KOÇ UNIVERSITY
EQUR 121
FIRST EXAM March 3, 2014
Burak Özbağcı
Duration of Exam: 75 minutes

INSTRUCTIONS: No calculators may be used on the test. No questions, and talking allowed. You must always explain your answers and show your work to receive full credit. Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: UZMAN, Ali Alp

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>POINTS</th>
<th>SCORE</th>
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Problem 1 (10 pts)
(a) Determine whether the given argument is valid or invalid by drawing a Venn diagram.

All humans are mammals
No humans are reptiles
/ No mammals are reptiles

(b) In order to show that the following argument is invalid, give a counterexample using necessarily the words “mothers”, “human” and “men”.

All humans are mammals
No humans are reptiles
/ No mammals are reptiles

Solution 1
(a)

We are given that the set of humans are included in the set of mammals, and the set of humans and the set of reptiles are disjoint. But we have no reason to have the set of mammals and the set of reptiles disjoint.

(b)
All mothers are human
No mothers are men
/ No human is a man.

Problem 2 (10 pts)
Construct the truth table for the formula:

\[ (P \land Q) \rightarrow \neg R \rightarrow (Q \lor R) \rightarrow \neg P \]

Solution 2

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>A : ((P \land Q) \rightarrow \neg R)</th>
<th>B : ((Q \lor R) \rightarrow \neg P)</th>
<th>A \rightarrow B</th>
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<tr>
<td>T</td>
<td>T</td>
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<td>((T \land T) \rightarrow F \equiv F)</td>
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<td>F</td>
<td>((T \land T) \rightarrow T \equiv T)</td>
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**Problem 3 (10 pts)** In each of the following say whether the statement is True (T) or False (F):

i. In any valid argument the conclusion is true.

ii. In any factually correct argument, the conclusion is true.

iii. In order to show that an argument is invalid it is sufficient to find a counterexample to it.

iv. If an argument is valid and has all true premises, then its conclusion must be true.

v. If an argument is valid and has a false conclusion, then it must have at least one false premise.

**Solution 3**

i. False (In any valid argument the conclusion is true provided that all the premises are true.)

ii. False (In any factually correct argument, all the premises are true.)

iii. True

iv. True
Problem 4 (30 pts) Translate each of the following statements into the language of sentential logic. Use the suggested abbreviations (capitalized words).

(a) If JAY will go only if KAY goes, then we will CANCEL the trip unless KAY goes.

(b) I will GRADUATE, provided I pass both LOGIC and HISTORY.

(c) In order to be ADMITTED to law school, it is necessary to have GOOD grades, unless your family makes a large CONTRIBUTION to the law school.

(d) In order to be a BACHELOR it is both necessary and sufficient to be ELIGIBLE but not MARRIED.

(e) If you do not PAY, Jones will KILL you unless you ESCAPE

(f) If neither JAY nor KAY is home this weekend, we will go to the BEACH; otherwise, we will STAY home.

(g) If I am not FEELING well this weekend, I will not GO out unless it is WARM and SUNNY.

(h) If you do not CONCENTRATE well unless you are ALERT, then provided that you are not a MANIAC you will FLY an airplane only if you are SOBER.

(i) I am HAPPY only if my assistant is COMPETENT, but if my assistant is COMPETENT, then he/she is TRANSFERRED to a better job and I am not HAPPY.

(j) If you WORK hard only if you are THREATENED, then you will not SUCCEED.

Solution 4  

(a) \((J \rightarrow K) \rightarrow ((\sim K) \rightarrow C)\)

(b) \((L \land H) \rightarrow G\)

(c) \((\sim C) \rightarrow (A \rightarrow G)\)

(d) \(B \leftrightarrow (E \land \sim M)\)

(e) \(\sim P \rightarrow (\sim E \rightarrow K)\)

(f) \([((\sim J \land \sim K) \rightarrow B] \land [\sim (\sim J \land \sim K) \rightarrow S]\)

(g) \(\sim F \rightarrow [\sim (W \land S) \rightarrow \sim G]\)

(h) \((\sim A \rightarrow \sim C) \rightarrow [\sim M \rightarrow (\sim S \rightarrow \sim F)]\)
(i) \((\sim C \to \sim H) \land [C \to (T \land \sim H)]\)
(j) \((\sim T \to \sim W) \to \sim S\)

Problem 5 (10 pts) Fill in the blanks (one word for each blank!)
(a) A formula \(\mathcal{A}\) is a \underline{tautology} if and only if the truth table of \(\mathcal{A}\) is such that every entry in the final(output) column is \underline{true}.
(b) Formulas \(\mathcal{A}\) and \(\mathcal{B}\) are \underline{logically equivalent} \underline{tautology} if and only if the biconditional \(\mathcal{A} \leftrightarrow \mathcal{B}\) is a tautology.
(c) The following argument is an example of \underline{modus tollens}.
"If Xavier was born in Mexico, then he is Mexican. Xavier is not Mexican. Therefore Xavier was not born in Mexico."
(d) An argument form is invalid if it admits a \underline{counterexample}.
(e) An argument is \underline{valid} if and only if its \underline{conclusion} follows from its \underline{premises}.

Solution 5
(a) tautology, true
(b) logically equivalent
(c) modus tollens
(d) counterexample
(e) valid, conclusion, premises

Problem 6 (15 pts) You have to use the method of truth tables in this problem.
Let \(\mathcal{A}: P \leftrightarrow Q\) and \(\mathcal{B}: (P \land Q) \land (Q \to P)\)
(a) Does \(\mathcal{A}\) logically imply \(\mathcal{B}\)? (b) Does \(\mathcal{B}\) logically imply \(\mathcal{A}\)? (c) Are \(\mathcal{A}\) and \(\mathcal{B}\) logically equivalent?

Solution 6
\[\begin{array}{c|c|c|c|c|c}
P & Q & A & B & A \rightarrow B & A \leftarrow B \\
\hline
T & T & T & (T \land T) \land (T \rightarrow T) \equiv T & T & T \\
T & T & F & (F \land T) \land (F \rightarrow T) \equiv F & T & T \\
T & F & F & (F \land F) \land (T \rightarrow F) \equiv F & T & T \\
T & F & T & (F \land T) \land (F \rightarrow F) \equiv F & F(a) & T & F(c) \\
\end{array}\]

\(B\) logically implies \(A\), while the other two do not hold as shown above.

**Problem 7 (15 pts)** Use the method of truth tables to decide whether the given argument is valid or invalid \( P \rightarrow Q ; Q \rightarrow R / R \).

**Solution 7**

\[\begin{array}{c|c|c|c|c|c}
P & Q & R & P \rightarrow Q & Q \rightarrow R & R \\
\hline
T & T & T & T & T & T \\
T & T & F & T & F & F \\
T & F & T & F & T & T \\
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F & T & T & T & T & T \\
F & F & T & T & F & F \\
F & F & T & T & F & T \\
F & F & F & T & T & F \\
\end{array}\]

In the case that all three \(P, Q, R\) are false, we have both our premises true, but the conclusion is false. Thus this argument is invalid.