A motorcyclist is riding at a constant speed up a hill which has an inclination angle of \( \phi \). The power (in watts) generated by the engine of the motorcycle is \( P_E \), where \( m \) is the mass of the motorcycle (plus the rider) and \( g \) is the gravitational acceleration. Find the power lost to frictional forces in terms of \( m, g, \phi, v, \) and \( t \).

\[
\begin{align*}
\Delta W & = \frac{K_i - K_f}{t} - \frac{P_E}{t} \\
P_f & = P_{\text{in}} - \frac{W}{t} \\
& = mgv_0 - mgv\sin\phi \\
& = mg\beta' [1 - \sin\phi] \\
\Delta E & = \int mgv_i dt = mgv_i t \\
U & = \int mg\sin\phi \cdot dt = mg\sin\phi \cdot t \\
\Delta W_f & = mgv - mgv_0 - mg\sin\phi \cdot t \\
P_f & = mgv - mgv_0 - mg\sin\phi \cdot t \\
& = mg\beta' [1 - \sin\phi]
\end{align*}
\]
A small block with a mass \( m \) is attached to a cord passing through a hole in a frictionless, horizontal surface. The block is originally revolving at a distance \( r_1 \) from the hole with a speed \( v_1 \). The cord is then slowly pulled from below, shortening the radius to \( r_2 \). Throughout the process, \( r \) and \( v \) change, but their product satisfies \( r v = r_1 v_1 \) at all times. Find the work done by the person pulling the cord.

\[
W_{\text{done}} = K_1 - K_2
\]

\[
= \frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2
\]

\[\text{See } v_2 = \frac{r_1 v_1}{r_2}.\]

\[K_1 \text{ kinetic energy at } r_1.\]

\[K_2 \text{ kinetic energy at } r_2.\]

\[
v_2 = \frac{r_1}{r_2} v_1,
\]

\[
= \frac{1}{2} m v_1^2 - \frac{1}{2} m \left( \frac{r_1 v_1}{r_2} \right)^2
\]

\[\text{with } r_1 v_1 = r_2 v_2 \text{ given.}\]

\[
= \frac{1}{2} m v_1^2 \left[ 1 - \left( \frac{r_1}{r_2} \right)^2 \right]
\]

\[
W_{\text{done}} = \frac{1}{2} m v_1^2 \left[ 1 - \left( \frac{r_1}{r_2} \right)^2 \right]
\]
A textbook with mass $m$ is forced against a horizontal and ideal spring with force constant $k$, compressing the spring a distance $x$. When released from this position, the book slides on a horizontal tabletop with a coefficient of kinetic friction $\mu_k$ and comes to rest after moving by a distance $\frac{x}{2}$ (that is, when the spring is still compressed by $\frac{x}{2}$). Find $x$ in terms of $m$, $k$, $\mu_k$, and the gravitational acceleration $g$.

\[ W = K + U + W_f \]

\[ \frac{1}{2} k x^2 = m x g + \int_0^{x/2} \mu_k m g x \, dx \]

\[ \frac{1}{2} k x^2 = \frac{1}{2} k \left( \frac{x}{4} \right)^2 + \mu_k m g \left( \frac{x}{2} \right) \]

\[ \frac{1}{2} k x^2 \left[ 1 - \frac{1}{4} \right] = \mu_k m g \frac{x}{2} \]

\[ x = \frac{4 \mu_k m g}{3 k} \]
An object slides on a very large, horizontal ice ring with a variable coefficient of kinetic friction \( \mu_k(x) = e^{-x/b} \), where \( x \) is the distance from the entrance. If the object enters the ring with speed \( v \) and moves on a straight line, what is the minimum value of \( v \) so that it never stops?

\[
\mu_k(x) = e^{-x/b}.
\]

\[
\kappa = \frac{1}{2} mv^2.
\]

\[
w_x = \int mg e^{-v/x} \, dx \bigg|_0^\infty = mg e^{-x/b} \bigg|_0^\infty = \left[ mg e^{x/b} - mg e^{-x/b} \right].
\]

For object to never stop:

\[
\kappa > w_x.
\]

\[
\frac{1}{2} mv^2 > mg e^{x/b} - mg e^{-x/b}.
\]

\[
v^2 > \sqrt{2g} e^{x/b} + mg e^{-x/b}.
\]

At \( x = \infty \):

\[
v^2 > \sqrt{2g} e^{x/b} + mg e^{-x/b}.
\]

\[
v^2 = \sqrt{2mg}.
\]

Ans.
A car with mass \( m \) and moving at speed \( v \) enters an icy patch on the highway and starts skidding. The coefficient of kinetic friction between tires and the road decreases with the distance \( x \) from the beginning of the patch as \( \mu_k(x) = \frac{h}{h + x} \). The car stops at \( x_f \). Write down the work-energy theorem for the motion between \( x = 0 \) and \( x = x_f \), expressing the work done by friction as an integral. (Gravitational acceleration is \( g \).)

\[
W_I = k_I + U_I + W_{\text{fr}} \quad \text{[Initial]}
\]
\[
= \frac{1}{2} \, m \, v^2 + 0 + 0
\]

\[
W_f = k_f + U_f + W_{\text{fr}} \quad \text{[Final]}
\]
\[
= 0 + 0 + \int_0^{x_f} \mu_k \cdot mg \, dx = \int_0^{x_f} \frac{b}{b+x} \cdot mg \, dx = bmg \cdot \ln(x_f + b)
\]

\[
W_i = W_f
\]

\[
\frac{1}{2} \, m \, v^2 = \int_0^{x_f} \frac{b}{b+x} \cdot mg \, dx
\]
\[
= bmg \ln \left( \frac{x_f + b}{b} \right)
\]