KOÇ UNIVERSITY
EQUR 121
FINAL EXAM January 3, 2014
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Duration of Exam: 75 minutes

INSTRUCTIONS: No calculators may be used on the test. No questions, and talking allowed. You must always explain your answers and show your work to receive full credit. Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

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SOLUTIONS

Problem 1 (3x10=30pts) Translate into symbolic logic language.

(a) George does not like anyone, but everyone likes George.

$Lxy = x$ likes $y$, $g = George$

$\sim \exists xLgx \land \forall xLxg$

(b) Philosophers dislike politicians.

$Px = x$ is a philosopher, $Tx = x$ is a politician, $Dxy = x$ dislikes $y$

$\forall x(Px \rightarrow \forall y(Ty \rightarrow Dxy))$
(c) No student has read every book in the library.
\[ Sx = x \text{ is a student, } Bx = x \text{ is a book, } Lx = x \text{ is in the library, } Rxy = x \text{ read } y \sim \exists x (Sx \land \forall y ((By \land Ly) \rightarrow Rxy)) \]

(d) There are no unspeakable truths.
\[ Tx = x \text{ is a truth, } Sx = x \text{ is speakable} \sim \exists x (\sim Sx \land Tx) \]

(e) Anyone who lives in an apartment is a senior.
\[ Lxy = x \text{ lives in } y, Ax = x \text{ is an apartment, } Sx = x \text{ is a senior} \forall x (\exists y (Ay \land Lxy) \rightarrow Sx) \]

(f) All students like some professor who likes them.
\[ Sx = x \text{ is a student, } Px = x \text{ is a professor, } Lxy = x \text{ likes } y \forall x (Sx \rightarrow \exists y ((Py \land Lyx) \land Lxy)) \]

(g) The only dangerous pets are spiders or snakes.
\[ Dx = x \text{ is dangerous, } Px = x \text{ is a pet, } Sx = x \text{ is a spider, } Nx = x \text{ is a snake} \forall x ((Dx \land Px) \rightarrow (Sx \lor Nx)) \]

(h) Every play written by Shakespeare is better than any movie.
\[ Px = x \text{ is a play, } s = \text{ Shakespeare, } Bxy = x \text{ is better than } y, Wxy = x \text{ was written by } y, Mx = x \text{ is a movie} \forall x ((Px \land Wxs) \rightarrow \forall y (My \rightarrow Bxy)) \]

(i) Diligent students who study hard and like all their professors always pass.
\[ Dx = x \text{ is diligent, } Sx = x \text{ is a student, } Tx = x \text{ studies hard, } Lxy = x \text{ likes } y, Px = x \text{ is a professor, } Ax = x \text{ always passes} \forall x ((Sx \land Dx \land Tx \land \forall y (Py \rightarrow Lxy)) \rightarrow Ax) \]

(j) Anyone taller than Tim is taller than every basketball player shorter than the tallest jockey.
\[ Txy = x \text{ is taller than } y, t = \text{ Tim, } Bx = x \text{ is a basketball player, } Sxy = x \text{ is shorter than } y, Jx = x \text{ is a jockey} \forall x (Txt \rightarrow \forall y ((By \land \exists z (Jz \land \sim \exists w (Jw \land T wz) \land Syz)) \rightarrow Txy)) \]
Problem 2 (10 pts)

(a) Using $Mx = x$ is a mistake, $Axy = x$ makes $y$ angry, $m = mike$,
translate the formula $\forall x(Axm \rightarrow Mx)$ into English by filling in the blanks:

______ ______________ make Mike ______.

Only mistakes make Mike angry.

(b) Using $Sx = x$ is a student, $Fxy = x$ fails $y$, $Cx = x$ is a class, $Hxy = x$ has $y$
translate the formula

$\exists x[Sx \land \forall y[(Cy \land Hxy) \rightarrow Fxy]] \land \exists x[Sx \land \sim \forall y[(Cy \land Hxy) \rightarrow Fxy]]$

into English by filling in the blanks:

_____ _____ are failing _____ _____ they have, but _____ are _____.

Some students are failing every class they have, but some are not.

Problem 3 (10 pts)

(a) Construct the truth table for the formula:

$[ (P \land Q) \rightarrow \sim R ] \rightarrow [ (Q \lor R) \rightarrow \sim P ]$

You can do this on your own.

(b) In each of the following say whether the statement is True (T) or False (F):

- In any valid argument the conclusion is true. F
- The sentence “There is no politician who respects every citizen.” includes only two predicates. F
- In order to show that an argument is invalid it is sufficient to find a counterexample to it. T
- The sentence “There are many politicians who respect every citizen.” includes at most one quantifier. F
• If an argument is valid and has a false conclusion, then it must have at least one false premise. T

Problem 4 (10 pts)
(a) Determine whether the given argument is valid or invalid by drawing a Venn diagram.
   All humans are mammals
   No humans are reptiles
   /No mammals are reptiles
(b) In order to show that the following argument is invalid, give a counterexample using necessarily the words “mothers”, “human” and “men”.
   All humans are mammals
   No humans are reptiles
   /No mammals are reptiles

See Midterm 1 Solutions below.

Problem 5 (10 pts) Construct a derivation of the conclusion from the premises.
(1) $\forall x(Fx \rightarrow \exists y \sim Kxy)$ Pr
(2) $\exists x(Gx \land \forall yKxy)$ Pr
(3) SHOW: $\exists x(Gx \land \sim Fx)$ ID (use indirect derivation)

See Chapter 8 #67 in the book.

Problem 6 (10 pts) Construct a derivation of the conclusion from the premises.
(1) $\forall x\exists yRxy$ Pr
(2) $\forall x\forall y(Rxy \rightarrow \exists zRzx)$ Pr
(3) $\forall x\forall y(Ryx \rightarrow \forall zRxz)$ Pr
(4) SHOW: $\forall x\forall yRxy$

See Chapter 8 #72 in the book.

Problem 7 (10 pts) Construct a derivation of the conclusion from the premises.
(1) $\exists x(Fx \land Kxa)$ Pr
(2) $\exists x[Fx \land \forall y(Kya \rightarrow \sim Rxy)]$ Pr
(3) SHOW: $\exists x[Fx \land \exists y(Fy \land \sim Ryx)]$ DD (use direct derivation)

See Chapter 8 #78 in the book.
Problem 8 (10 pts) Fill in the blanks

(a) A formula \( \mathcal{A} \) is a \underline{tautology, TRUE} if and only if the truth table of \( \mathcal{A} \) is such that every entry in the final(output) column is \underline{TRUE}.

(b) Formulas \( \mathcal{A} \) and \( \mathcal{B} \) are \underline{logically equivalent} if and only if the biconditional \( \mathcal{A} \leftrightarrow \mathcal{B} \) is a tautology.

(c) The following argument is an example of \underline{modus tollens}.

“If Sam was born in Canada, then he is Canadian. Sam is not Canadian. Therefore Sam was not born in Canada.”

(d) An argument form is invalid if it admits a \underline{counterexample}.

(e) An argument is \underline{valid, conclusion, premises} if and only if its \underline{premises} follows from its \underline{conclusion}.