Practice Problems

1 T-bills

Assume a T-bill trades at BDR=3.03% where maturity is 182 days away from settlement. Calculate discount from par and price for $M=10,000.

Now, assume that the maturity date of the bill falls exactly six months away from the settlement date. Calculate the yield to maturity on the bill (semi-annual). Convert the YTM to an annualized rate using the Bond Equivalent Yield convention.

2 Treasury Auctions

Suppose the Federal Reserve auctions $10 million (worth of maturity value) of 5-year notes.

The competitive bids are:

A: $5 million at 5.60%
B: $2 million at 5.592%
C: $3 million at 5.5%

Note that the yields above are quoted on a bond equivalent basis.

Noncompetitive offers total $1 million.

The auction is conducted using the Dutch system (single-price auction). Calculate the stop yield on the accepted competitive bids.

Assume the annualized coupon rate on the bond is 5.5%. Calculate the price that bidders pay for $100,000 of par value.

3 Discount Factors, Spot Rates, and Forward Rates

Consider the three coupon bonds from the WSJ shown in Table 1. (Settlement: 8/15/98)
Calculate discount factors \( d_1, d_2, \) and \( d_3 \) spot rates \( y_1, y_2, \) and \( y_3 \), and implied six-month forward rates \( f_1 \) and \( f_2 \). Annualize your semi-annual rates to obtain bond equivalent yields (BEY=2\( y \)). (In doing your calculations assume $100,000 face value.)

4 Spot Prices and Forward Prices

Consider a forward contract for the delivery of a six-month STRIPS, six months from now: in six months the short side will deliver a six-month STRIPS. The forward price of the STRIPS is 97:00. What is the forward rate implied by the forward price?

Consider now a coupon bond maturing in twelve months, with a coupon rate of 5\( \frac{1}{2} \) and a price of 100:16. Use this price information, together with the forward rate, to calculate the one-period spot rate \( y_1 \).

5 Replicating Portfolios and Coupon Bonds

Consider the three bonds quoted in Table 2. (Settlement: 2/15/94.)

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<th>Table 1</th>
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<td>Bond</td>
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Construct a portfolio of the 6\(7/8\) and the 7\(3/4\) replicating the cash flows of the 5\(1/2\). (Note that the replicating portfolio may have negative quantities of a bond: it means that you need to short that bond to do the replication.)

Compare the price of the replicating portfolio to the price of the 5\(1/2\).
6 Expectations Hypothesis

Assume the following term structure of spot rates: \( y_1 = 1.55\% \), \( y_2 = 1.82\% \), and \( y_3 = 2.03\% \). Use the return-to-maturity EH to calculate the implied expected one-period spot rates \( E(y_1') \), and \( E(y_1'') \). Also, calculate the implied expected two-period spot rate \( E(y_2') \).

7 Duration, Price Changes, and Rates of Return

Assume a flat term structure at BEY=5.625\%. (BEY: Bond Equivalent Yield.) Consider a 55/8 note maturing 8/15/99 (settlement 2/15/94). Calculate modified duration.

Now assume all interest rates suddenly increase to BEY=6.625\%. What is the percentage change in the price of the bond predicted by your measure of duration?

Now, assume you hold the bond of the previous problem for a period equal to its Macaulay duration rounded to the closest integer. Calculate the realized return on the bond assuming reinvestment rates and the yield at sale to be BEY=6.625\%.

8 Duration and Hedging

Consider a corporate bond with 10 years until maturity, trading at par, with yield to maturity BEY=7\%. Also, consider a Treasury bond with 7 years until maturity, also trading at par with yield to maturity BEY=6\%.

Suppose you want to buy the corporate bond and hedge it with a short position in the Treasury bond. What is the dollar amount of the short position? How many Treasury bonds should you sell short?

Now assume that the Treasury yield increases to 7\%, but the credit spread on the corporate yield narrows to 50bps (on a BEY basis). What is the change in the net value of your portfolio? (Use durations to calculate it).

9 Credit Risk

Assume the Treasury yield to equal \( y = 0.05 \). Also, assume the yield on bonds issued by corporation XYZ to be \( y_c = 0.09 \) and the recovery rate to be \( \alpha = 0.6 \). Calculate the implied survival rate and default rate, assuming that, in expectation, investors are indifferent between Treasury and corporate bonds.