The bond contract fixes the coupon rate, par value, and maturity date.

Example: Bond A is the 5 ¾ Oct 02
Coupon = 5.75\% of par value \[\$57.50\] annually; actually \$28.75 per half year
Maturity = 2 years, or 4 semi-annual periods
Final, par, or face value = \$1,000

The price of the bond is determined in the marketplace.

\[
\text{P}_{\text{ask}} = 99:23 = 99 \frac{23}{32} = 99.71875\% \text{ of par} = \$997.1875
\]

\[
\text{P}_{\text{ask}} > \text{P}_{\text{bid}}
\]

The last column gives the yield to maturity. An (imperfect) measure of the average return over the entire life of the bond. YTM is calculated assuming you buy the bond for the ask price.

Scan T-bond handout

- Bond versus note.
- Callable versus non-callable. (No new callable bonds since 1982)
- Notice that while bonds of similar maturities may have very different coupon rates, the yields to maturity are nearly identical.

- Notable exception: inflation indexed bonds (identified with an i next to date), e.g., the 3 \(\frac{7}{8}\) April 2029 bond (Bond G) selling for a seemingly low yield of only 3.92\%. Why?
We infer the YTM from the bond price by setting price equal to present value.

Equate the bond price to the present value of cash flows:

\[
\frac{997.1875}{(1 + y) + \frac{28.75}{(1 + y)^2} + \frac{28.75}{(1 + y)^3} + \frac{1000 + 28.75}{(1 + y)^4}}
\]

\[
= \frac{28.75}{y} \left[1 - \frac{1}{(1 + y)^4}\right] + \frac{1000}{(1 + y)^4}
\]

PV Annuity          PV “Balloon”

The YTM is considered the implied rate of return on the bond investment, like an IRR.

Solution to this equation: \( y = 0.0295 = 2.95\% \), which is a semiannual yield.
How can we solve for YTM using financial calculators?

\[
n = 4; \quad \text{PMT} = 28.75; \quad \text{FV} = 1000; \quad \text{PV} = -$997.1875; \quad \text{Compute } i = 2.95\%
\]

In Excel: \[ \text{RATE}(n, \text{pmt}, \text{PV}, \text{FV}, \text{type}) \]
\[ = \text{RATE}(4, 28.75, -997.1875, 1000, 1) = .0295 \]

1 = annuity; 0 = annuity due

Why is PV entered as a negative number?

Annualizing the semiannual yield:
Bond equivalent yield (BEY) uses simple interest: \( 2.95\% \times 2 = 5.90\% \), which is the value reported in the *Wall Street Journal*.

Effective annual yield (EAY) uses compound interest:

\[
(1 + \text{EAY}) = (1.0295)^2 = 1.0599.
\]

Therefore, \( \text{EAY} = .0599 = 5.99\% \).
**Prices versus yields**

Current yield = Annual coupon/Price = $57.50/$997.188 = 0.0577 = 5.77%.

Illustrates general rule that for *discount* bonds:

<table>
<thead>
<tr>
<th>coupon rate</th>
<th>current yield</th>
<th>YTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.75%</td>
<td>5.77%</td>
<td>5.90%</td>
</tr>
<tr>
<td>coupon/100</td>
<td>coupon/price</td>
<td>Average rate of return</td>
</tr>
</tbody>
</table>

If P < par, then there is built-in price appreciation to augment income yield and therefore YTM > current yield > coupon rate.

Similarly, if P > par then YTM < current yield < coupon.

What must be true if a bond is selling at par?

That YTM = coupon rate = current yield.

What does all of this say about the relationship between price and yields?

Higher interest rates imply lower bond prices.

Careful: a common mistake is to think that higher interest rates imply bonds pay higher coupon so bond price is not adversely affected.

But remember that the coupon rate paid on a given bond is fixed as part of the bond contract. Given the *fixed* coupon rate of a particular bond, a rise in *market* interest rates makes that bond less valuable.
**Bond maturity and interest rate risk**

Will long-or short-term bonds be more sensitive to changes in market yields? Example: 8% coupon paid annually.

<table>
<thead>
<tr>
<th>Bond price as a percent of par value</th>
<th>YTM=7%</th>
<th>YTM=8%</th>
<th>YTM=9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 1 year</td>
<td>100.93</td>
<td>100</td>
<td>99.08</td>
</tr>
<tr>
<td>t = 10 years</td>
<td>107.02</td>
<td>100</td>
<td>93.58</td>
</tr>
<tr>
<td>t = ∞</td>
<td>114.28</td>
<td>100</td>
<td>88.89</td>
</tr>
</tbody>
</table>

Note:

1. The longer-term bonds’ prices are more sensitive to changes in yields.

2. The price increase from falling rates is greater than the price decrease from rising yields for a given maturity. The slope of the bond price curve flattens out: this is called **convexity**.

![Graph of bond price vs. yield](attachment:image.png)
Strips are zero coupon bonds.

Example: Bond C [Nov 2009]

\[ c = 0 \]

\[ n = 9 \text{ years, 1 month} = 9 \frac{1}{12} \text{ years} = 18.167 \text{ semi-annual periods} \]

(the standard cash flow "period" in the U.S. is \( \frac{1}{2} \) year)

\[ P = \frac{58.3}{100} = 58.9375\% \text{ of par} \]

Solve for \( y = 2.953\% \) \([n = 18.167; \ PV = -589.375; \ FV = 1000; \ PMT = 0]\)

Annualize: \( 2.953 \times 2 = 5.906\% \text{ (BEY)} \)

You can solve for yields on zeros directly:

\[ \frac{589.375}{1000} = \frac{1000}{(1 + y)^{18.167}} \Rightarrow 1 + y = \left(\frac{1000}{185.625}\right)^{1/18.167} = 1.02953 \]

Notice: Longer maturity implies lower price
How and why are strips created?

Suppose you are an investment banker. You notice that there are two ways to obtain one-year coupon bonds:

**Method 1:** You can buy $1 million face value (i.e., 1,000 bonds each with $1,000 par value) of Bond D, the 7 ½ Nov 2001 bond, selling at 101:11 = 101.3438% of par value. You pay $1,013,438 for the bonds.

What are your cash flows?
- $37,500 in May 2001
- $1,037,500 in November 2001

**Method 2:** Use strips (zero coupon bonds) to obtain the same CFs. The following bundle of zeros is a synthetic coupon bond.

- 37.50 zero-coupon bonds with $1000 face value and maturity May 2001
  Price of each = 96:20 = $966.25 [see Bond E]

- 1037.50 bonds with $1000 face value and maturity November 2001
  Price of each = 93:22 = $936.875 [see Bond F]

\[
\text{Total} = 37.50 \times 966.25 + 1037.50 \times 936.875 \\
= \underbrace{36,234}_{\text{37.50 zeros}} + \underbrace{972,008}_{\text{1037.50 bonds}} = $1,008,242
\]

Total is less than the price of the coupon bond [$1,0134,438].

*Arbitrage strategy:* Parts sell for less than the whole ⇒ *buy* parts and *sell* whole: i.e., buy zeros for $1,008,242; bundle into a coupon bond, then sell bundle for $1,0134,438.
Strips illustrate the concept of financial engineering

Financial engineering:
• Treat securities as bundles of cash flows that can be rearranged into attractive packages.
• Coupon bond = “portfolio” of zero-coupon bonds. Here, the cash flows originally derive from the coupon bonds, but ultimately are "passed through" to the strip.
• Each cash flow can stand on its own as a separate security.

Arbitrage:
• Since the sum of zero prices < coupon prices we could buy the pieces and sell the bundle for a profit. This is called bond reconstitution.
• Process can work in reverse. If the sum of strip prices is more than bond price, you can strip a bond. You buy the whole bond and sell off the pieces (i.e., the coupons and final payment) as a series of zeros.

Anomaly: np prices are systematically higher than ci prices at the long maturities. Why?
(i) reconstitution of bonds can use only principal strips so there can be a "shortage" value of np
(ii) stale prices? Note that some bid-ask spreads are zero.
More financial engineering:
Mortgage-backed and other asset-backed securities

Example of another type of passthrough security:

Agency buys mortgage loans from loan originator, and combines them into pools. Sells passthrough certificates which are claims to the cash flows. Mortgage derivative products like CMOs re-allocate the cash flows even further into various classes or "tranches."

Mortgage-backed securities:

<table>
<thead>
<tr>
<th>$100 K</th>
<th>$100 K</th>
<th>$100 K</th>
<th>101K ??</th>
</tr>
</thead>
<tbody>
<tr>
<td>←</td>
<td>←</td>
<td>←</td>
<td>←</td>
</tr>
<tr>
<td>Homeowner</td>
<td>Originator</td>
<td>Agency</td>
<td>Investor</td>
</tr>
<tr>
<td>→</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
<tr>
<td>P&amp;I</td>
<td>P&amp;I – Servicing</td>
<td>P&amp;I – servicing – guarantee fee</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, investor's coupon on MBS < weighted average coupon rate (the "WAC") of the underlying mortgage pool. Difference equals the sum of the servicing spread and guarantee fee.
Rates of return on bond investments

Rate of return depends on coupon income as well as any price change. Price change in turn depends on changes in the bond's YTM.

Case 1: a one-period investment.

Suppose that a 7.5% coupon 25-year maturity annual payment bond sells for $988.95 today (YTM = 7.6%). In one year, if the bond is selling at a YTM of 8% what will be the investor's rate of return?

The bond will sell for $947.36 \([n = 24, i = 8, FV = 1000, PMT = 75]\)

\[
\text{Rate of return} = \frac{\text{Coupon} + \text{Capital Gain}}{\text{Initial Price}}
\]

\[
= \frac{75 + (947.36 - 988.95)}{988.95} = .0338 = 3.38\%
\]

If YTM had remained unchanged, then rate of return would equal the YTM:

If YTM were still 7.6%, bond price would have been $989.11, implying:

\[
\text{Rate of return} = \frac{75 + (989.11 - 988.95)}{988.95} = .0760 = 7.60\% = \text{YTM}
\]

Conclusion:

- YTM equals the expected one-period return if you believe the current YTM is the best estimate of next period's YTM.
- If you believe the YTM will rise, then the expected holding period return is less than the current YTM; if YTM will fall, then holding period return exceeds YTM.
Another rate of return example: Strips

If the interest rate originally is 10%, a 30-year zero will be issued at a price of:

\[
\frac{1000}{(1.10)^{30}} = 57.31.
\]

The following year, if the YTM is still 10%, the bond will sell for

\[
\frac{1000}{(1.10)^{29}} = 63.04
\]

This is simply 10% more than the original price. This makes sense -- why?

\[
\text{Holding period return, } HPR = \frac{\text{Coupon + Capital Gain}}{\text{Initial Price}} = \frac{0 + (63.04 - 57.31)}{57.31} = .10 = 10\
\]

\text{Illustrates general rule: if YTM is unchanged, } HPR = \text{YTM. A big "if" however.}

If YTM actually falls, let’s say to 9.9%, then

\[
\text{Price} = \frac{1000}{(1.099)^{29}} = 64.72,
\]

\[
\text{HPR} = \frac{\text{Coupon + Capital Gain}}{\text{Initial Price}} = \frac{0 + (64.72 - 57.31)}{57.31} = .129 = 12.9\
\]

\text{Illustrates general rule: because YTM falls, } HPR > \text{original YTM.}

The IRS imputes interest income based on the built-in price appreciation of the bond. It uses a constant yield method to infer price gain, meaning that it calculates price path assuming YTM will remain constant over time. So in first year, taxes owed will be $63.04 – $57.31 = $5.73, even if the YTM falls to 9.9%.

\text{If the bond is sold, then the difference between $64.72 and $63.04 will be treated as capital gains income and taxed at the capital gains tax rate. If the bond is not sold, then the price difference is an unrealized capital gain, and does not result in taxes in that year.}

\text{IRS calculates imputed taxable income on all original issue discount (OID) bonds, not just zeros.}
Multi-period return versus yield to maturity

Yield to maturity will equal the holding period return over the entire life of the bond if all coupons can be reinvested at an interest rate equal to the bond's yield to maturity.

Consider for example, a two-year bond selling at par value paying a 10% coupon once a year. The yield to maturity is 10%.

Panel A: The $100 coupon payment is reinvested at an interest rate of 10%. The $1000 investment in the bond grows after two years to $1210.

The realized compound yield (also called the horizon return) therefore is calculated from

\[ 1000 (1 + y_{\text{realized}})^2 = 1210 \]

\[ y_{\text{realized}} = .10 = 10\% \]

Conclusion: With a reinvestment rate equal to yield to maturity, horizon return equals YTM.

Panel B. The interest rate at which the coupon can be invested is only 8%. The investor purchases the bond for par at $1000 and the investment grows to $1208.

\[ 1000 (1 + y_{\text{realized}})^2 = 1208 \]

\[ y_{\text{realized}} = .0991 = 9.91\% \]

The reinvestment rate is less than YTM, and therefore, so is the realized compound return.
Multi-period return: Horizon analysis

Suppose you buy a 30-year, 7.5% coupon bond for $980 [YTM = 7.67%], and plan to hold it for 20 years. Then the return will depend on both the yield to maturity of the bond when you sell it and the rate at which you were able to reinvest coupon income. The reinvestment of coupons distinguishes the multiperiod case from the one-period case.

Example: Suppose the forecast is that the bond's YTM will be 8% when sold, and that the reinvestment rate on the coupons will be 6%. In this case:

Bond price will be $966.45 [n = 10, i = 8]

The 20 coupon payments will grow with compound interest to:

$2,758.92 [n = 20, i = 6, PV = 0, PMT = 75; compute FV]

So your $980 investment grows in 20 years to $966.45 + $2,758.92 = $3,725.37.

This corresponds to an annualized compound return calculated as follows:

$980 \times (1 + r)^{20} = 3,725.37 \quad \Rightarrow \quad r = 6.90\%$

[n = 20, PV = 980, FV = 3,725.37, PMT = 0]

Note steps: (i) forecast bond's YTM at horizon date; (ii) calculate sales price of bond; (iii) forecast reinvestment rate; (iv) calculate future value of coupons; (v) find compound growth rate of funds from initial investment to total future value.

Suppose reinvestment rate were 7%:

The future value of coupons would be $3074.66 and total proceeds would be $3074.66 + $966.45 = $4041.11,

which is $315.74 more than in the first scenario.

Reinvestment rate risk is greater the longer the horizon.
Summary of return measures for bond investments

**Horizon return** (also called *realized compound return*):

Rate of return accounting for both price change and reinvestment of coupon income.

Advantage: explicitly takes into account reinvestment rate risk. Disadvantage: requires forecast of future yields and reinvestment rates.

**Yield to maturity:**

The investor’s realized compound rate of return will equal the bond's YTM if

(i) the reinvestment rate equals the YTM

*and either*

(ii-a) the investor either holds the bond to maturity,

*or,

(ii-b) if sold prior to maturity, the bond's YTM on the sale date is the same as when the investor bought it.

Note: while YTM can be calculated without *explicit* assumptions regarding future YTM and reinvestment rates, you *implicitly* assume these values equal the current YTM if you use YTM as a measure of expected return.
Corporate bonds are generally more complex than Treasury issues

Mobil bond p. 440
Callable bonds provide a valuable option to the issuer.

Example: consider a bond with an 8% coupon and 30-year maturity that sells for $1,150 and is callable in 10 years at a call price of $1,100.

<table>
<thead>
<tr>
<th>Input</th>
<th>Yield to Call</th>
<th>Yield to Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupon payment</td>
<td>pmt $40</td>
<td>$40</td>
</tr>
<tr>
<td>Semi-annual periods</td>
<td>n 20</td>
<td>60</td>
</tr>
<tr>
<td>Final payment</td>
<td>FV $1,100</td>
<td>$1,000</td>
</tr>
<tr>
<td>Price</td>
<td>PV $1,150</td>
<td>$1,150</td>
</tr>
</tbody>
</table>

Yield to call = 6.64% [BEY]

Yield to maturity = 6.82% [BEY]

YTM is too optimistic: it doesn't reflect the possibility of call.

All else equal, a bond with a call provision will sell at a higher YTM (be issued with a higher coupon) than a non-callable bond.

Discount bonds provide a form of *implicit* call protection.
Promised yields on corporate bonds are subject to default risk.

n = 6 years; PMT = 97.50; Price = 125; FV = 1000 \implies YTM = 91.3\%

Perhaps you expect to receive only $260 in 6 years but nothing until then.
Your \textit{expected} YTM is then only 11.3\%: \quad 125 \times (1.113)^6 = 260

Conclusion: default risk implies that the \textit{reported} yield to maturity overstates the \textit{expected} YTM, the more so the greater is that risk.
Municipal bonds offer tax advantages

Interest income (but not realized capital gains) are exempt from federal, state, and local taxes.

Suppose combined tax rate is 40% and the interest rate on municipal bonds is 5%. Then the *equivalent taxable yield* is defined as

\[ r_{ETY}(1 - t) = r_{muni} \]

\[ r_{ETY} = \frac{r_{muni}}{1 - t} = \frac{5\%}{1 - 0.4} = 8.3\% \]

How do we find the combined tax rate?

Your net of tax rate of interest is

\[ r \times (1 - t_{state}) \times (1 - t_{federal}) \]

\[ = r \times [1 - t_{federal} - t_{state}(1 - t_{federal})] \]

Therefore,

\[ t_{combined} = t_{federal} + t_{state}(1 - t_{federal}) \]

For example, if \( t_{federal} = 28\% \) and \( t_{state} = 6\% \), then \( t_{combined} = 32.32\% \), not 34%.

The equivalent taxable yield on the muni paying 5% interest is thus:

\[ \frac{5\%}{1 - 0.3232} = 7.38\% \]

Obviously, for any given rate on municipal bonds, the equivalent taxable yield is higher for higher-tax-bracket investors.