The wire shown in the figure is infinitely long and carries a current $I$. Starting from the Biot-Savart law, find the magnetic field (both magnitude and direction) at point P.

\[ dB = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \]

For horizontal part, $\vec{B} = 0$.

For vertical part:

\[ dB = k \frac{M_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \]

\[ \vec{B} = \hat{k} \frac{M_0 I}{4\pi} \int \frac{d\vec{l} \sin \theta}{r^2} = \hat{k} \frac{M_0 I}{4\pi} \int \frac{\sin \theta \, dy}{r^2} \]

\[ \vec{B} = \hat{k} \frac{M_0 I}{4\pi} \int_0^{\pi/2} \sin \theta \, d\theta \cdot \frac{\sin \theta}{\sin \theta} \cdot \frac{(x^2)}{x^2} \]

\[ \vec{B} = \hat{k} \frac{M_0 I}{4\pi x} \int_0^{\pi/2} \sin \theta \, d\theta = \hat{k} \frac{M_0 I}{4\pi x} \left[ -\cos \theta \right]_0^{\pi/2} \]

\[ \vec{B} = \hat{k} \frac{M_0 I}{4\pi x} \]

\[ \frac{dy}{d\theta} = d\left( \frac{x}{\tan \theta} \right) = \frac{1}{\tan \theta} \]

\[ \frac{d}{dy} \frac{dy}{d\theta} = \frac{d}{dy} \frac{x}{\tan \theta} = \frac{1}{\sin^2 \theta \, \cos \theta} \]

\[ \frac{d}{dy} \frac{dy}{d\theta} = \frac{1}{\tan \theta} \]
Using Ampere's law, find the direction and magnitude of the net current passing from the rectangular region shown in the figure, if the resulting magnetic field is \( \vec{B} = Ay \hat{i} \), where \( A > 0 \) is a constant. ( \( \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \) )

\[ \vec{B}(y) = Ay \hat{i} \]

- at \( y = 0 \) \( \vec{B}(0) = Ao \hat{y} = 0 \)
- at \( y = b \) \( \vec{B}(b) = Ab \hat{i} \)

\( \vec{B} \perp d\vec{l} \) means \( \vec{B} \) and \( d\vec{l} \) are perpendicular so dot product gives us zero.

\[ \oint \vec{B} \cdot d\vec{l} = \int_0^b \vec{B} \cdot d\vec{i} + \int_0^c \vec{B} 
\int d\vec{i} + \int_0^b \vec{B} \cdot d\vec{i} = 0 + \int \vec{B} \cdot d\vec{i} + 0 + 0 \]

\[ \oint \vec{B} \cdot d\vec{l} = \int_0^b \vec{B} \cdot d\vec{i} = \int (Ab \hat{i}) \cdot (dx \hat{i}) = Ab \int_0^b dx = Ab a = \mu_0 I_{\text{enc}} \]

\[ \text{length of } bc \]

\[ I_{\text{enc}} = \frac{Ab a}{\mu_0} \text{ (direction is in to the page, } -\hat{k}, \text{ from right hand rule) } \]

look at \( d\vec{l} \)
A circular loop has radius $R$ and carries a current $I$ in a clockwise direction. Starting from the Biot–Savart law, find the magnetic field (both magnitude and direction) at its center.

\[
 dB = \frac{\mu_0 I \, dl \times \hat{r}}{4\pi \, r^2}
\]

Using the right hand rule, the direction of $dB$ is into the page.

\[
 dB = \frac{\mu_0 I}{4\pi} \, \frac{dl \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \, \frac{R \, d\theta}{R^2}
\]

\[
 B = \int dB = \frac{\mu_0 I}{4\pi} \, \int_0^{2\pi} \frac{d\theta}{R} = \frac{\mu_0 I}{4\pi R} \, 2\pi R = \frac{\mu_0 I}{2R}
\]
A cylindrical wire with radius $R$ carries a current $I$. The current is uniformly distributed over the cross-sectional area of the conductor. Using Ampere's law, find the magnetic field (both magnitude and direction) everywhere (for $r < R$ and $r > R$),

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc} \]

**For $r < R$**

$I_{enc} = \pi r^2$, \[ \frac{I}{\pi R^2} = \frac{I}{r^2} \]

\[ B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I r}{2\pi R^2} \]

**For $r > R$**

$I_{enc} = I$, \[ B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I}{2\pi r} \]
The wire shown in the figure is infinitely long and carries a current I. Starting from the Biot-Savart law, find the magnetic field (both magnitude and direction) at point P.

\[ (d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{l} \times \hat{r}}{r^2}) \]

For the straight part, \(d\vec{r} \times \hat{r} = 0\) since \(\sin(0) = \sin(\pi) = 0\).

For curvature:

\[ dB = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}}{r^2} \cdot \sin\left(\frac{\theta}{2}\right) = \frac{\mu_0 I}{4\pi} \frac{d\theta}{r^2} \]

\[ dB = \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{R \, d\theta}{R^2} = \frac{\mu_0 I}{4\pi R} \int_0^{\pi} d\theta = \frac{\mu_0 I}{4R} \]

Direction is into the page, from right hand rule.