A thin, 1 m wire has a mass of 20 g. One end is tied to a nail, and the other end is attached to a screw that can be adjusted to vary the tension in the wire. To what tension (in newtons) must you adjust the screw so that a transverse wave of wavelength 4 cm makes 500 vibrations per second?

\[ L = 1 \text{ m}, \quad m = 20 \text{ g}, \quad \lambda = 4 \text{ cm} = 4 \cdot 10^{-2} \text{ m}, \quad f = 500 \text{ Hz} \]

\[ F = MV^2 = \frac{m}{L} (\lambda f)^2 = \frac{20 \cdot 10^{-3}}{1} (500 \cdot 4 \cdot 10^{-2})^2 \]

\[ = 2 \cdot 10^{-3} \cdot 400 = 8 \text{ N} \]

\[ F = 8 \text{ N} \]
A 1.5 m string of weight 1.5 N is tied to the ceiling at its upper end, and the lower end supports a weight $W$. When you disturb the string slightly, the waves traveling up the string obey the equation $y(x, t) = 8.5 \text{ mm} \cos \left( \frac{140 \text{ m}^{-1} x - 2800 \text{ s}^{-1} t}{L} \right)$. What is the weight $W$? (Take $g = 10 \text{ m/s}^2$ and ignore the effect of the weight of the string on the tension)

\[
L = 1.5 \text{ m} \quad W_s = 1.5 \text{ N}
\]

\[
y(x, t) = 8.5 \text{ mm} \cos \left( \frac{140 \text{ m}^{-1} x - 2800 \text{ s}^{-1} t}{L} \right)
\]

\[
W = F = MV^2 = \frac{W_s}{g} \cdot \frac{(\omega)}{k} = \frac{1.5}{9} \cdot \frac{2800}{1400} = 1.4 \text{ N}
\]

\[
W = 40 \text{ N}
\]
Transverse waves on a string have wave speed 8 m/s, amplitude 0.07 m, and wavelength 0.32 m. The waves travel in the -x-direction, and at t = 0 the x = 0 end of the string has its maximum upward displacement. Write a wave equation describing the wave.

\[ v = 8 \text{ m/s} \quad \lambda = 0.32 \text{ m} \quad t = 0, x = 0 \quad y = A \]

Let's write general wave equation:

\[ y(x, t) = A \cos(kx + \omega t) \]

Note that \( y(0, 0) = A \)

\[ A = 0.07 \text{ m} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.32} \]

\[ \omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{2\pi \times 8}{0.32} = \frac{2\pi}{0.04} \]

Thus:

\[ y(x, t) = 0.07 \cos\left(2\pi \left(\frac{1}{0.32} x + \frac{1}{0.04} t\right)\right) \]
Show by direct substitution that $y(x,t) = A\cos(kx - \omega t)$ satisfies the wave equation.

Let's substitute given expression to wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(-A k \sin(kx-\omega t)\right) = -A k^2 \cos(kx-\omega t)$$

$$\frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{\partial}{\partial t}\left(+A \omega \sin(kx-\omega t)\right) = \frac{1}{v^2} (-A \omega^2) \cos(kx-\omega t)$$

$$-A k^2 \cos(kx-\omega t) = -A \frac{\omega^2}{v^2} \cos(kx-\omega t)$$

$$k^2 = \frac{\omega^2}{v^2} \Rightarrow \text{This equation is correct}$$

Therefore wave equation is satisfied.
Let's check whether it satisfies the wave equation:

\[
\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \left( A k \cos(kx) \right) = -A k^2 \sin kx
\]

\[
\frac{1}{V^2} \frac{\partial^2 y}{\partial t^2} = \frac{1}{V^2} \frac{\partial}{\partial t} \left( -A \omega \sin \omega t \right) = -\frac{A}{V^2} \omega^2 \cos \omega t
\]

But:

\[-A k^2 \sin kx \neq -\frac{A}{V^2} \omega^2 \cos \omega t
\]

(since \( \sin kx \neq \cos \omega t \))

Therefore the wave equation is not satisfied.