A circular loop of flexible iron wire has an initial circumference of 150.0 cm, but its circumference is descending at a constant rate of 10.0 cm/s due to a tangential pull on the wire. The loop is in a constant, uniform magnetic field oriented perpendicular to the plane of the loop and with magnitude 0.5000 T. (Take $\pi=3$).

(a) Find the emf induced in the loop at the instant when 9.0 s have passed.
(b) Find the direction of the induced current in the loop as viewed looking along the direction of the magnetic field.

\[ L = 2\pi r \]
\[ A = \pi r^2 \]
\[ A = \frac{c^2}{4 \pi} \]
\[ \frac{dA}{dt} = \frac{2c}{4 \pi} \frac{dc}{dt} = \frac{c}{2 \pi} \frac{dc}{dt} \]

\[ \frac{d}{dt} \left( \frac{dA}{dt} \right) = \frac{8c}{2 \pi} \left| \frac{dc}{dt} \right| \]

At \( t = 9 \text{s} \), \( c = 150 \text{cm} - 10 \text{cm/s} \cdot 9 \text{s} = 60 \text{cm} \)

\[ \frac{dE}{dt} = \frac{0.5 \text{T}}{2.3} (0.6 \text{m}) (0.1 \text{m/s}) = 0.005 \text{V} = 5 \text{mV} \]

\[ \text{b) Taking } \vec{\mathbf{A}} \text{ into the page, then ind } \mathbf{E} \text{ is clockwise.} \]

To oppose the change, by the Lenz law.
A circular loop of wire with radius $r=0.0500 \, m$ and resistance $R=0.150 \, \Omega$ is in a region of spatially uniform magnetic field, as shown in the figure. The magnetic field is directed out of the plane of the figure. The magnetic field has an initial value of $8.00 \, T$ and is decreasing at a rate of $dB/dt=-0.600 \, T/s$. (Take $\pi=3$).

(a) Is the induced current in the loop clockwise or counterclockwise?
(b) What is the rate at which electrical energy is being dissipated by the resistance of the loop?

\[ I = \frac{\varepsilon}{R} = \frac{\pi r^2}{R} \left| \frac{dB}{dt} \right| \]

\[ = \frac{3 \cdot (0.050 \, m)^2}{0.150 \, \Omega} \cdot (0.600 \, T/s) = 0.034 \, A \]

\[ P = I^2 R \]

\[ = (0.034 \, A)^2 \cdot (0.150 \, \Omega) \]

\[ = 1.3 \times 10^{-4} \, W \]
A circular loop of wire with radius $r=0.02 \, \text{m}$ and resistance $R=0.40 \, \Omega$ is in a region of spatially uniform magnetic field, as shown in the figure. The magnetic field is directed into the plane of the figure. At $t=0$, $B=0$. The magnetic field then begins increasing, with $B(t)=(0.30 \, T/\text{s}^3) \, t^3$. What is the current in the loop (magnitude and direction) at the instant when $B=2.40 \, T$? (Take $\pi=3$).

\[ \begin{align*}
|\varepsilon| &= \left| \frac{d\Phi_B}{dt} \right| = \pi r^2 \left| \frac{dB}{dt} \right| \\
\frac{dB}{dt} &= 3 \left( 0.30 \, \text{TR}^3 \right) t^2 \\
|\varepsilon| &= 3 \cdot (0.02 \, \text{m})^2 \left( 3 \cdot 0.30 \, \text{TR}^3 \right) t^2 \\
&= \left( 1.08 \times 10^{-3} \, \text{V/} \text{s} \right) t^2
\end{align*} \]

When $B=2.40 \, T$, $B = (0.3 \, \text{TR}^3) t^2 \Rightarrow t = 2 \, \text{s}$

\[ t = 2 \, \text{s} ; \quad I = \frac{|\varepsilon|}{R} = \frac{(1.08 \times 10^{-3} \, \text{V/} \text{s}) \, (2 \, \text{s})}{0.4 \, \Omega} \]

\[ = 10.8 \, \text{mA} \]

induced current - counterclockwise
A rectangular circuit is moved at a constant velocity of 3.0 m/s into, through, and then out of a uniform 1.50 T magnetic field, as shown in the figure. The magnetic field region is considerably wider than 50.0 cm. Find the magnitude and direction (clockwise or counterclockwise) of the current induced in the circuit as it is
(a) going into the magnetic field;
(b) totally within the magnetic field, but still moving; and
(c) moving out of the field.

\[ \varepsilon = BLv \]
\[ I = \frac{\varepsilon}{R} = \frac{BLv}{R} = \frac{(1.5 \, T)(0.95 \, m)(3 \, m/s)}{10 \, \Omega} \]
\[ = 0.3375 \, A \]

magnetic flux is increasing, induced current clockwise.

b) The flux is not changing, so no induced current.

c) \[ I = \frac{\varepsilon}{R} = 0.3375 \, A \], but the direction of induced current is counterclockwise since the time flux is decreasing.
A metal ring 5.00 cm in diameter is placed between North and South poles of large magnets with the plane of its area perpendicular to the magnetic field. These magnets produce an initial magnetic field of 1.12 T between them but are gradually pulled apart, causing this field to remain uniform but decrease steadily at 0.250 T/s.

(a) What is the magnitude of the electric field induced in the ring?

(b) In which direction (clockwise of counterclockwise) does the current flow as viewed by someone on the south pole of the segment?

\[ a) \oint E \cdot dl = -\frac{d\Phi_B}{dt} \]

\[ = 2\pi r = 7\pi r^2 \left| \frac{dB}{dt} \right| \]

\[ = \left| \frac{5}{2} \left| \frac{dB}{dt} \right| \right| = \left( \frac{0.025}{2} \right) \left( 0, 2.50 \times 10^{-1} \right) \]

\[ = 3.125 \times 10^{-3} \text{ V/m} \]

b) counterclockwise
A parallel-plate, air filled capacitor is being charged as shown in the figure. The circular plates have Radius $2.00\ cm$, and at a particular instant the conduction current in the wires is $0.30\ A$. (Take $\pi=3$).

(a) What is the displacement current density $j_d$ in the air space between the plates?
(b) What is the rate at which the electric field between the plates is changing?
(c) What is the induced magnetic field between the plates at a distance $2.00\ cm$ from the axis?

\[ a) \quad \vec{j}_d = \vec{i}_c \quad \frac{\vec{j}_d}{A} = \frac{i_d}{A} = \frac{i_c}{A} = \frac{0.3A}{\pi r^2} = \frac{0.3A}{3 \cdot (0.02m)^2} = 250\ A/m^2 \]

\[ b) \quad \frac{\partial \vec{E}}{\partial t} = \frac{\partial \vec{E}}{\partial r} = \frac{\vec{j}_d}{\varepsilon_0} = \frac{250\ A/m^2}{8.85 \times 10^{-12}\ V/m^2} = 2.8 \times 10^{13}\ V/m \]

\[ c) \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 \vec{H}_d \]
\[ \vec{B} = \mu_0 \left( \oint_{\partial \Sigma} \vec{J}_d r \right) \]
\[ \vec{B} = \mu_0 \left( \frac{4\pi}{2} \right) \left( 8.85 \times 10^{-12}\ T/m \right) \left( 2.8 \times 10^{13}\ V/m \right) (0.02m) \]
\[ = 3 \times 36 \times 10^8\ \text{T} \]