An idealized ammeter is connected to a battery as shown in the figure. Find (a) the reading of the ammeter, and (b) the current through the 4\,\Omega resistor.

(a) and (b)

Since the ammeter is ideal it has no resistance, so the current will only circulate at the upper circuit. Therefore the current at \( R=4 \,\Omega \) would be zero.

\[
I_{\text{ammeter}} = \frac{V}{R_{\text{total}}} = \frac{10}{2+0} = \frac{10}{2} = 5 \,\text{A}
\]
When switch S is open, the voltmeter of the battery reads "v". When the switch is closed, the voltmeter reading drops to "y", and the ammeter reads "z". Find the emf, and the circuit resistance R in terms of x, y and z.

\[ \mathcal{E} = ? \]
\[ R = ? \]

\[ S \text{ is open } \Rightarrow \quad V = \mathcal{E} - Ir \]

since the circuit is open we have \( I = 0 \)

\[ \Rightarrow \quad V = \mathcal{E} \]
\[ \Rightarrow \quad (\mathcal{E} = x) \]

\[ S \text{ is closed } : \quad V = \mathcal{E} - Ir \quad \text{and} \quad \begin{cases} V = y \\ I = z \end{cases} \]

\[ \Rightarrow \quad (y = \mathcal{E} - z \mathcal{R}) \quad (1) \]

Also:

\[ R = \frac{V}{I} = \frac{y}{z} \quad \Rightarrow \quad (R = \frac{y}{z}) \]

And:

\[ I = \frac{\mathcal{E}}{R + r} \quad \Rightarrow \quad z = \frac{x}{\frac{y}{z} + r} \quad \Rightarrow \quad x = \frac{z}{x + r} (\frac{y}{z} + r) \]

\[ \Rightarrow \quad x = y + z r \quad \Rightarrow \quad (r = \frac{x - y}{z}) \quad (2) \]

(1) & (2) \Rightarrow \quad \mathcal{E} = y + z r = y + z (\frac{x - y}{z}) = y + x - y = x \quad \Rightarrow \quad (\mathcal{E} = \mathcal{X})
According to the figure the current will flow radially inward.

Take the resistance element $dR$ as an spherical shell of thickness $dr$ and surface $A$.

\[
dR = \rho \frac{\ell}{A}
\]

\[
\begin{align*}
\rho &= \frac{c}{4\pi} \\
\ell &= dr \\
A &= 4\pi r^2
\end{align*}
\]

\[
\Rightarrow \quad dR = \rho \frac{\ell}{A} = c \frac{dr}{4\pi r^2}
\]

\[
\Rightarrow \quad R = \int_{a}^{b} dR = \frac{c}{4\pi} \int_{a}^{b} \frac{dr}{r^2} = \frac{-c}{4\pi} \left[ \frac{1}{r} \right]_{a}^{b}
\]

\[
\Rightarrow \quad R = \frac{-c}{4\pi} \left( \frac{1}{b} - \frac{1}{a} \right) = \frac{c}{4\pi} \left( \frac{1}{a} - \frac{1}{b} \right)
\]

\[
\Rightarrow \quad R = \frac{c}{4\pi} \frac{b-a}{ab}
\]

\[
\Rightarrow \quad R = \frac{c(b-a)}{4\pi ab}
\]
What is the potential difference $V(a) - V(d)$?

First we need to calculate the current. Specify an arbitrary direction for current such as the one shown above.

Start from any point say "a" and circulate in the direction of current:

$V(a) - 6I - 0.5I - 4 - 9I + 8 - 0.5I - 8I = V(a)$

$\Rightarrow -24I + 4 = 0$

$\Rightarrow I = \frac{4}{24} = \frac{1}{6} \ A > 0$  \quad \Rightarrow \quad \text{The direction we have chosen is correct.}$

$V(a) + 8I + 0.5I - 8 = V(d)$

$I = \frac{1}{6} \ A \quad \Rightarrow \quad V(a) - V(d) = 8 - 8.5 \times \frac{1}{6} = 6.5 \ V$

$\Rightarrow (V(a) - V(d)) = 6.5 \ V$
What is the potential difference $V(b) - V(d)$?

First calculate current. Choose an arbitrary direction for $I$. Start from a certain point say "a" and circulate along the direction of current.

\[ V(a) - 6I - 0.5I - 4 - 9I + 8 - 0.5I - 8I = V(a) \]

\[ -24I + 4 = 0 \quad \Rightarrow \quad I = \frac{1}{6} \quad A \quad \Rightarrow \quad \text{The direction of } I \text{ is correct.} \]

\[ V(b) - 0.5I - 4 - 9I = V(d) \]

\[ I = \frac{1}{6} \]

\[ \Rightarrow \quad V(b) - V(d) = 9.5I + 4 = 9.5 \times \frac{1}{6} + 4 \]

\[ \Rightarrow \quad V(b) - V(d) = 5.58 \text{ V} \]