Math 207: Quiz #1B
Fall 2004

- You have 35 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 2 points will be deducted from your grade (You may or may not get an answer to your question(s)).

1. Determine which of the following series converges. Explain your reasoning.

1.a) \[ \sum_{n=0}^{\infty} \frac{\ln(n+1)}{(n+1)} \] (3 points)

Hint: Use the integral test.

\[ f(x) = \frac{\ln(x+1)}{x+1}, \quad I = \int_{0}^{\infty} \frac{\ln(x+1)}{x+1} \, dx = \int_{0}^{\infty} (-1)^n \ln(n+1) \, dx \]

\[ I = \int_{0}^{\infty} \frac{\ln(x+1)}{x+1} \, dx = \left. \frac{(\ln(x+1))^2}{2} \right|_{0}^{\infty} = \infty \]

So, the series diverges.

1.b) \[ \sum_{n=0}^{\infty} (-1)^n \ln(n+1) \] (2 points)

This series diverges because \((-1)^n \ln(n+1) \to \infty\).
2. Use the power series expansion of $e^t$ about $t = 0$ to estimate the value of 

$$f(x) = \int_0^x e^t \, dt$$

at $x = 0.1$. (7 points)

$$e^t = 1 + t + \frac{(t^2)}{2!} + \frac{(t^3)}{3!} + \cdots$$

$$\int_0^x e^t \, dt = x + \frac{t^2}{2} + \frac{t^3}{3 \times 2} + \frac{t^4}{4 \times 3} + \cdots$$

$$\int_0^{0.1} \sim 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3 \times 2} + \frac{(0.1)^4}{4 \times 3 \times 2} + \cdots$$

$$\sim 0.1 + \frac{1}{3000} + \frac{1}{100000}$$

$$\sim 0.100000 + 0.0000333 + 0.0000001$$

$$\sim 0.1000334$$

$$\frac{1}{3000} = 0.000333$$
3. Let \( u = 1 - \sqrt{3}i \) and \( w = \sqrt{3} - i \). Find the modulus \(|z|\), the principal argument \( \arg_p(z) \), the real part \( \text{Re}(z) \), and the imaginary part \( \text{Im}(z) \) of

\[
z = \frac{u^2}{w^3}.
\]

(8 points)

Hint: First determine the polar form of \( u \) and \( w \).

\[
|u| = \sqrt{1+3} = 2, \quad |w| = \sqrt{3+1} = 2
\]

\[
\arg_p(u) = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) = \tan^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{5\pi}{3}
\]

\[
\arg_p(w) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}
\]

\[
u = 2 \ e^{i \frac{5\pi}{3}} \quad \omega = 2 \ e^{i \frac{2\pi}{3}}
\]

\[
\overline{\omega} = 2 \ e^{-i \frac{2\pi}{3}}
\]

\[
z = \frac{1}{2} \ e^{i \frac{5\pi}{6}} + \frac{1}{2} \ e^{i \frac{11\pi}{6}} + \frac{1}{2} \ e^{i \frac{11\pi}{3}} + \frac{1}{2} \ e^{i \frac{3\pi}{2}}
\]

\[
\left( \frac{2\pi + 2\pi}{6} = \frac{5\pi}{3} = \frac{4\pi + \pi}{6} = 8 + \frac{5\pi}{6} \right)
\]

\[
z = \frac{1}{2} \ e^{i \frac{5\pi}{6}}
\]

so

\[
|z| = \frac{1}{2}, \quad \arg_p(z) = \frac{5\pi}{6}
\]

\[
\text{Re}(z) = \frac{1}{2} \ \text{Cn}\left(\frac{5\pi}{6}\right) = -\frac{1}{2} \ \text{Cn}\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{4}
\]

\[
\text{Im}(z) = \frac{1}{2} \ \text{Sn}\left(\frac{5\pi}{6}\right) = \frac{1}{2} \ \text{Sn}\left(\frac{\pi}{6}\right) = \frac{1}{4}
\]