Math 207: Quiz # 4A
Fall 2004

• You have 35 minutes.
• You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 2 points will be deducted from your grade.

1. Let \( f(z) = \frac{e^{(z+2)^2}}{z^2 + 4z + 5} \) and \( C := \{ z \in \mathbb{C} | |z| = 3 \} \).

1.a) Find the poles of \( f \) and determine their order. (8 points)

\[
\lim_{z \to 2} \frac{e^{(z+2)^2}}{z^2 + 4z + 5} = \lim_{z \to 2} \frac{e^{(z+2)^2}}{2z} = \frac{e^{-2}}{2} = \frac{1}{2e}
\]

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\lim_{z \to 2} \frac{e^{(z+2)^2}}{z^2 + 4z + 5} = \lim_{z \to 2} \frac{e^{(z+2)^2}}{z} = \frac{e^{-2}}{2} = \frac{1}{2e}
\]

\[
-1 \quad R(2^-) = \frac{1}{2e} \quad \text{Both poles are simple.}
\]

1.b) Calculate \( \oint_C f(z) \, dz \). (5 points)

\[
\oint_C f(z) \, dz = 2\pi i \left( R(2-) + R(2+) \right)
\]

\[
= 2\pi i \left( \frac{i}{2e} + \frac{1}{2i e} \right) = 0
\]
2. Use the method of contour integration to calculate:

\[ I = \int_0^{2\pi} \frac{d\theta}{e^{i\theta} - 5}. \]

(15 points)

Let \( z = e^{i\theta} \), \( dz = ie^{i\theta} d\theta = i \, d\theta \). \( \theta = \frac{d\theta}{i} \)

\[ I = \oint_C \frac{dz}{iz(\frac{1}{z} - 5)} = \oint_C \frac{dz}{z(1 - 5z)} \]

\( z = \frac{1}{5} \) is a pole and it is inside \( C \).

\[ \text{Residue at } z = \frac{1}{5} \quad \text{is } \frac{1}{z(-5)} = -\frac{1}{5z} \quad \text{is finite and nonzero} \]

\[ z = \frac{1}{5} \text{ pole is simple} \quad R\left(\frac{1}{5}\right) = -\frac{1}{5} ; \]

\[ I = 2\pi i \, R\left(\frac{1}{5}\right) = -\frac{2\pi}{5}. \]