A projectile is launched from the origin with initial speed $v_0$ on an inclined surface. The launch angle with the horizontal is $\alpha_0 = \frac{\pi}{3}$. The inclination angle of the surface is $\theta = \frac{\pi}{4}$.

Determine the distance of the landing point from the launch point in terms of the given parameters. The gravitational acceleration is $g$.

Notes: $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos \frac{\pi}{3} = \frac{1}{2}$, $\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

\[ x = (v_0 \cos \alpha_0) t \]
\[ y = (v_0 \sin \alpha_0) t - \frac{1}{2} g t^2 \]
\[ \sin \alpha_0 = \frac{\sqrt{3}}{2}, \quad \cos \alpha_0 = \frac{1}{2} \]

\[ d = \sqrt{x^2 + y^2} \]
\[ y \geq -x \]
\[ (v_0 \cos \alpha_0) t = \frac{1}{2} g t^2 - (v_0 \sin \alpha_0) t \]
\[ t = \frac{2(v_0 \cos \alpha_0)}{g} - \frac{1}{2} \left( \frac{v_0}{g} (1+3) \right)^2 \]
\[ y = \frac{v_0^2}{2g} (1+3) \]
\[ d = \sqrt{2} \frac{v_0^2}{2g} (1+3) = \frac{v_0^2}{\sqrt{2} g} (1+3) \]
Consider the motion shown in the figure, where a ball starts moving at time $t = 0$ with speed $v_0$ and making an angle of $\pi/4$ radians with the horizontal axis. If the horizontal and vertical accelerations of the ball are given by $a_x = -2A$ and $a_y = -B$, respectively, find its vertical and horizontal displacements with respect to the initial position as a function of time. Here $A$ and $B$ are positive constants. You may ignore the effects of gravity.
A missile is thrown at an angle $53^\circ$ into air from the ground with initial speed $v_0 = 720$ km/h (there is no air resistance). Find the altitude of the missile from the ground at the instant when its velocity vector makes an angle $-45^\circ$ with the $x$-axis. (Hint: $\tan(53^\circ) = \frac{4}{3}$, $g = 10 \text{m/s}^2$).

$$y = \left(\frac{200 \text{ m}}{5}\right) - \frac{1}{2 \left(\frac{10 \text{ m/s}^2}{52}\right)} \left(\cos^2 53^\circ - x^2 53^\circ\right)$$

$$y = 560 \text{ m}$$
A vertical launch ramp is \( d = 10 \text{ m} \) long from the ground. During the launch, the ramp applies a constant vertical acceleration \( a_0 = 5 \text{ m/s}^2 \) to an object until it leaves the ramp. (\( g = 10 \text{ m/s}^2 \)).

1. The launch starts at \( t = 0 \text{ s} \) with the object at rest on the base of the ramp at the ground. When does the object return to the ground?

2. Plot the velocity vs. time graph of the object during this motion qualitatively. Indicate the important data points on the graph.

\[
\begin{align*}
\text{Object's velocity just after launch:} & \quad v_y = v_{y0} + 2a_0 \left( \frac{d}{2} \right) \\
v_{y0} & = 10 \text{ m/s} \\
y - y_0 = d = (v_{y0} t + \frac{1}{2} a_{y0} t^2) \\
10 \text{ m} = 0 + \frac{1}{2} (5 \text{ m/s}^2) t^2 \Rightarrow t = 2 \text{ s} \quad \text{Object stays launcher} \\
\left( v_{y0} = v_y - g t \Rightarrow \left( \frac{g}{2} \text{ m/s}^2 \right) \right) & \Rightarrow t = 1.5 \text{ s} \quad \text{Object reaches max. height} \\
h = \Delta y + d \\
\Delta y = u_y t - \frac{1}{2} g t^2 \Rightarrow \Delta y = \left( \frac{10 \text{ m}}{5} \right) (1.5) - \frac{1}{2} \left( \frac{10 \text{ m}}{5^2} \right) (1.5)^2 \\
\Delta y = 5 \text{ m} \\
h = 5 \text{ m} + 10 \text{ m} = 15 \text{ m} \\
\text{Object reaches ground from } h = 15 \text{ m} \text{ in} \\
h = v_{y0}^2 t + \frac{1}{2} g t^2 = \frac{1}{2} \left( \frac{10 \text{ m}}{5^2} \right) t^2 = 15 \text{ m} \\
t = 3 \text{ s}
\end{align*}
\]
In total, object reaches ground in $2t = 3 + \sqrt{3}$ s.
The position of a particle as a function of time is given as
\[ r(t) = R [ \cos(\omega t) \hat{i} + \sin(\omega t) \hat{j} ]; \] where \( R \) and \( \omega \) are some constants.

1. Determine the instantaneous velocity vector, \( \dot{r}(t) \) (using unit vectors \( \hat{i} \) and \( \hat{j} \)).
2. Determine the instantaneous acceleration vector, \( \ddot{r}(t) \) (using unit vectors \( \hat{i} \) and \( \hat{j} \)).
3. What is the angle between \( \dot{r}(t) \) and \( \ddot{r}(t) \)?
4. Based on the information obtained in previous parts, what is the name for this particular motion?

A useful identity: \( \sin^2 \theta + \cos^2 \theta = 1 \)

\[ a) \quad \dot{r}(t) = \frac{dr(t)}{dt} = -R \omega^2 \cos(\omega t) \hat{i} + R \omega^2 \sin(\omega t) \hat{j} \]

\[ b) \quad \ddot{r}(t) = \frac{d\dot{r}(t)}{dt} = -R \omega^2 \sin(\omega t) \hat{i} - R \omega^2 \cos(\omega t) \hat{j} \]

\[ c) \quad \ddot{r} \cdot \dot{r} = 10\| \dot{r} \| \cos \alpha \]

\[ \cos \alpha = \frac{\ddot{r} \cdot \dot{r}}{\| \dot{r} \| \| \ddot{r} \|} = \frac{R^2 \omega^4 \sin^2(\omega t) \cos(\omega t) - R^2 \omega^4 \sin^2(\omega t) \cos(\omega t)}{R^2 \omega^4} = \frac{\omega^2 R^2}{\omega^2 R^2} \]

\[ \cos \alpha = \frac{\omega^2}{\omega^2} \implies \alpha = \pi/2 \implies \ddot{r} \perp \dot{r} \]

\[ d) \quad |\ddot{r}(t)| = R \sqrt{(1 + \cos(\omega t))^2 + \sin^2(\omega t)} = 2R \cos(\omega t/2) \]

\[ |\ddot{r}| = \omega R \quad , \quad |\dot{r}| = \omega^2 R \]

Uniform circular motion