Q. Calculate the magnetic force on the triangular loop whose corners are at \((0,0,0), (a,0,0), (0,a,0)\) if the loop carries a clockwise current \(I\) in a nonuniform magnetic field \(\vec{B}(x,y) = Axy \hat{k}\).

\[\text{SOLUTION:}\]

Force on the edges along the axes is zero since the magnetic field is zero there. Along the diagonal, \(d\vec{l} = dx\hat{i} - \hat{j}\) and \(y = a - x\), so that

\[
F = \int_0^a I d\vec{l} \times \vec{B} = I (-\hat{i} - \hat{j}) \int_0^a A(x(a-x))dx = IA^3 \frac{a^3}{6} (-\hat{i} - \hat{j}).
\]
Q. A straight current-carrying wire of length $4a$ is placed on the border between two regions with uniform and opposing magnetic fields, as shown. Calculate the magnitude and the direction of the net force on the wire and the net torque with respect to the wire’s midpoint.

SOLUTION:

The net force on the right half of the wire is zero by symmetry. Therefore $F_{\text{net}} = 2aiB$ in the upward direction in the plane of the paper. For the torque, consider the system as a superposition of two: (the full wire in a uniform field $B \otimes$) + (left quarter of the wire in a field $2B \odot$). First carries zero torque. Second yields a torque generated by a perpendicular force $ai(2B)$ acting at a distance $3a/2 \Rightarrow \tau = 3a^2iB$. 
Q. Find the magnitude of the magnetic force acting on a 60° arc of a circular wire with radius $R$, if the wire carries a current $I$ in a perpendicular, uniform magnetic field $\vec{B}$. (Hint: Use the fact that the total magnetic force on any current loop in a uniform field is zero.)

SOLUTION:

If the loop were closed by a straight wire from the right end to the left, the net force on the loop would be zero. Then, the force on the arc is the same as the force on a straight wire which connects the two ends and carries the same current. The length of this straight wire would be $R$, yielding a downward force with magnitude $F = iRB$. 

$F_{\text{net}} = 0$. 
Q. A straight wire of length $a$ with end points at $(0, 0, 0)$ and at $(a, 0, 0)$ carries a current $I$ in a nonuniform magnetic field $\vec{B}(x) = Ax^2 \hat{j}$ ($A$ is a constant with units $T/m^2$). Find the magnitude of the total force acting on the wire.

SOLUTION:

The total force is the sum of forces acting on infinitesimal current elements:

$$ F = \int_0^a IB(x)dx = IA \int_0^a x^2 dx = IAa^3/3. $$
Q. Q. A straight wire of length $a$ with end points at $(0, 0, 0)$ and at $(a, 0, 0)$ carries a current $I$ in a nonuniform magnetic field $\vec{B}(x) = Ax \hat{j}$ ($A$ is a constant with units T/m). Find the magnitude of the total torque acting on the wire, with respect to the origin.

SOLUTION:

The total torque is the sum of torques due to each infinitesimal current element. Since the magnetic force is perpendicular to the wire at all points,

$$\tau = \int_0^a xIB(x)dx = IA \int_0^a x^2 dx = IAa^3/3.$$