Q. Use Biot-Savart law \((d\vec{B} = I\vec{dl} \times \hat{r}/r^2)\) to calculate the magnitude and the direction of the magnetic field produced at the origin by the current wire in the figure.

SOLUTION:

Due to symmetry and superposition principle we note that the half circle part of above shape will produce magnetic field of half of that of the complete circle. The two straight segments will not produce any magnetic field at the origin.

Expressing the magnetic field \(d\vec{B}\) due to each infinitesimal current element by Biot-Savart law as \(d\vec{B} = (\mu_0 I\vec{dl}/4\pi R^2)\hat{j}\), we find

\[
\vec{B} = \frac{\mu_0 I}{4\pi R^2} \hat{j} \int d\vec{l} = \frac{\mu_0 I}{4R} \hat{j}
\]
Q. Use Biot-Savart law \( d\vec{B} = I\vec{dl} \times \hat{r}/r^2 \) to calculate the magnitude of the y-component of the magnetic field produced at \( P \) by the current wire in the figure.

SOLUTION:

Due to symmetry and superposition principle we note that the half circle part of above shape will produce magnetic field of half of that of the complete circle. The two straight segments will not produce any net magnetic field, as their magnetic fields at point \( P \) are equal but in opposite direction, therefore they will cancel each other.

Use Biot-Savart law to express the y-component of magnetic field \( d\vec{B} \) at \( P \) due to an infinitesimal current element on the semicircle as: \( dB_y = \mu_0 I \vec{dl} \sin \theta / 8\pi l^2 \), where \( \sin \theta = 1/\sqrt{2} \). Integration gives

\[
B_y = \frac{\mu_0 I \sin \theta}{8\pi l^2} \int dl = \frac{\mu_0 I}{8\sqrt{2}l}
\]
Q. Calculate the magnitude and the direction of the magnetic field produced at $P$ by the current wire in the figure. (Hint: Use the result we obtained for the infinite current wire and some reasoning.)

\[ \vec{B} = \left( B_\infty / 2 \right) (-\hat{k}) = \left( \mu_0 I / 4\pi l \right) (-\hat{k}) \]

where the direction is determined by the right-hand rule.
Q. Use Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ to calculate the magnitude and the direction of the magnetic field produced at $P$ by the current wire in the figure.

SOLUTION:

For a straight, infinite current-carrying wire, we know from Ampere's law that $B_P = \frac{\mu_0 I}{2\pi r_P}$, where $\vec{r}_P$ is the vector pointing from the wire to $P$.

The direction of the magnetic field for the wire on the $z$-axis is $-\hat{k} \times \vec{r}_P = -\hat{k} \times \hat{i} = -\hat{j}$.

The direction of the magnetic field for the other wire is $\hat{k} \times \vec{r}_P' = -\hat{k} \times (\hat{i} - \hat{j})/\sqrt{2} = (\hat{i} + \hat{j})/\sqrt{2}$.

Adding up the fields generated by the two currents, we obtain

$$\vec{B}_P = \frac{\mu_0 I}{2\pi} \left[ \frac{-\hat{j}}{a} + \frac{\hat{i} + \hat{j}}{2a} \right] = \frac{\mu_0 I}{4\pi a} (\hat{i} - \hat{j})$$
Q. Use Ampere’s Law (\( \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \)) to calculate the magnitude and the direction of the magnetic field produced at point \( P \) by the currents in the figure.

SOLUTION:

For a straight, infinite current-carrying wire, we know from Ampere’s law that \( B_P = \frac{\mu_0 I}{2\pi r_P} \), where \( \vec{r}_P \) is the vector pointing from the wire to \( P \).

The direction of the magnetic field for the wire parallel to the x-axis is \( \hat{i} \times \hat{r}_P = \hat{i} \times \hat{j} = \hat{k} \).

The direction of the magnetic field for the wire parallel to the z-axis is \( \hat{k} \times \hat{r}_P = -\hat{k} \times (\hat{i} - \hat{j})/\sqrt{2} = (\hat{i} + \hat{j})/\sqrt{2} \).

Adding up the fields generated by the two currents, we obtain

\[
\vec{B}_P = \frac{\mu_0 I}{2\pi a} \left[ \frac{i}{2} + \frac{j}{2} + \hat{k} \right]
\]