_t^A$ and $K_t^B$. The general analysis of these games is difficult. We first restrict our study to the single-period case. Then in Section 6, through numerical examples, we explore the pricing decision in multi-period settings. In the following, $F$ and $f$ stand for $F_1$ and $f_1$ respectively.

### 5.1 Characterizing The Capacity Reservation Price

We start by defining a distribution with an increasing generalized failure rate (IGFR) as in Lariviere and Porteus (2001). A distribution is said to have an IGFR if

$$h(\epsilon) = \frac{\epsilon f(\epsilon)}{G(\epsilon)}, \tag{15}$$

is weakly increasing for all $\epsilon$ such that $F(\epsilon) < 1$. This property is satisfied by common distributions like the normal, uniform, gamma (Erlang), Weibull, etc.

**Proposition 3** *Suppose that $F$ has IGFR with a finite mean and support $[a, b)$. A Stackelberg equilibrium exists and the corresponding capacity levels $K^{A*}$, $K^{B*}$ and the reservation price $\gamma^*$ satisfy:*

$$\begin{aligned}
K^{B*} &= 0 \\
G(K^{A*})\left(1 - \frac{f(K^{A*})}{G(K^{A*})}\right) &= c^A/\gamma \\
\gamma^* &= rG(K^{A*}).
\end{aligned}$$

**Proof:** For any given $\gamma$, Proposition 1 implies that the corresponding optimal capacity levels $K^A$ and $K^B$ are equal to $\kappa^1$ and 0 respectively, since $\kappa_1 > \phi^1$ from $c^B > \gamma$. But when $K^B$ is zero, the contract is equivalent to the supply chain wholesale contract for a supplier-vendor in Lariviere and Porteus (2001), where the operator is a retailer, the contractor a manufacturer, and $K^A$ a quantity of products. Applying Theorem 1 of Lariviere and Porteus (2001) leads then to the result. $\square$

Given this equivalence result between the single-period case of Contract 1 and the problem in Lariviere and Porteus (2001), we can further draw on their Lemma 1 and conclude that $\gamma^\star$ will decrease as a function of the coefficient of variation of certain demand distributions (for example uniform, gamma, normal). This is demonstrated in the numerical analysis section.

## 5.2 Characterizing The Price Per Call

A complete characterization of the optimal price is difficult in this case, however the following bounds can be established on the optimal value of $p$. Let $c_0 = (c^A + c^B)/2$ and denote by $\rho$ the ratio $\rho = rc^A/c_0^2$.

**Proposition 4** *The price per call at the equilibrium $p^*$ verifies*

$$\rho c_0 \leq p^* \leq r$$

*If furthermore $\rho \leq 1$ then*

$$p^* \geq c_0 \left(1 + \sqrt{1-\rho}\right)$$

**Proof:** $p^*$ is the maxima of $\pi^A = p \min((D - K^B(p), K^A(p))^+ - c^A K^A(p))$, subject to $\pi^B(p)$ being larger than the profit of the call center operator when it does not outsource any calls. We can then restrict $p$ such that $p \geq rc^A/c^B$, since $K^A$ and then $\pi^A$ are zero otherwise from the expression of $K^A$ of Proposition 2.

Note then that

$$\begin{aligned} \frac{\partial \pi^A}{\partial p} &= \int_{K^B}^{K} G(x)dx + p\left(G(K)\frac{\partial K}{\partial p} - G(K^B)\frac{\partial K^B}{\partial p}\right) \\ &\quad -c^A \frac{\partial (K - K^B)}{\partial p} \\ &= \int_{K^B}^{K} G(x)dx + p\left(F(K^B) - F(K)\right)\frac{\partial K^B}{\partial p} \end{aligned} \tag{16}$$

where the last equality is true since $pG(K) = c^A$ from Proposition 2. Since $p \geq rc^A/c^B$, we have $F(K^B) - F(K) \leq 0$. As a result, if $\partial K^B/\partial p$ is negative for a given $p$ then $\partial \pi^A/\partial p$ is positive, and $p$ cannot be an equilibrium. A direct computation leads then to

$$\frac{\partial K^B}{\partial p} = 2\left(\frac{c_0}{p^2} - \frac{rc_A}{p^3}\right)\frac{1}{f(K^B)} \tag{17}$$

which is negative or zero if and only if $p \leq rc_A/c_0$. It follows that $p^* \geq rc_A/c_0$.

Assume now that $rc^A \leq c_0^2$. From Proposition 2, $K^B$ is zero if and only if $p \in [p_1, p_2]$ where

$$\begin{aligned} p_1 &= c_0\left(1 - \sqrt{1 - \frac{rc^A}{c_0^2}}\right) \\ p_2 &= c_0\left(1 + \sqrt{1 - \frac{rc^A}{c_0^2}}\right) \end{aligned}$$

and the corresponding derivative of $\pi^A$ with respect to $p$ is reduced to

$$\frac{\partial \pi^A}{\partial p} = \int_0^K G(x)dx \tag{18}$$

which is positive or null. It follows then that $p \notin [p_1, p_2]$. Furthermore since $rc^A/c_0 \in [p_1, p_2]$, $\pi^A$ is increasing for $p < p_1$ from (16 ) and (17), and we have $p^* \geq p_2$.

Finally, let us show that $p^* \leq r$. From the definition of $\pi^B$ and Proposition 2, we have

$$\frac{\partial \pi^B}{\partial p} = (r-p)G(K)\frac{\partial K}{\partial p} + \frac{p-r}{p}c^A\frac{\partial K^B}{\partial p} - \int_{K^B}^K G(x)dx. \tag{19}$$

which is negative for $p \geq r$ ($\partial K^B/\partial p$ is negative since $p \geq r \geq rc_A/c_0$). In this case, $\pi^B(p)$ is then less or equal to $\pi^B(r)$ which is also the profit of the call center operator when it does not outsource calls. Hence $p^* \leq r$. $\square$

On a more technical note, Proposition 4 refined the relationship between Capacity $K^B$ and the price $p$ provided by Proposition 2. More precisely the price needs to satisfy Proposition 2, and can take one of the following two values $c_0(1+/-\sqrt{1-\rho G(K^B)})/G(K^B)$. Note however that $c_0(1-\sqrt{1-\rho G(K^B)})/G(K^B) \leq \rho c_0$. Hence, from Proposition 4 the price must be equal to

$$p = c_0\left(\frac{1+\sqrt{1-\rho G(K^B)}}{G(K^B)}\right). \tag{20}$$

We next partially characterize the Stackelberg game for distributions with non-increasing failure rate (DFR). A distribution is said to have a DFR if $f(\epsilon)/G(\epsilon)$ is non-increasing. Characterizations of similar games have been provided before in Lariviere and Porteus (2001) for demand distributions having IGFR, and Dong and Rudi (2004) for normally distributed demand. Note that the problem being considered herein is significantly more difficult, due to the fact that the contractor's profit function (corresponding to the manufacturer in the mentioned papers) is not deterministic. As a result, the first order conditions one gets for $p^*$ depend on both the density and the distribution of the demand (and functions thereof) in a complicated way, thus rendering the analysis less tractable. Changes of variables, as in the above papers, do not circumvent the problem because of the complex relationship between $K^B$ and the price $p$ as illustrated by equation (20).

In the following we provide sufficient conditions for which the equilibrium price is equal to $r$ so that the operator never prefers outsourcing the fluctuation (Contract 2).

**Proposition 5** *If $F$ has DFR, the equilibrium $p^*$ is always equal to $r$ and the call center operator never prefers outsourcing the fluctuation (Contract 2) as a contract.*

**Proof:** If $F$ has DFR then for all $K^B \geq x$, $G(K^B)/f(K^B) \leq G(x)/f(x)$. As a result,

$$\int_{K^B}^{K} G(x)dx = \int_{K^B}^{K} \frac{G(x)}{f(x)} f(x)dx \geq \frac{G(K^B)}{f(K^B)} \Big(F(K) - F(K^B)\Big) \tag{21}$$

From Equation (16) and Proposition 2 we have

$$\frac{\partial \pi^A}{\partial p} \geq \frac{F(K) - F(K^B)}{f(K^B)} \frac{rc^A}{p^2} > 0 \tag{22}$$

Hence $\pi^A$ is strictly increasing and $p^* = r$, which is also the operator profit when it does not outsource calls and the operator never prefers outsourcing the fluctuation (Contract 2). □

Depending on their parameters, the gamma, Weibull and lognormal distributions can all have DFR. (The gamma and Weibull distributions always have IGFR so that Proposition 3 applies in these case.) In particular the exponential distribution has a constant failure rate, and $p^* = r$ from Proposition 5. When the failure rate is increasing, the existence of $p^*$ is not easy to show. Our numerical results suggest that $\pi^A$ can be concave or convex depending on the distribution, that is

$$\frac{\partial^2 \pi^A}{\partial^2 p} = \frac{c^A}{p} \frac{\partial K}{\partial p} - \frac{c^A + c^B}{p} \frac{\partial K^B}{\partial p} + \frac{rc^A - c^B p}{p^2} \frac{\partial^2 K^B}{\partial^2 p} \tag{23}$$

is either negative or positive for $p \in [\rho c_o, r]$ (from Proposition (4)). As a consequence, when the distribution has an increasing failure rate, $p^*$ can be less than $r$. For instance, the uniform distribution has an increasing failure rate and $p^* = 3rc^A/c^B$ as stated by the following proposition.

**Proposition 6** *If demand is uniformly distributed, the equilibrium $p^*$ is always equal to*

$p^* = \min(3rc^B/c^A, r)$.

**Proof:** Assume that demand has an uniform distribution over $[a, b]$. It follows that when $x \in [a, b]$, $G(x) = (b - x)/d$ with $d = b - a$, $\int_0^x G(u)du = (1 - x/2)x/d$ and $f(G^{-1}(x)) = dx + a$. Using (16), (17) and Proposition 2, we compute

$$\frac{\partial \pi^A}{\partial p} = \frac{F(K) - F(K^B)}{2ap^2}(3rc^A - c^B p)$$

which is positive if and only if $p < 3rc^A/c^B$. From Proposition 4, $p^* \in [\rho c_0, r]$ where $3rc^A/c^B > \rho c_0$ and the result directly follows. □

## 6 Numerical Analysis

This Section will explore optimal prices, profits, and capacities for the two contracts under different environments. These environments will be described by the variability of demand and the economic parameters $r, c^A, c^B$ that establish the margins for each party. We consider two types of demand variability: *within period variability*, which is determined by the demand distribution of a given time period and *between period variability*, which refers to the demand pattern across multiple periods, captured through the change in the parameters of a particular demand distribution. Our first objective is to develop a general understanding of each contract under steady demand (i.e. no between period variability). We then explore the impact between period variability has on contract behavior. Restricting our attention to a particular level of within period variability, we then explore the role economic parameters have on these contracts. For all settings, our ultimate objective is to address the contract choice problem posed in the Introduction: under what conditions does each party prefer a particular contract?

Viewed in a different context, the contract choice problem resembles the outsourcing versus contract manufacturing problem faced by OEMs in electronics, as described by Plambeck and Taylor (2001). OEM outsourcing refers to the case where an OEM only outsources part of its production, whereas contract manufacturing constitutes the case where the entire production is outsourced. In a single period setting, we show that under *Contract 1* all capacity is outsourced, like in contract manufacturing, whereas under *Contract 2*, some capacity is kept in-house, like in OEM outsourcing. Our numerical examples will explore the multi-period setting, and illustrate that when there is between period demand variability both parties can invest in some capacity under both contracts.

## 6.1 The role of within period variability

We consider the simplest multi-period setting with three time periods. In order to isolate the effect of within period variability, this section considers an identical demand distribution in each time period. For the experiments we use the Erlang family of distributions. The m-stage Erlang distribution (referred to as Erlang-m) is completely characterized by its mean and the number of stages $m$, and has the following probability density function:

$$f(x) = \frac{x^{m-1}e^{-x/\theta}}{\theta^m \, (m-1)!} \quad x \geq 0$$

where $m$ is positive integer and $\theta$ is a positive real number. The mean of the distribution is $m\theta$. Note that the Erlang-1 distribution is the simple exponential distribution. Focusing on the Erlang family enables to systematically investigate the effect of variability since an Erlang-m distribution is more *variable* than an Erlang-m' distribution with the same mean when $m' > m$ according to a convex stochastic order. This, naturally, implies that the variance is decreasing in $m$ (for identical means). Finally, Erlang distributions possess the IGFR property required by Proposition 3.

We analyze four demand distributions, Exponential, Erlang-2, Erlang-10, and Erlang-100, going from high (H) within period variability, to low (L) within period variability. The exponential (Erlang-1) represents a highly variable demand, the Erlang-10 resembles a Normal Distribution with a small coefficient of variation, and Erlang-100 is used as a test case for extremely low variability.

When an identical demand distribution is assumed in each period, the multi-period problem is structurally equivalent to the single-period problem. In order to compute the optimal capacities we make use of Propositions 1 and 2. The optimal price for each contract is then calculated via a numerical search (using discrete intervals of 0.01). For Contract 2, we also make use of the bounds established in Proposition 4 to restrict the search region. Finally, for the exponential distribution Proposition 5 directly yields the optimal price for Contract 2.

Demand is steady (independent and identically distributed in each period) with a mean of 30. In these examples $c^A = 2$, $c^B = 5$, and $r = \{6,\ 10\}$. For each contract Tables 1, 2, and 3 tabulate optimal prices, profits, and capacities.

| | $(r, c^A, c^B)$=**(6,5,2)** | | | | | $(r, c^A, c^B)$=**(10,5,2)** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\gamma^*$ | $\pi^{A^*}$ | $\pi^{B^*}$ | $K^{A^*}$ | $K^{B^*}$ | $\gamma^*$ | $\pi^{A^*}$ | $\pi^{B^*}$ | $K^{A^*}$ | $K^{B^*}$ |
| **Exp.** | 3.76 | 74.02 | 43.45 | 14.02 | (0,0,0) | 5 | 187.15 | 138.06 | 20.79 | (0,0,0) |
| **Erl-2** | 4.30 | 108.88 | 48.65 | 15.78 | (0,0,0) | 5 | 226.22 | 214.12 | 25.22 | (0,0,0) |
| **Erl-10** | 4.99 | 188.10 | 52.76 | 20.97 | (0,0,0) | 5 | 260.39 | 338.9 | 29.03 | (0,0,0) |
| **Erl-100** | 4.99 | 243.17 | 77.77 | 27.2 | (0,0,0) | 5 | 268.29 | 415.04 | 29.91 | (0,0,0) |

Table 1: Contract 1 for steady demand

The following observations can be made for Contract 1:

- $\gamma^*$ can take values that are strictly less than $c^B$, implying that B can earn more than its participation profit under this contract.
- Both $\pi^{A^*}$ and $\pi^{B^*}$ are decreasing in demand variability.
- $\gamma^*$ is decreasing in demand variability.
- B makes no capacity investment under steady demand.
- Total capacity investment is decreasing as a function of demand variability.

| Demand Dist. | $p^*$ | $K^{A*}$ | $K^{B*}$ | $\pi^{A^*}$ | $\pi^{B^*}$ |
|---|---|---|---|---|---|
| **Exp.** | 6 | (27.49, 27.49, 27.49) | (5.47, 5.47, 5.47) | 105.07 | 7.95 |
| **Erl-2** | 6 | (23.37, 23.37, 23.37) | (10.97, 10.97, 10.97) | 97.38 | 20.53 |
| **Erl-10** | 4 | (29.00,29.00,29.00) | (0,0,0) | 135.21 | 154.63 |
| **Erl-100** | 4 | (29.90, 29.90, 29.90) | (0,0,0) | 165.66 | 172.53 |

Table 2: Contract 2 for steady demand: $(r, c^B, c^A)$=(6,5,2)

The above observations are consistent with Proposition 3 and the results in Lariviere and Porteus (2001).

| Demand Dist. | $p^*$ | $K^{A*}$ | $K^{B*}$ | $\pi^{A^*}$ | $\pi^{B^*}$ |
|---|---|---|---|---|---|
| **Exp.** | 10 | (27.49, 27.49, 27.49) | (20.79, 20.79, 20.79) | 105.07 | 138.08 |
| **Erl-2** | 10 | (19.74, 19.74, 19.74) | (25.18, 25.18, 25.18) | 78.04 | 213.37 |
| **Erl-10** | 10 | (8.55, 8.55, 8.55) | (29.00, 29.00, 29.00) | 35.16 | 338.03 |
| **Erl-100** | 10 | (2.59, 2.59, 2.59) | (29.90, 29.90, 29.90) | 10.88 | 414.15 |

Table 3: Contract 2 for steady demand $(r, c^B, c^A)$=(10,5,2)

The following observations can be made for Contract 2 from Tables 2 and 3:

- The optimal price $p^*$ is equal to $r$ except in two instances involving low variability (Erlang-10 and Erlang-100) distributions. Thus even for increasing failure rates, i.e. when these rates are low, we find the same result as shown in Proposition 5. As a result B can rarely earn more than its participation constraint.
- $\pi^{B^*}$ is increasing in demand variability. $\pi^{A^*}$ is decreasing (in general) in demand variability but can be non-monotonic when the optimal price changes (as observed in two instances).
- $p^*$ is non-decreasing in demand variability.
- B invests in some capacity unless $p^* < c^B$.
- Total capacity investment by both parties is higher under Contract 2 in all cases (compared to Contract 1).
- The relationship between demand variability and total capacity investment is not monotonic under this Contract.

The first observation implies that Contract 2 will rarely be preferred by B. Contrasting this to the first observation for Contract 1, we expect Contract 1 to be preferred by B more frequently. Coupled with the differences in the response of $\pi^{A^*}$ and $\pi^{B^*}$ to variability, we anticipate contract choice to change as a function of demand variability. This will be explored below.

## 6.2 The impact of between period variability

In this section, we drop the assumption that demand is steady and consider the effect of between period variability. The calculations are done in a similar manner using the analytical results in combination with a numerical search for the optimal prices. Mean demands are assumed to be $(15, 60, 15)$. Note that the total mean demand is the same as before, however the way it is distributed over time is different. All other parameters are the same. For each contract Tables 4, 5 and 6 tabulate optimal prices, profits, and capacities.

| | $(r, c^A, c^B)$=**(6,5,2)** | | | | | $(r, c^A, c^B)$=**(10,5,2)** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\gamma^*$ | $\pi^{A^*}$ | $\pi^{B^*}$ | $K^{A^*}$ | $K^{B^*}$ | $\gamma^*$ | $\pi^{A^*}$ | $\pi^{B^*}$ | $K^{A^*}$ | $K^{B^*}$ |
| **Exp.** | 3.68 | 52.46 | 32.50 | 10.41 | (0, 0.53, 0) | 4.79 | 136.40 | 138.13 | 16.30 | (0, 25.29, 0) |
| **Erl-2** | 4 | 64.14 | 44.74 | 10.69 | (0, 11.24, 0) | 4.99 | 113.20 | 213.74 | 12.62 | (0, 37.73, 0) |
| **Erl-10** | 4.99 | 94.18 | 52.45 | 10.5 | (0, 31.33, 0) | 4.99 | 130.24 | 338.47 | 14.52 | (0, 43.49, 0) |
| **Erl-100** | 4.99 | 121.63 | 77.36 | 13.56 | (0, 40.63, 0) | 4.99 | 134.19 | 414.59 | 14.96 | (0, 44.84, 0) |

Table 4: Contract 1 under fluctuating demand

| Demand Dist. | $p^*$ | $K^{A*}$ | $K^{B*}$ | $\pi^{A^*}$ | $\pi^{B^*}$ |
|---|---|---|---|---|---|
| **Exp.** | 6 | (13.74, 54.98, 13.74) | (2.73, 10.94, 2.73) | 105.07 | 7.95 |
| **Erl-2** | 6 | (11.68, 46.74,11.68) | (5.48, 21.93, 5.48) | 97.38 | 20. 53 |
| **Erl-10** | 4 | (14.50,58.01,14.50) | (0,0,0) | 135.21 | 154.63 |
| **Erl-100** | 4 | (14.95, 59.80, 14.95) | (0,0,0) | 165.66 | 172.53 |

Table 5: Contract 2 under fluctuating demand: $(r, c^B, c^A)$=(6,5,2)

Comparing the results in this Section to the previous one, we note the following:

- $\gamma^*$ is also decreasing as a function of between period demand variability.
- $\pi^{A^*}$ and $\pi^{B^*}$ under Contract 2 do not depend on how demand is allocated between periods.
- B invests in some capacity even under Contract 1, due to the fluctuation in demand. However total capacity investment is still higher under Contract 2.

| Demand Dist. | $p^*$ | $K^{A*}$ | $K^{B*}$ | $\pi^{A^*}$ | $\pi^{B^*}$ |
|---|---|---|---|---|---|
| **Exp.** | 10 | (13.74, 54.98, 13.74) | (10.40, 41.59, 10.40) | 105.07 | 138.08 |
| **Erl-2** | 10 | (9.87, 39.48, 9.87) | (12.59, 50.35, 12.59) | 78.04 | 213.37 |
| **Erl-10** | 10 | (4.28, 17.10, 4.28) | (14.50,58.01,14.50) | 35.16 | 338.03 |
| **Erl-100** | 10 | (1.29, 5.18, 1.29) | (14.95,59.80,14.95) | 10.88 | 414.15 |

Table 6: Contract 2 under fluctuating demand: $(r, c^B, c^A)$=(10,5,2)

These observations imply that there is an important effect that between period variability has on contract characteristics. This in turn suggests that between period variability will play a role in determining contract choice.

### 6.3 Contract choice

Based on the profits tabulated for the previous set of examples, Table 7 summarizes contract choice by each party. Whenever B appears in both the columns for Contract 1 and Contract 2, this means that both contracts force B to its participation constraint, and it is assumed that B is indifferent between these contracts. The following observations can be made:

- Focusing on the steady demand cases in the first two columns, we observe that within period variability can change preferences for contracts.
- As seen in the first row, between period variability can induce a complete switch in preferences by both parties.
- Rows two through four show that between period variability can change only one party's preference as well, thus enabling a clear preference for one of the contracts by both parties.
- Contract preferences differ as a function of margins, as demonstrated by the cases for $r = 6$ and $r = 10$.
- In these examples, the margin effect seems to dominate the earlier mentioned variability effects. For the high margin cases, B is forced to its participation contraint in both contracts. Assuming B is indifferent, we can state that Contract 1 is preferred in all these cases.
- In the low margin cases, joint contract preferences are more difficult to obtain.
- Contract 2 is preferred by both parties only in cases that combine low margins, very low within period variability, and high between period variability.

| | | (30,30,30) | | (15,60,15) | |
|---|---|---|---|---|---|
| $(r, c^B, c^A)$ | | **Contract 1** | **Contract 2** | **Contract 1** | **Contract 2** |
| **(6,5,2)** | **Exp.** | B | A | B | A |
| | **Erl-2** | A,B | - | B | A |
| | **Erl-10** | A | B | - | A,B |
| | **Erl-100** | A | B | - | A,B |
| **(10,5,2)** | **Exp.** | A,B | B | A,B | - |
| | **Erl-2** | A,B | B | A,B | B |
| | **Erl-10** | A,B | B | A,B | B |
| | **Erl-100** | A,B | B | A,B | B |

Table 7: Contract choice by A and B

## 6.4 The impact of economic parameters

In this Section we focus on the Exponential demand distribution case, and explore the role of cost parameters $c^A$, $c^B$ and revenue $r$ further. As shown before, $p^* = r$ in all cases. Rather than reporting detailed profit and capacity results, we tabulate contract choice for these examples. The examples in rows 1-3 and 4-6 have $r$ and $c^A$ fixed for different values of $c^B$. A comparison of each party's choice demonstrates that the relative value of $c^B$, through its impact on both A's and B's margins, can change contract preferences. These examples also show that $c^B$ needs to be high enough (like in 6,5,2 or 10,8,2) to make outsourcing worth pursuing for B. Rows 1 and 6 as well as 5 and 7 keep the cost structure the same while varying $r$. These examples confirm earlier observations that changing the value of $r$ impacts contract preferences. Finally, rows 2 and 5 demonstrate how A's preference can change as a function of between period variability. Based on these examples, we

| | (30,30,30) | | (15,60,15) | |
|---|---|---|---|---|
| $(r, c^B, c^A)$ | **Contract 1** | **Contract 2** | **Contract 1** | **Contract 2** |
| (6,5,2) | B | A | B | A |
| (6,4,2) | A,B | B | B | A,B |
| (6,3,2) | A,B | B | A,B | B |
| (10,8,2) | B | A | B | A |
| (10,6,2) | A,B | - | B | A |
| (10,5,2) | A,B | B | A,B | B |
| (20,6,2) | A,B | - | A,B | B |

Table 8: Contract choice under an Exponential demand distribution

can state that all economic parameters have an important role in determining contract choice by defining profit margins available to each party.

### 6.5 Summary

The different types of demand variability that were considered are observed to have two different effects in these examples. The within period demand variability has an impact on the optimal price. Thus, under Contract 1, as also in Proposition 3, lower within period demand variability implies higher $\gamma$ values in equilibrium, approaching the $c^B$ upper limit. The optimal price $p$ under Contract 2 on the other hand, approaches its upper value $r$ for higher values of $r$ and for environments with high within period demand variability. Whenever $p = r$ or $\gamma$ approaches $c^B$ the operator earns profits equal to the case when everything is performed in-house. As a result, whenever price is at its upper-bound for one contract, the operator prefers outsourcing with the other contract. This is so as long as the other contract enables an optimal price $p$ or $\gamma$ that is below their upper bound values. When $\gamma^* = c^B$ and $p^* = r$, $\gamma^* < p*$ so that for such a case Contract 1 with subcontracting the base is preferred by the operator. The contractor's preference also depends on the between period variability effect. Indeed we find both in some low within period, low revenue cases and some high within period, high revenue cases that the contractor's preference switches from one contract to the other as a function of the between period demand variability. This example underlines the importance of considering the multi-period effect in such outsourcing contract choice problems.

Our examples demonstrate that contract choice depends on between period demand fluctuations, as our operations management intuition would suggest, but also on the within period demand variability through its effect on pricing. Similarly economic parameters, through their effect on the optimal pricing decision, impact this choice. Thus the operator's choice of Contract 1 turns out to be also the contractor's choice in high margin settings, or under particular variability conditions when margins are low. In the example, total capacity investment by the operator and contractor in each period is higher under Contract 2, thus customer service is better when this Contract is preferred by both parties. However conditions that ensure this are found to be quite restrictive. Despite our initial intuition that the contract which subcontracts the fluctuation is structurally more appropriate for a setting where the contractor's capacity cost is lower, due to pricing effects (driven in part by within period variability) this contract is not preferred by both in most settings. As is also evident from Proposition 1, in the multi-period setting with fluctuating demand, the operator may choose to invest in capacity under both contracts. Thus we find that the between period variability effect can induce a switch from a contract manufacturing to outsourcing type of solution in the electronics manufacturing context. For call centers, our examples suggest that both contract types will lead to a co-sourcing solution whereby calls are shared by both parties in most cases. Unless call volumes are steady across periods (under Contract 1) or within period variability and margins are low (under Contract 2), B will not prefer to outsource all calls to A.

In summary, we find that none of the contracts are universally preferred, and that preferences change as a function of within period variability, between period variability, and profit margins determined by the values of $r$, $c^A$, and $c^B$. Our results further demonstrate that the pricing decision plays an important role in this choice, and that taking price as an exogenously given parameter could be misleading. The analysis points to the need for a good understanding of the operating environment of service companies, before contract decisions are made for outsourcing.

# 7 Concluding Remarks

This paper is the first to model call center outsourcing contracts, and to explore these in terms of design and contract choice. While the motivating example came from a call center, the results would also be applicable to other types of service outsourcing like the outsourcing of back-office functions. Key distinguishing features of the model are the presence of multiple decision makers, uncertain demand, endogenous pricing decisions, and a multi-period decision horizon. We find that all of these features have an important impact on the contract choice problem, and that the qualitative nature of the results change as a function of these features. This points to the importance of taking them into account in answering questions about service outsourcing contracts. From a modeling standpoint, one can conclude that the endogenous pricing feature is the one that complicates the analysis the most. Whenever the outsourcing vendor is a price taker, our analysis fully characterizes both contracts.

For managers who face these types of contract design and choice problems, our analysis demonstrates that in addition to a knowledge of economic parameters like costs, managers need to have a very good understanding of the underlying demand uncertainties. None of the contracts are preferred under all conditions, and different choices lead to different outcomes in terms of profits and customer service provided. Evaluation of different contract choices should not be simplified to a cost per transaction basis. Our analysis suggests that when margins are relatively high, subcontracting the base is preferred by both parties. Subcontracting the fluctuation is only preferred by both parties when margins are low, inherent demand variability is low but between period fluctuations are high. Since this contract typically leads to higher capacity investment, further research may explore hybrid contracts that ensure these high service levels as well as preference by both parties under more general conditions.

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