**Interest rate expectations and bond returns**

We've noted the inverse relationship between bond prices and yields, and that long-term bonds are more sensitive to changes in yields than short-term bonds.

Does this mean that if you believe rates are going to rise you should prefer short term rather than long-term bonds? Surprisingly, answer is "not necessarily."

**Example:**

\[ r_1 = 10\% \text{ and } E(r_2) = 14\%. \]

The "short-term" one-year bond will provide a rate of return of 10%:

It's priced today at \( \frac{100}{1.10} = $90.91 \) and will be cashed in for $100 in one year.

What about the longer-term bond? Is its expected HPR lower if rates are expected to rise?

*Today's* price of a 2-year zero coupon bond is \( \frac{100}{(1.10)(1.14)} = $79.74. \)

Based on the expectation of a 14% interest rate, the expected price of the 2-year bond *next* year is \( \frac{100}{1.14} = 87.72, \)

The expected HPR on the long-term bond is

\[
\frac{0 + (87.72 - 79.74)}{79.74} = .10 = 10\%, \text{ the same as for the one-year bond.}
\]

Conclusion: The expectation of rising rates already is built into bond prices. *Given expectations*, HPR is equal.
The importance of *relative* expectations.

Long-term bond returns will be worse than returns on short-term bonds only if the interest rate rises *more than expected*.

What if *you* think that \( r_2 \) will be 13%? Notice that you also believe that rates will rise, but not by as much as the rest of market anticipates. You are *relatively* optimistic. You value the two-year zero-coupon bond at

\[
\frac{100}{(1.10)(1.13)} = 80.45 \text{ [versus market price of $79.74]}
\]

So *you* think the bond is *underpriced*.

Which bond will give the higher expected holding period return based on *your* expectations?

The expected rate of return on the "long-term" two-year bond depends on the forecast for the interest rate next year. Since you believe that the rate will be 13%, you forecast a bond price of \( \frac{100}{1.13} = 88.49 \), implying an expected HPR of

\[
\frac{0 + (88.49 - 79.74)}{79.74} = .11 = 11\%
\]

So *despite* your expectation that rates will rise, you still believe that long-term bonds will provide a higher expected rate of return than the ST bond.
You are relatively optimistic about rates compared to market expectations. So your forecast for the rate of return on the LT bond actually is higher than that of the ST bond, even though you agree that rates will rise. This is because the price of the 2-year bond in your view reflects a forecast for interest rates that is excessively pessimistic.

**Lessons**

1. Expectations of rising rates already are reflected in bond prices. So even if those market expectations are correct, the return on the long-term bond will not necessarily be any worse than the short-term bond. The current price already reflects the "bad news." Future rates of return are "fair."

2. If you wish to speculate on rates, your position will depend on your expectations relative to those of the rest of the market. It doesn't matter that you think rates will rise — only that you believe they will rise by more or less than rest of market anticipates.

This is why bond traders are interested in yield curve — it provides a way to gain a sense of the anticipations of the rest of the market.
Yield curve

Yield curve: graph of YTM as a function of time to maturity.

This curve is downward sloping, but typically, the yield curve is upward sloping: Long-term bonds have higher yields than short-term bonds.

Why might it slope upward? (i) expectations (ii) risk premiums on long-term bonds. Let's examine role of expectations first.

Suppose these are expectations:

\[
\begin{array}{c|c|c|c}
0 & 1 & 2 & 3 \\
\hline
r_1 = 10\% & r_2 = 11\% & r_3 = 12\% \\
\end{array}
\]

These are called short rates -- they are ST rates that will prevail in future years.

Ignoring risk, the price of a 3-year zero will be

\[
\frac{100}{(1.10)(1.11)(1.12)} = 73.125
\]

and YTM = 10.997%: 

\[
73.125 = \frac{100}{(1+y)^3} \Rightarrow y = 0.10997
\]

Question: what is the YTM of the 2-year zero-coupon bond?
Notice that YTM is nearly the average of the short-term rates expected to prevail over the next few years. Why does this make sense? Because you should make about the same 3-year return on a 3-year investment as on 3 one-year investments. Ignoring compounding,

$$3 \times y_3 = 10\% + 11\% + 12\% \text{ implies } y_3 = 11\%$$

With compounding,

$$(1 + y_3)^3 = (1.10)(1.11)(1.12)$$

$$\Rightarrow 1 + y_3 = [(1.10)(1.11)(1.12)]^{1/3} = 1.10997$$

Conclusion: The yield is a geometric average of the expected short rates.

<table>
<thead>
<tr>
<th>T</th>
<th>r</th>
<th>YTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>10.499</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>10.997</td>
</tr>
</tbody>
</table>

Graphically,

![Graph showing yield curve and expected short rates.](image)

Conclusion: one factor explaining slope of yield curve is expectations of future short rates. If rates are expected to rise, yield curve will slope upward.
Using yield curve to infer expected future short rates

If prices are determined without risk premiums (as we have so far assumed), we can turn this analysis on its head to infer expectations of future rates from the yield curve. The reason is that if we ignore risk, the only reason that the yield curve would slope upward would be that investors believe that short rates will be rising over time.

Remember that YTM is the average of the expected short rates over coming periods. If the yield curve is rising, this must indicate that as we go out to longer maturities the average rate is rising, so we must be averaging in higher expected short rates. Therefore, slope of yield curve tells us effect of each new short rate that we include in the average.

The yield curve in our example was:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>YTM</th>
<th>Short rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.0</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>10.499</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>10.997</td>
<td>12</td>
</tr>
</tbody>
</table>

Careful: pay attention to the difference between short rates and yields to maturity (also called spot rates). Spot rates are rates today for various investment periods.
Now use yield curve to back out expectations of future short rates

Consider two investment strategies:
1. Invest $1 in a three-year zero. Lock in a total 3-year final value of $1 \times 1.10997^3$
2. Invest $1 in a two-year zero. Then reinvest proceeds in a one-year bond. The total return will be $1 \times (1.10499)^2 \times (1+r_3)$.

$$(1+y_2)^2 (1+f_3) = (1+y_3)^3$$

$$(1.10499)^2 (1+f_3) = (1.10997)^3$$

$$f_3 = .12 = 12\%.$$

Conclusion: if slope of yield curve is due only to expectations of future short rates, then $f_3 = E(r_3)$. 
Similarly, can obtain forecast of second-year rate by equating

\[(1+y_1)(1+f_2) = (1+y_2)^2\]
\[1.10 \times (1+f_2) = (1.10499)^2\]
\[f_2 = .11 \text{ which is in fact equal to } E(r_2)\]

Expectations hypothesis:

1. Since prices are based only on expectations of future rates, the inferred future interest rate equals the expectation of the future rate. Hence, we can use yield curve to infer market's expectations for rates and compare them to our own.
2. The expected holding period return on all bonds is equal. No risk premium present in prices or expected returns.
Liquidity Preference Theory

But what if people do care about risk? Then life is far more complicated. For example, suppose r₁ = E(r₂) = E(r₃) = … = 10%

Then 1-year zero would sell for $90.91

Ignoring risk, a two-year zero would sell for $82.64.

In one year, the two-year zero will become a 1-year zero. Its expected price = 100/1.10 = 90.91 so its expected HPR = 10%. Actual HPR can be more or less: Actual return will be greater if r₂ turns out < 10%, less if r₂ turns out > 10%.

But: would you buy the 2-year zero with an expected return of 10% with interest rate risk, when you could buy a 1-year zero with a sure rate of return of 10%? Probably not. You would buy the two-year zero only if you could buy it at a price less than $82.64.

Suppose you could buy the bond for $80. Then its expected HPR would be

\[
\frac{0 + (90.91 - 80)}{80} = 0.1364
\]

Because LT bond is perceived as riskier, its price is bid down, and its expected HPR is bid up.
If the 2-year bond sells for $80, what is its YTM and the corresponding forward rate for the second year?

The yield to maturity of the 2-year bond is determined by:

\[
\frac{100}{(1+y)^2} = 80 \quad \Rightarrow \quad y_2 = 0.118
\]

Notice, the yield curve is upward sloping even though expectations are for *no change* in the short rate. Rather, upward slope and higher yields are reflection of a risk premium.

The forward rate is no longer an unbiased estimate of expected short rate. The forward rate is

\[
1+f_2 = \frac{(1.118)^2}{1.10} = 1.136
\]

The difference between the forward rate, 13.6% and the expected short rate, 10%, is called a *liquidity premium*. Such premiums mean that use of yield curve to infer rates is more questionable than one would think from expectations hypothesis.

Nevertheless, expectations hypothesis can be useful in some circumstances. E.g.,
- downward sloping curve
- unusually steep curve
- slope of yield curve is one of the most important components of the index of leading economic indicators.
# Forward contracts and forward rate agreements

<table>
<thead>
<tr>
<th>T</th>
<th>y</th>
<th>Zero-coupon bond price</th>
<th>Forward rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>$92.593 = 100/1.08</td>
<td>8.00%</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>$84.168 = 100/(1.09)^2</td>
<td>10.01%</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>$75.131 = 100/(1.10)^3</td>
<td>12.03%</td>
</tr>
</tbody>
</table>

Now consider this strategy:

- **Buy $100 of one year ZCB**: $100 \times 1.08
- **Sell $100 of 2-year ZCB**: $100 \times (1 + y_2)^2 = 118.81

Initial CF = 0

Your cash flow stream is:

So you have arranged to borrow $108 one year from now and to pay back $118.81 in two years, implying an interest rate of 10.01%, exactly the forward rate, $f_2$. In fact, this is why it’s called a forward rate of interest. You can lock in a borrowing (or lending) rate equal to the forward rate.
Try another example. Suppose you want to lock in a lending rate between 
t = 3 and t = 5. So you want a two-year zero that commences three years from now. 
You observe that 3-year zeros sell for $839.62 (YTM = 6%) and that 5-year zeros sell for $733.36 (YTM = 6.399%)

Since you want to invest between t = 3 and t = 5, intuitively you should buy the 5-
year zero which causes you to lend over the entire 5-year period, and then sell a 3-
year zero, which causes you to unload the lending position over the first three 
years.

But we don't buy and sell an equal number of zeros -- we buy and sell an equal
dollar value of zeros so that our initial CF will be zero. Because each 5-year zero 
costs less, we can buy more of them.

If we sell $1000 face value [equivalently, $839.62 market value] of the 3 year zero, 
we can buy \( \frac{839.62}{733.36} \times 1000 = 1,144.90 \) face value of the 5-year zeros. Our initial 
cash flow is zero. Our future cash flows are:

\[
\begin{array}{c|c}
\text{time} & \text{CF} \\
3 & -1,000 \\
5 & +1,144.90 \\
\end{array}
\]

We have locked in a two-year cumulative return of 14.49%. The annualized rate on the forward loan is obtained from \( 1,000 \times (1 + y)^2 = 1,144.90 \Rightarrow y = 7\% \).

This is a synthetic forward contract on a two-year zero with delivery in 3 years.

Notice that the rate on the 2-year forward agreement equals the 2-year forward rate from the yield curve. Thus the yield curve is the key to pricing forward contracts.

\[
(1 + f_{3,5})^2 = \frac{(1 + y_3)^2}{(1 + y_3)^3} = \frac{(1.06399)^5}{(1.06)^3} = 1.1449
\]
The "long" position in the forward contract, who will purchase the zero with a locked-in interest rate or price, benefits if rates fall. Receives a better-than-market rate for the borrowing period. Alternatively, pays less than the market price at future date for a zero coupon bond that commences at that date. So this is a bet (or a hedge) on the rates that will prevail on some date in the future.

A variation on this forward contract is a forward rate agreement. In fact FRAs are very commonly traded in capital markets. The forward rate agreement also is an exchange of fixed for variable rates, but the contract is written directly on the rate rather than on the price of short-term securities.

E.g., you might agree to exchange in two years a cash flow equal to the difference between 6% and LIBOR. Choose some amount of notional principal.

The fixed payer pays \( .06 \times \text{notional principal} \)

The floating payer pays \( \text{LIBOR} \times \text{notional principal} \).

Net payment (from fixed to floating) = \( [.06 - \text{LIBOR}] \times \text{notional principal} \).

Like a forward contract, the FRA provides a profit that depends on the outcome for the short rate at the maturity of the contract. If rates rise, fixed payer/floating receiver benefits.
Swaps: Multiperiod version of a FRA.

Exchange series of cash flows based on notional principal. One party pays fixed rate, the other pays floating rate. E.g., if swap is 7.5% fixed for LIBOR, and notional principal = $10 million, then cash flows would be as follows, if sequence of LIBOR in the next 3 periods turns out to be 7%, 7.5%, 8%:

<table>
<thead>
<tr>
<th>LIBOR</th>
<th>7%</th>
<th>7.5%</th>
<th>8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td></td>
<td>↑</td>
<td></td>
</tr>
<tr>
<td>↓</td>
<td></td>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>Fixed rate</td>
<td>7.5%</td>
<td>7.5%</td>
<td>7.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Net cash flow</th>
<th>if:</th>
<th>L = 7%</th>
<th>L = 7.5%</th>
<th>L = 8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed payer receives</td>
<td>-50,000</td>
<td>0</td>
<td>+50,000</td>
<td></td>
</tr>
<tr>
<td>Floating payer receives</td>
<td>+50,000</td>
<td>0</td>
<td>-50,000</td>
<td></td>
</tr>
</tbody>
</table>

Why would the parties be interested in such a swap? An easy and cheap way to do balance sheet restructuring.

Consider a firm that has borrowed $10 million fixed rate at a coupon rate of 8%, but wishes to convert to floating rate because it thinks rates might fall.

It can enter a pay-floating/receive-fixed swap and transform its fixed-rate debt into synthetic floating rate debt.

<table>
<thead>
<tr>
<th>Cash flow</th>
<th>if:</th>
<th>L = 7%</th>
<th>L = 7.5%</th>
<th>L = 8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>As debt payer</td>
<td>-800,000</td>
<td>-800,000</td>
<td>-800,000</td>
<td></td>
</tr>
<tr>
<td>As floating payer receives</td>
<td>+50,000</td>
<td>0</td>
<td>-50,000</td>
<td></td>
</tr>
<tr>
<td>Net payment</td>
<td>-750,000</td>
<td>-800,000</td>
<td>-850,000</td>
<td></td>
</tr>
</tbody>
</table>

Its net debt payment is now simply:

\[ [0.08 - (\text{LIBOR} - 0.075)] \times \text{principal} = [\text{LIBOR} + 0.50\%] \times \text{principal} \]

So the firm has transformed its fixed rate debt into synthetic floating rate debt.
Conventional story about swaps is the comparative advantage story:

<table>
<thead>
<tr>
<th></th>
<th>Fixed</th>
<th>Floating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company A</td>
<td>10.0%</td>
<td>6-month LIBOR + .30%</td>
</tr>
<tr>
<td>Company B</td>
<td>11.2%</td>
<td>6-month LIBOR + 1.0%</td>
</tr>
</tbody>
</table>

Company B is lower credit so higher rates, but it *appears* that B has a comparative advantage in the floating market, where the spread is .70% and a comparative *disadv* in the fixed market, where the spread is 1.2%.

What if A wants to borrow $1 million at a floating rate, while B wants to borrow $1 million at a fixed rate, precisely in opposition to their comparative advantages? Firm A can borrow fixed, Firm B can borrow floating, and they can enter a swap in which A agrees to pay B the 6-month LIBOR rate while B agrees to pay A a fixed rate of 9.95%.

Here are Company A's CFs:

Pay .10 x 1 million on its fixed rate debt \(- .10 x 1\) million 
Rcv .0995 x 1 million on notional principal \(+ .0995 x 1\) million 
Pay LIBOR x 1 million on notional principal \(- \text{LIBOR} x 1\) million 
Total \(- (\text{LIBOR} + .0005) x 1\) million 

Creates synthetic floating rate borrowing with rate of LIBOR + .05%. Savings versus directly borrowing at floating rate is .25% of 1 million each year.

Here are Company B's CFs:

Pay (LIBOR + .01) x 1 million on its fixed rate debt \(- (\text{LIBOR} + .01) x 1\) m. 
Pay .0995 x 1 million of notional principal \(- .0995 x 1\) m. 
Rcv LIBOR x 1 million of notional principal \(+ \text{LIBOR} x 1\) m. 
Total \(- (.1095) x 1\) million 

Creates synthetic fixed rate borrowing with rate of 10.95%. Savings versus directly borrowing at fixed rate is .25% of 1 million each year.
The net savings are \(0.25\% + 0.25\% = 0.50\%\), exactly equal to the comparative advantage: the 1.20\% spread in the fixed market versus the 0.70\% spread in the floating market.

In practice both parties would deal with a financial intermediary rather than with each other. The deal may look like this:

\[
\begin{array}{ccc}
\text{Company A} & \text{Financial Inst} & \text{Company B} \\
\text{LIBOR} & \text{LIBOR} & \\
\end{array}
\]

\[\begin{array}{c}
\diamond 9.9\% \\
\diamond 10.0\% \\
\end{array}\]

Notice that the intermediary takes a cut of 5 bp from each party -- pays 9.9 fixed to A but rcvs 10.0 fixed from B instead of original 9.95\% transfer in direct deal. This is in effect a bid-ask spread. So net advantage is reduced to 40 bp.

The intermediary bears credit risk of both parties and it *warehouses* swaps until it can find counterparties, so it must either hedge the interest rate risk, or bear that risk.

**Weakness of the comparative advantage argument**

Why does the market "allow" different spreads in the fixed and floating markets? Could be that creditworthiness of B is expected to deteriorate, or that the probability of insolvency rises over time resulting in higher default premiums on long than on short-term loans. In this case, the spread over LIBOR would be expected to increase over time.

**Other kinds of swaps:** Foreign exchange swaps, Commodity swaps, Equity swaps