A bowling ball of mass $M$ and radius $R$ rolls without slipping down a ramp, which is inclined at an angle $\theta$ to the horizontal.

a. Draw the free body diagram for the bowling ball.
b. What is the acceleration of the center of mass of the bowling ball?  
c. What is the magnitude of the friction force on the bowling ball?  

Your results should be in terms of the given quantities ($M$, $g$, $\theta$)  
$I$(sphere)=$(2/5)MR^2$  

\[ Mgsin\theta - f = M{a_{cm}} \]
\[ \Sigma \tau = \int R = I_{cm} \alpha = \frac{2}{5}MR^2 \frac{a_{cm}}{R} \]

(1) \[ \int R = \frac{2}{5}MR_{acm} \]

(2) \[ Mgsin\theta - f = M{a_{cm}} \]

From eqn. 2: \[ \int = M(gsin\theta - a_{cm}) \]

Insert in eqn. 1

\[ MR(gsin\theta - a_{cm}) = \frac{2}{5}MR_{acm} \]
\[ gsin\theta = \left( \frac{2}{5} + 1 \right) a_{cm} \]
\[ a_{cm} = \frac{5}{7}gsin\theta \]

\[ c) \int = M (gsin\theta - \frac{5}{7}gsin\theta) = \frac{2}{7}Mgsin\theta \]
A primitive yo-yo is made by wrapping a massless string around a solid cylinder of mass $M$ and radius $R$. You hold the free end of the string stationary and release the cylinder (yo-yo) from rest. The string unwinds but does not slip or stretch as the cylinder (yo-yo) descends and rotates.

a. Draw the free body diagram for the yo-yo.
b. What is the acceleration of the cylinder (yo-yo)?
c. What is the tension in the string?

Your results should be in terms of the given quantities $(M, g)$.

\[
\sum F_y = Mg - T = Ma_{cm, y}
\]

\[
\sum \tau = TR = I_{cm} \alpha = \frac{1}{2} MR^2 \alpha
\]

\[
\alpha = \frac{a_{cm, y}}{R}
\]

\[
TR = \frac{1}{2} MR^2 \frac{a_{cm, y}}{R}
\]

\[
T = \frac{1}{2} M a_{cm, y}
\]

\[
Mg - T = Ma_{cm, y}
\]

\[
T = \frac{1}{3} Mg
\]
A hollow, spherical shell with mass $3 \text{ kg}$ rolls without slipping down a ramp, which is inclined at an angle $30^0$ to the horizontal.

a. Draw the free body diagram for the spherical shell.

b. What is the acceleration of the center of mass of the spherical shell?

c. What is the magnitude of the friction force on the spherical shell?

$I$(hollow, spherical shell)=$(2/3)MR^2$, $(\sin 30^0 = \cos 60^0 = 1/2)$, $g = 10 \text{ m/s}^2$.

\[ a_{cm} = \frac{3g \sin \theta}{5} = \frac{3 \times 10 \times \frac{1}{2}}{5} = 3 \text{ m/s}^2 \]

\[ f = \frac{2}{3}Macm = \frac{2}{3} \times 3.3 = 6.6 \text{ N} \]
m₁=15 kg box is resting on a horizontal, frictionless surface is attached to a m₂=4 kg weight by a thin, light wire that passes over a frictionless pulley. The pulley has the shape of a uniform solid disk of radius R and mass M=2kg. System is released from rest.

a. Draw the free body diagram and show the forces acting on box m₁, weight m₂, and the pulley with mass M.

b. Find the tension in the wire on both sides of the pulley.

c. Find the acceleration of the box.

For the pulley let clockwise rotation be positive.
Take I=(1/2)MR²

\[ \sum F = m \ddot{a} \text{ to each mass}, \]

\[ \sum \tau = I \ddot{a} \text{ to the pulley}, \]

\[ a = R \alpha \]

\[ T₁ = m₁a \]

\[ m₂g - T₂ = m₂\ddot{a} \]

\[ T₂R - T₁R = \frac{1}{2} MR² \]

Add 3 equations side by side

\[ \frac{1}{N} = m₁a \]

\[ m₂g - T₂ = m₂\ddot{a} \]

\[ T₂ - T₁ = \frac{1}{2} MR² \]

\[ T₁ = (15)(2) = 30 \text{N}, T₂ = 32 \text{N} \]
We wrap a light, non-stretching cable around a solid cylinder with mass $M$ and radius $R$. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass $m$ and release the block from rest at a distance $h$ above the floor. As the block falls, the cable unwinds without stretching or slipping.

a. Draw the free body diagram for the block and the cylinder.

b. What is the acceleration of the falling block?

c. What is the tension in the cable?

Your results should be in terms of the given quantities ($M$, $m$, $g$)

$I_{cylinder} = \frac{1}{2}MR^2$.

\[
\begin{align*}
\mathbf{a)} & \quad N \downarrow \\
& \quad T \downarrow \\
& \quad M \mathbf{g} \downarrow \\
& \quad m \mathbf{g} \downarrow
\end{align*}
\]

\[
\begin{align*}
\mathbf{b)} & \quad \sum \mathbf{F} = m\mathbf{a} \\
& \quad m\mathbf{g} - (-T) = ma_y \quad \checkmark \\
& \quad \sum \mathbf{F} = I\alpha \mathbf{z} \quad , \quad a_y = R\alpha \mathbf{z} \\
& \quad R\mathbf{T} = I\alpha \mathbf{z} \quad = \left(\frac{1}{2}MR^2\right)\alpha \mathbf{z} \\
& \quad T = \frac{1}{2}MR^2\alpha \mathbf{z} = \frac{1}{2}Ma_y \\
& \quad \text{From (x)} \quad m\mathbf{g} - \frac{1}{2}Ma_y = ma_y \\
& \quad a_y = \frac{g}{1 + \frac{M}{2m}} \quad \checkmark
\end{align*}
\]

\[
\begin{align*}
\mathbf{c)} & \quad T = mg - ma_y \\
& \quad = mg - m\left(\frac{g}{1 + \frac{M}{2m}}\right) \\
& \quad = \frac{mg}{1 + \frac{2m}{M}} \quad \checkmark
\end{align*}
\]